Risk-Limiting Audits – Practical Application

Jerome Lovato – U.S. Election Assistance Commission
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Introduction

In 2007, Dr. Philip B. Stark (Associate Dean, Division of Mathematical and Physical Sciences at the University of California – Berkeley) introduced the concept of risk-limiting audits (RLAs). The term "risk-limiting audit" was formalized in his 2009 paper titled "Risk-Limiting Postelection Audits: Conservative *P*-Values From Common Probability Inequalities." In 2010, the U. S. Election Assistance Commission (EAC) made grant awards totaling $1,463,074 to county and state organizations to support research, development, documentation and dissemination of a range of procedures and processes for managing and conducting high-quality logic and accuracy testing and post-election audit activities. California, Colorado, and Ohio used the awarded grant money to conduct research on RLAs, and from 2008 until present, RLA pilots have been conducted in jurisdictions in California, Colorado, Indiana, Ohio, and Virginia.

There have been many discussions, papers published, and presentations given about risk-limiting audits in the elections community over the past decade. The intent of this paper is to provide a foundation about how RLAs work and things to consider before conducting pilots or mandating RLAs via legislation. This paper provides detailed information about the practical application of RLAs, details of the RLA conducted in Colorado for the 2017 Coordinated Election, lessons learned from conducting pilot and live RLAs, and factors to study for jurisdictions as they consider implementing RLAs. This paper also provides a high-level overview of post-election audits conducted throughout the United States for those who are getting acquainted with post-election audits.

Post-Election Audit Overview

Simply, a post-election audit verifies that the voting equipment used to count ballots during an election properly counts a sample of voted ballots after an election. Exact definitions of post-elections and all that they encompass (methodologies, voting equipment, processes, and procedures) vary from state to state. Some post-election audits are designed to change the outcome of an election if enough discrepancies are discovered. However, most post-election audits that are currently conducted do not have a mechanism that alters the outcome of an election. Risk-limiting audits are post-election audits with one distinct difference between traditional post-election audits: the purpose of the risk-limiting audit is to provide strong statistical evidence that the winner of a contest is the winner and that the loser is the loser, and has high probability of correcting the outcome of an election if it is wrong.

Currently, post-election audits are required in 35 states and the District of Columbia. One state has a conditional post-election audit and 14 states do not require a post-election audit. Although some states have similar methods for conducting post-election audits, the requirements for how post-election audits are conducted are unique to each state. See Appendix A for a more comprehensive view of how states conduct post-election audits.

Risk-Limiting Audit Overview

The intent of this overview is to highlight what a risk-limiting audit is, pros and cons of conducting an RLA, and methods used and proposed. More in-depth technical information about RLAs mentioned in this paper can be found in the References section.

What is a risk-limiting audit?

A risk-limiting audit is a post-election audit that provides strong statistical evidence that the election outcome is correct, and has a high probability of correcting a wrong outcome. An RLA manually checks a sample of ballots, or voter-verifiable paper records, until there is sufficient evidence that the reported outcome is correct. An RLA could eventually lead to a full manual recount if there is not enough evidence to prove that the reported outcome is correct.

Definitions

Ballot manifest: A list that indicates how the ballots in an election are organized and stored. For instance, a ballot manifest might list the ballot containers used for an election, the number of batches in each container, and the number of ballots in each batch.

Coordinated Election (Colorado-specific definition): To be coordinated, the election must:
- Have more than one political subdivision holding an election (state, county, municipality, school district, or special district).
- Take place on the same day in November, and
- Have eligible voters that are either the same for each election or live in overlapping subdivision boundaries.

When these requirements are satisfied, the county clerk and recorder will coordinate and conduct the elections on behalf of all political subdivisions. The November odd-year election is generally referred to as the coordinated election. Coordinated elections are conducted by mail ballot.

Cast Vote Record (CVR): Archival record of all votes produced by a single voter. Cast vote records may be in electronic, paper, or other form.

Diluted margin: The smallest reported margin (in votes), divided by the number of ballots cast. Dividing by the number of ballots, rather than by the number of valid votes, allows for the possibility that the vote tabulation system mistook an undervote or overvote for a valid vote, or vice versa.

Risk limit: The largest statistical probability that, if an outcome is wrong, the RLA does not correct that outcome. For example, assume the reported outcome of an election contest is wrong, and the risk limit for the audit is 5%. In this instance, there is at most a 5% chance that the audit will not correct the wrong outcome, and at least a 95% chance that the audit will correct the wrong outcome. The risk limit is a number between 0 and 1 that limits the risk of certifying an incorrect outcome, and is chosen by the RLA administrative authority before the audit is conducted.

RLA Tool: An online software utility used by the Colorado Secretary of State (SOS) and local election officials to conduct the risk-limiting audit. The county-facing side allows the local election official to upload the ballot manifest and CVR, and allows the county audit board to input ballot content information and submit the results of the audit. The state-facing side allows the SOS to input the random seed, randomly select ballots, examine the audit results, export audit data, and provide feedback to the local election officials.

Methods

As of the date of this paper, all four RLA methods have been piloted and two have been used to audit live election results. Colorado implemented comparison and ballot-polling RLAs for the November 2017

Coordinated Election. Rhode Island and Virginia will be conducting RLAs for the 2018 General Election, and Washington will draft rules and procedures to conduct RLAs. Other local jurisdictions continue to pilot and implement RLAs. Table 2 provides a brief description for each method.

Table 1 - Risk-Limiting Audit Methods

RLA Method	Description
Ballot-level comparison	Individual ballots are randomly selected and compared to the voting system's cast vote record (CVR) for each ballot.
Batch-level comparison	Batches of ballots are randomly selected and compared to batch subtotals produced by the voting system.
Ballot-polling	A random sample of ballots are selected and the results for the selected contest(s) are tallied; the audit stops if it produces strong enough evidence to support the reported outcome.
Batch-polling	A random sample of batches are selected and the results for the selected contest(s) are tallied; the audit stops if it produces strong enough evidence to support the reported outcome.

Pros
- Adaptable: Can be adapted to any voting system or contest as long as a paper trail exists whether it be hand-marked paper ballots, voter-verifiable paper records, or voter-verifiable paper audit trails
- Efficient: The comparison RLA provides efficiency by allowing election officials to compare a ballot to the voting system's CVR and generally allows jurisdictions to audit fewer ballots compared to other audit methods. A ballot-polling RLA is also efficient for contests with a margin of 10% or greater.
- Time-Saving: A jurisdiction can complete an audit in two hours or less for contests with relatively large margins (30% or greater) versus four hours or more for most standard post-election audits. Most counties in Colorado experienced a time savings after conducting RLAs for the 2017 Coordinated Election compared to their previous random machine audit.
- Accurate: Provides assurance that the election outcome is correct (the declared winner is truly the winner and the declared loser is truly the loser)
- Independent: Ballot-polling and batch-polling RLAs can be conducted without knowing how the voting system interpreted a particular ballot or batch.

Cons
- Limited: Cannot be used by jurisdictions that use currently deployed DREs without a voter-verifiable paper record, and does not audit every part of the election process, such as voter registration and ballot programming.
- Unpredictable: A very close contest may need additional resources that were not accounted for in a jurisdiction's budget.

- Time-Consuming: Can lead to a 100% manual recount if a contest has a very slim margin (approximately 0.5% for comparison; less than 10% for ballot-polling) and/or enough discrepancies are found during the audit that continues to increase the number of ballots to audit.
- Dependent: Comparison RLAs are dependent upon maintaining ballot order (or marking ballots with a unique identifier) and CVRs from the voting system.

Workload

The number of ballots required to conduct an RLA will vary based on the margin of a selected contest and the risk limit. The total number of ballots cast for a contest will not change the number of ballots to audit, but it may help determine if a full hand recount is more practical. Table 3 breaks down different scenarios using a comparison RLA. The RLA tools of Dr. Philip B. Stark can be used to simulate real-life workload scenarios:

- [Comparison RLA tool](#)

- [Ballot-polling RLA tool](#)

Table 2 - Comparison RLA Scenarios

Ballots Cast	Margin	Risk Limit	Initial Sample Size (Ballots)
1,000,000	1%	1%	1,067
1,000,000	10%	1%	102
100,000	1%	1%	1,067
100,000	10%	5%	68
10,000	1%	1%	1,067
10,000	1%	10%	534
1,000	1%	1%	1,067 ballots*
1,000	20%	1%	51

In this case, a full hand recount would be required since the initial sample size is greater than the total number of ballots cast.

The following figures provide a graphical representation of workload examples. Figures 1 and 2 illustrate the relationship between margin and risk limit for comparison RLAs. Figures 3 and 4 illustrate the relationship between margin and risk limit for ballot-polling RLAs. Figure 5 illustrates the difference in workload between the two methods with a fixed risk limit, and Figure 6 illustrates the difference in workload between the two methods with a fixed margin. The margin dramatically changes the workload of an RLA as it should – more ballots should be examined for closer contests.
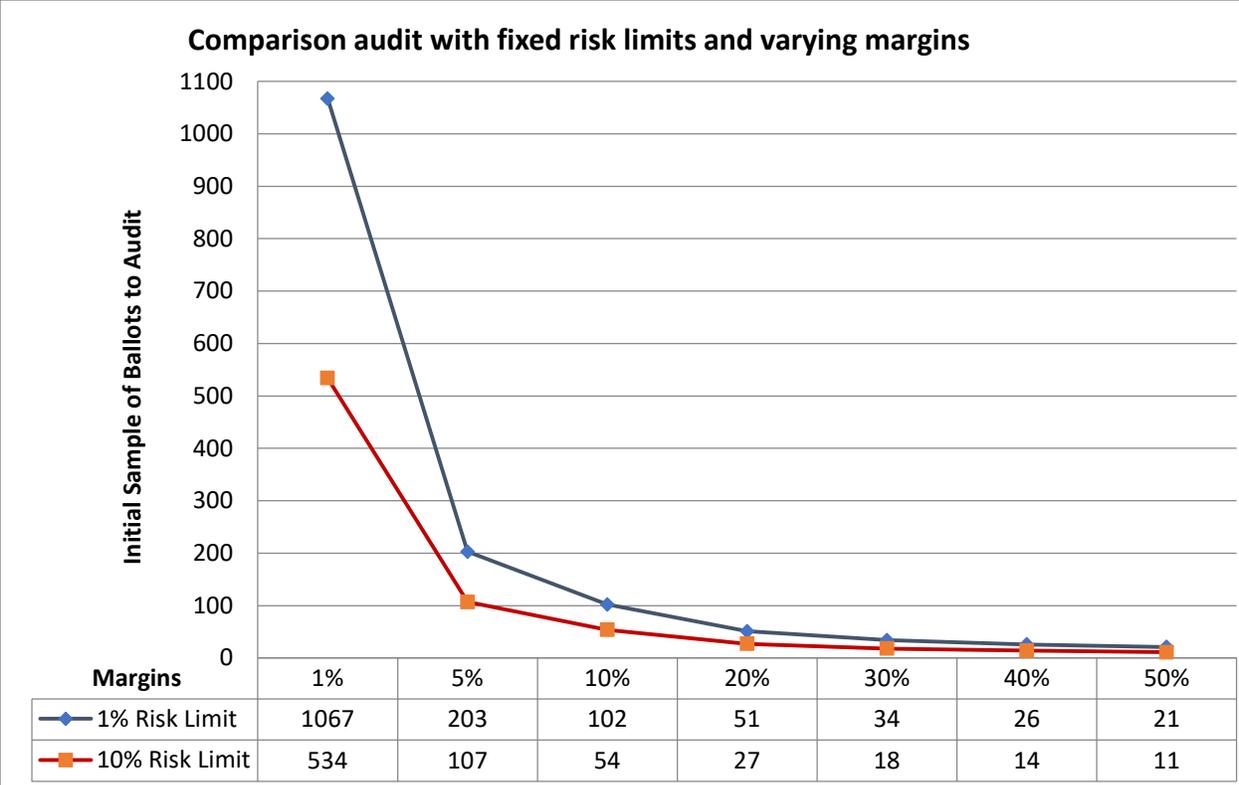
Comparison audit with fixed risk limits and varying margins

Margins	1%	5%	10%	20%	30%	40%	50%
1% Risk Limit	1067	203	102	51	34	26	21
10% Risk Limit	534	107	54	27	18	14	11

Figure 1 - Comparison audit with fixed risk limits and varying margins

Comparison audit with fixed margins and varying risk limits

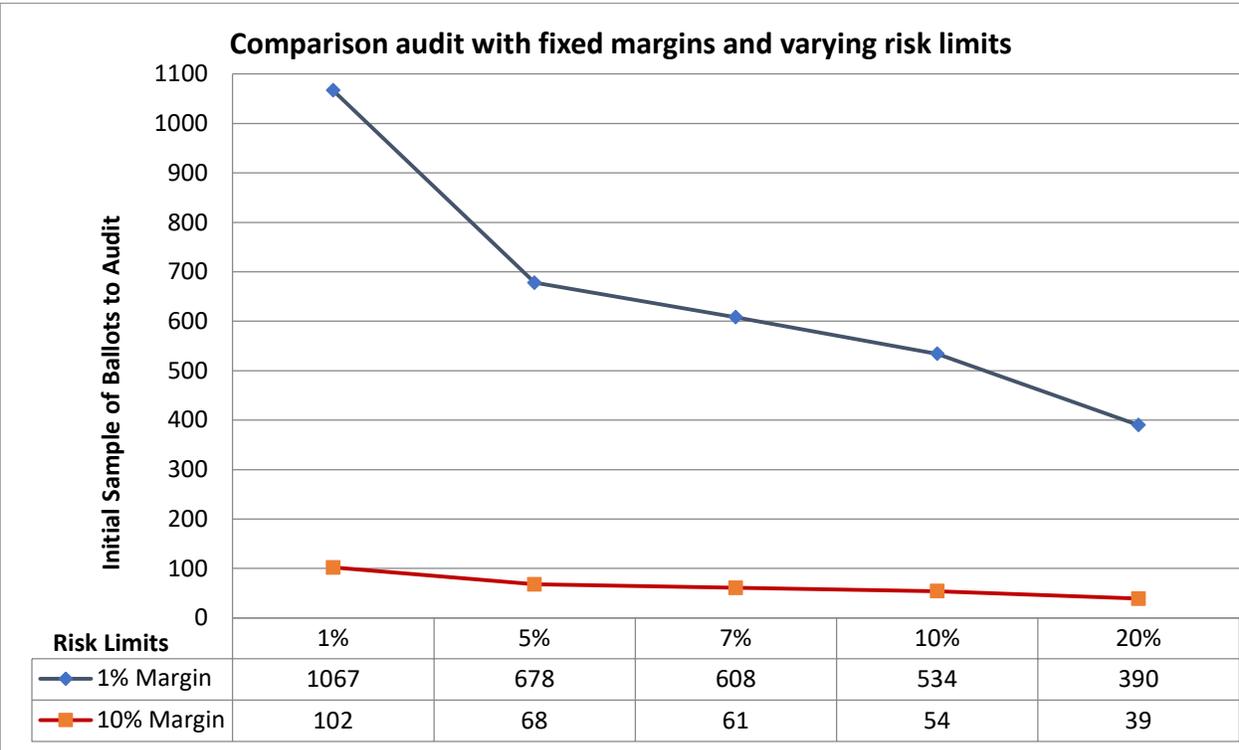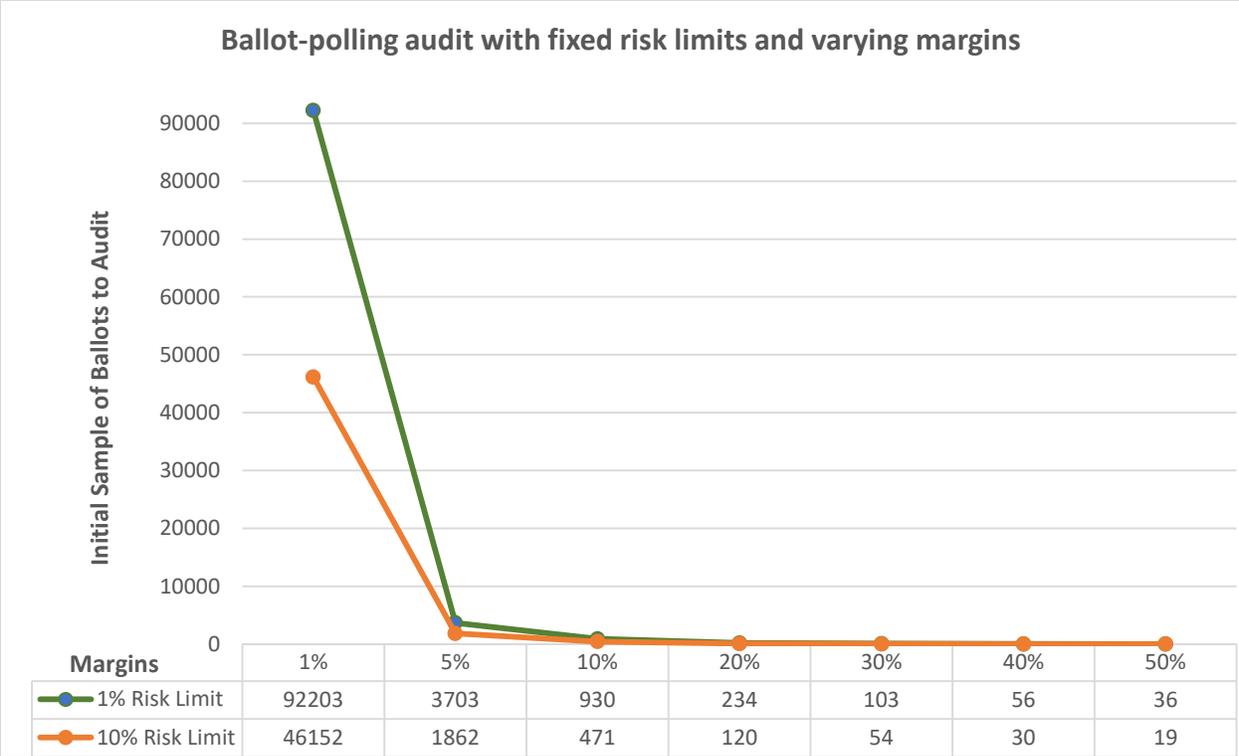| Risk Limits	1%	5%	7%	10%	20%
1% Margin	1067	678	608	534	390
10% Margin	102	68	61	54	39

Figure 2 - Comparison audit with fixed margins and varying risk limits

Figure 3 - Ballot-polling audit with fixed risk limits and varying margins

Ballot-polling audit with fixed risk limits and varying margins

Margins	1%	5%	10%	20%	30%	40%	50%
1% Risk Limit	92203	3703	930	234	103	56	36
10% Risk Limit	46152	1862	471	120	54	30	19
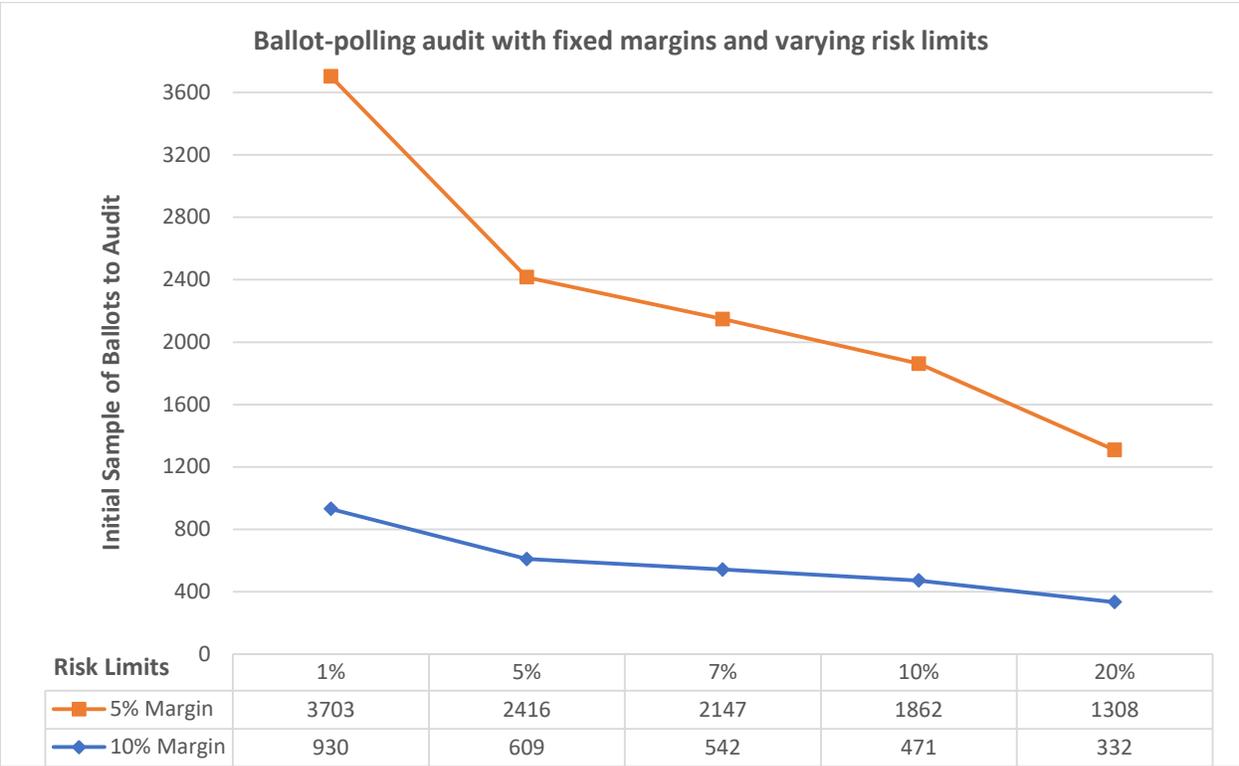
Figure 4 - Ballot-polling audit with fixed margins and varying risk limits

Ballot-polling audit with fixed margins and varying risk limits

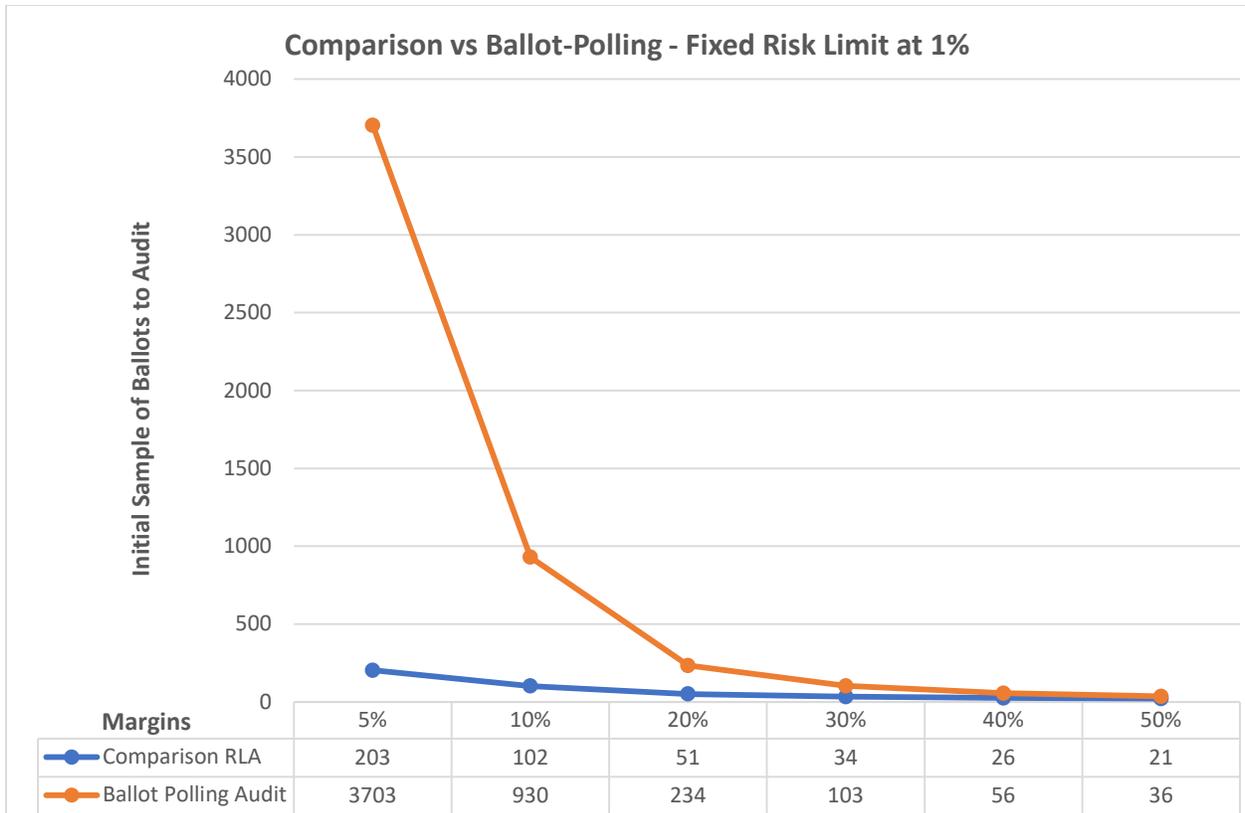| Risk Limits	1%	5%	7%	10%	20%
5% Margin	3703	2416	2147	1862	1308
10% Margin	930	609	542	471	332

Comparison vs Ballot-Polling - Fixed Risk Limit at 1%

Margins	5%	10%	20%	30%	40%	50%
Comparison RLA	203	102	51	34	26	21
Ballot Polling Audit	3703	930	234	103	56	36

Figure 5 - Comparison vs Ballot-Polling - Fixed Risk Limit at 1%

Comparison vs Ballot-Polling - Fixed Margin at 10%

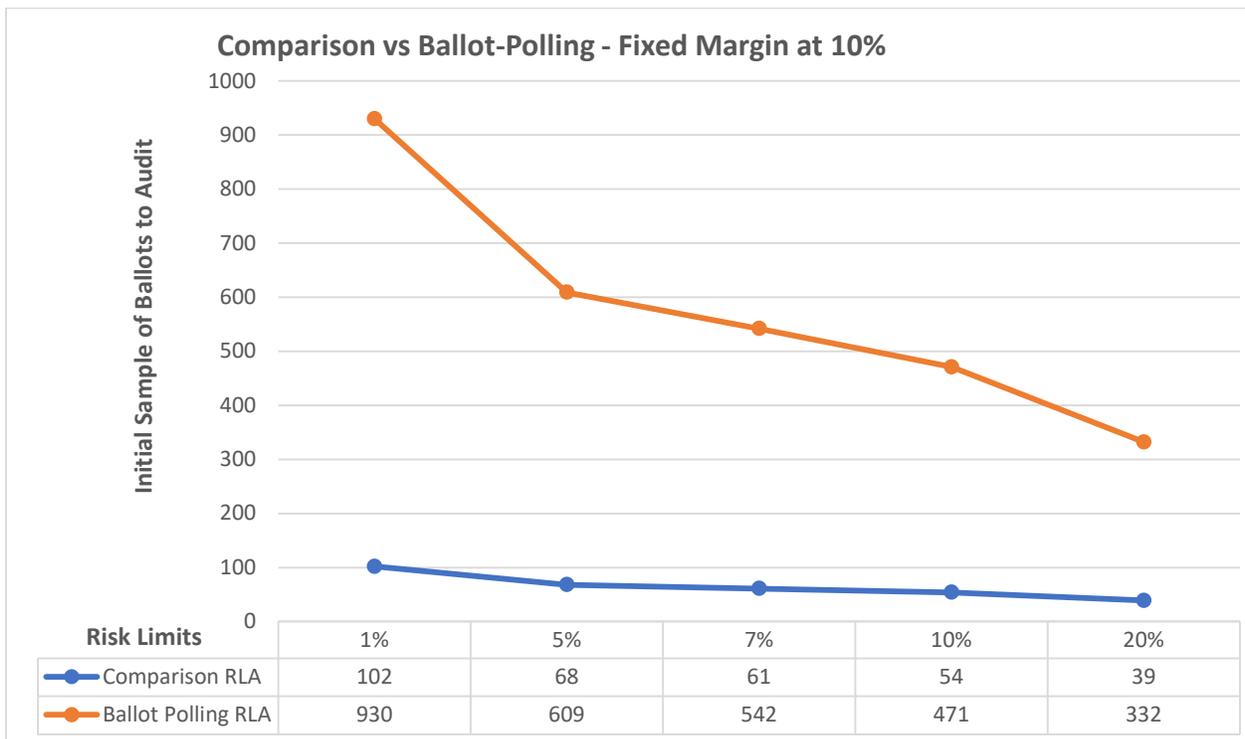| Risk Limits	1%	5%	7%	10%	20%
Comparison RLA	102	68	61	54	39
Ballot Polling RLA	930	609	542	471	332

Figure 6 - Comparison vs Ballot-Polling - Fixed Margin at 10%

Practical Application

Background

Prior to implementing risk-limiting audits, Colorado used a random machine audit for every contest in a county's election. The Secretary of State randomly selected at least 5% of the central count ballot scanners, at least one ballot scanner used at a polling location, and 5% of DREs from the voting devices used in the election. For instance, if a county used DREs and Central Count Scanners then at least one DRE and one Central Count Scanner would be selected to conduct the audit. The Secretary of State then issued a form to each county with a list of devices to audit, including the contests to audit on each device (see Appendix C for a sample of this form). The ballots were hand tallied and that tally was then compared to the machine report. See Appendix D for a history of RLAs in Colorado.

November 2017 Election

Colorado conducted the first risk-limiting audit for a live election for the 2017 Coordinated Election. Fifty-six of the 64 counties in Colorado conducted a risk-limiting audit. Fifty counties conducted a comparison RLA (47 completed the audit in the first round and 3 completed the audit in the second round). Six counties conducted a ballot-polling RLA (3 completed the audit in the first round and 3 completed the audit in second round). Six counties did not have elections and 2 counties hand-counted ballots.

The Colorado RLA process consisted of the following steps:

1. Thirty-two days before Election Day, the Secretary of State established separate risk limits for the comparison audit (9%) and the ballot-polling audit (20%).

2. Fifteen days before the election, designated election officials appointed a bipartisan audit board to conduct the RLA. Prior to Election Day, the Secretary of State provided the counties with a ballot manifest form.

3. During ballot scanning, designated election officials were required to maintain the order in which ballots were scanned and fill out the ballot manifest form.

4. Three days after the election, the Secretary of State selected one contest for each county that was solely in the county. Colorado did not have a statewide contest otherwise a statewide contest plus a county contest would have been chosen.

5. Nine days after the election, counties that conducted a comparison audit uploaded their ballot manifests and CVR export to the state's RLA software; counties that conducted a ballot-polling audit submitted their ballot manifests and cumulative tabulation report (a total of votes tabulated) to the Secretary of State via email.

6. Ten days after the election, a random seed was drawn in a public meeting. This was performed by having the public roll a ten-sided die and having that number recorded in a public ledger. This was performed 20 times to generate a random seed. This random seed is used as an input into a pseudo-random number generator, which is used to randomly select ballots to be audited. The Secretary of

State notified each county of the ballots to audit based off of the ballot manifests and the pseudo-random number generator.

7. For counties that conducted a comparison audit, the audit board examined each ballot and replicated the vote selections for each ballot on the RLA software. For counties that conducted a ballot-polling audit, the audit board examined each ballot and reported the vote selections (for only the selected contest) in a state-provided tally spreadsheet.

8. After the audit board replicated all of the selected ballots on the RLA software, the designated election official submitted the information to the Colorado SOS via the RLA software. The RLA software compared the submitted data to the CVRs submitted by the counties. If there were no discrepancies, then the audit was concluded. If there were discrepancies, then the RLA software calculated a number of additional ballots to be audited based off of Dr. Philip Stark's comparison audit algorithm, and a new list of ballots were sent to the counties.

9. Step 7 was repeated for counties that had to audit additional ballots.

Random Machine Audit vs Risk-Limiting Audit Comparison

The risk-limiting audits in Colorado have proven to be more efficient than the random machine audits that were conducted prior to the 2017 Coordinated Election. Counties were able to complete the audit in a fraction of the time it took in previous years using fewer resources. Table 3 provides a comparison between the two audit methods as conducted in Colorado.

Table 3 - Random Machine Audit vs Risk-Limiting Audit

Random Machine Audit	Risk-Limiting Audit
Starts 3 days after the election	Starts 10 days after the election
Every contest is audited	At least one statewide contest and at least one countywide contest
Voting devices are randomly selected	Ballots are randomly selected
Ballot order is irrelevant	Comparison: Must maintain ballot order Ballot-Polling: For DREs, all of the ballots on the Voter Verifiable Paper Audit Trail (VVPAT) are tallied
The lesser of these two options: 20% of ballots on each scanner or 500 ballots 100% of ballots cast on DREs 2016 Presidential Contest – ~32,000 ballots were audited statewide	Varies on the risk limit and the margin of the selected contests 2016 Presidential Contest – 117 ballots statewide using 9% risk limit and comparison audit; 2,139 statewide using 9% risk limit and ballot-polling audit
Canvass earlier than the state deadline	Canvass after all counties have completed their audit
Not dependent on other counties to submit ballot manifest and CVR data	All counties must submit ballot manifests and CVRs or cumulative reports in order for the audit to start

Not dependent on software to upload data and perform calculations	Dependent on external software to upload election data and perform audit calculations
County election officials filled out state-issued audit form	County election officials used a software tool to complete their audit

Figure 7 illustrates the time that can be saved by conducting a risk-limiting audit using the same resources. The blue bar shows how long it took to audit the 2015 Coordinated Election using the random machine audit; the orange bar shows how long it would have taken to audit the same contest using an RLA. Two things to note are 1) the number of ballots to audit is considerably less and 2) how long it would have taken to audit the same election with an RLA.
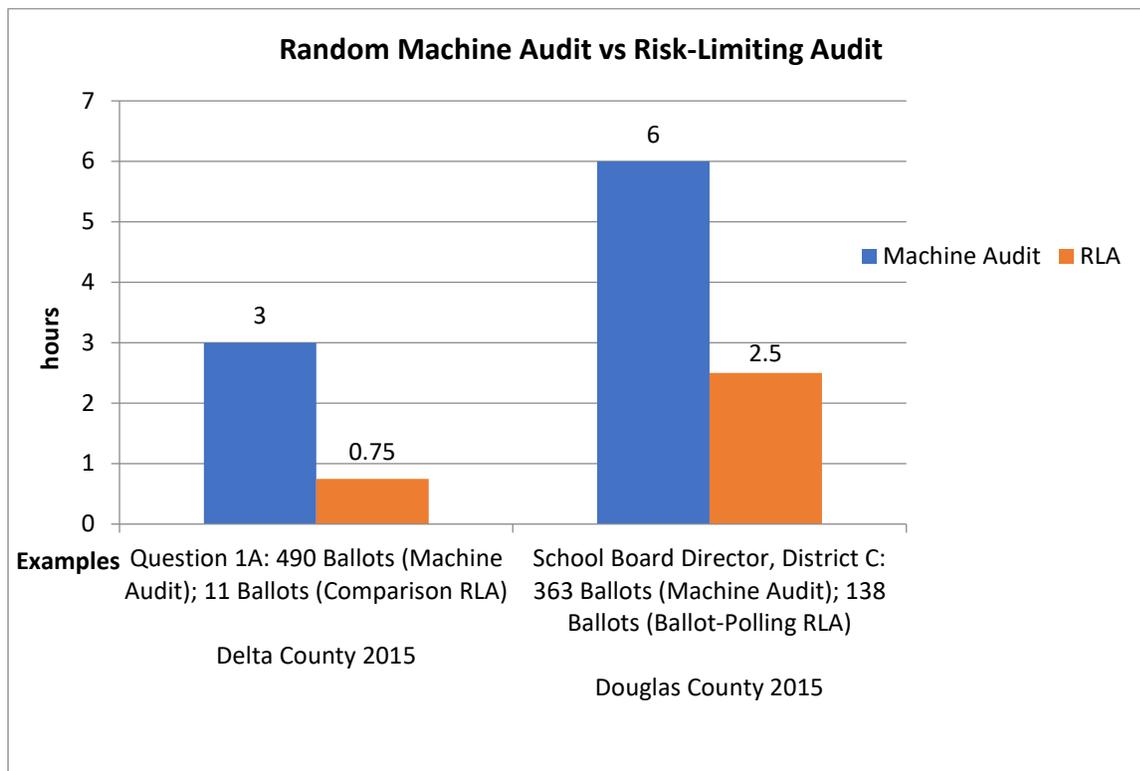

Random Machine Audit vs Risk-Limiting Audit

Examples:
Question 1A: 490 Ballots (Machine Audit); 11 Ballots (Comparison RLA)

Delta County 2015

School Board Director, District C: 363 Ballots (Machine Audit); 138 Ballots (Ballot-Polling RLA)

Douglas County 2015

Figure 7 - Random Machine Audit vs Risk-Limiting Audit

Lessons Learned

A. Challenges

Introducing a new post-election audit, along with a new software utility (RLA Tool), presented some challenges for state and local election officials in Colorado. Some issues that came up and solutions used to remedy them included:

- Communication
  - Ensure that there is clear communication to all stakeholders (local election officials, public, state election officials, advocacy groups, commissioners, legislators, candidates).Example: The Colorado SOS formed and facilitated a public participation

group that allowed stakeholders to provide input, and assistance, with drafting rules and procedures to conduct an RLA.

- o Train election officials on how to communicate RLA information (what it is, how it will be conducted, the time and location) to their constituency.
  Example: The Colorado SOS Voting Systems team conducted regional trainings for election officials and created a web page on the Colorado SOS website dedicated to RLA information.
- o Effectively communicating what the audit data represents.
  Example: Provide a glossary of terms and descriptive text for graphs and spreadsheets that are used.

- Procedures
  - o Maintaining ballot order of scanned ballots is essential for comparison audits. Most of the counties used either automatic or manual imprinting. The counties that used imprinters had to fine tune their process to account for ballot misfeeds, which led to some ballots having multiple numbers imprinted on them. Some counties experienced problems with maintaining ballot order and worked with the Colorado SOS to resolve these problems.

- Schedule
  - o Provide sufficient time for local and state officials to train, conduct end-to-end mock RLAs, and test any software/utilities that will be used for a live RLA.
    Example: The Colorado SOS conducted trainings in the spring and summer while the RLA rules were being drafted. The rules were published in August 2017, which gave local election and state official two months to conduct mock RLAs, in addition to their other election-related duties.
  - o Provide the software vendor and state IT staff time to test the software and network security.
    Example: The RLA Tool is an online utility that required development and end-to-end testing in a three-month time frame, which required additional time and resources from the Colorado SOS IT and elections staff.

B. Successes
  - The Colorado SOS as the central audit authority was vital to the success of the Colorado RLA.
  - The Colorado SOS and Colorado County Clerks Association collaborated to produce documentation to assist counties.
  - Open communication among the county clerks resulted in best practices being developed.
  - Colorado SOS staff conducted regional trainings to get county clerks familiar with the RLA concept and terminology, and conducted trainings on how to use the RLA software.
  - Allowing audit board members to replicate the ballot content on the RLA software simplified the audit for the comparison audit.
  - Using the state-provided ballot selection and tally spreadsheets assisted counties who conducted the ballot-polling audit. See Appendix E.
  - Each participant had clearly defined roles and responsibilities.

- The Colorado RLA was completely transparent (the public was invited to view and participate in generating the random seed at the SOS office and the RLAs at the county level can be observed by the public) and audit data with its associated hash value is available to the public on the SOS' website.

C. Unknowns
- Risk-limiting audits have not been conducted with a statewide contest or a cross-jurisdictional contest.
- The length of time an audit will continue before it reaches the full manual recount threshold.
- What escalation looks like if there are discrepancies across the state.

Factors to consider for implementing an RLA

A. Law
The laws of the jurisdiction set the scope of the RLA. Laws should be flexible to allow local election officials to develop processes and procedures and gives them time to get comfortable with a new post-election audit.

B. Administration
Determine who will oversee the audit and how the results will be reported. Post-election audit administration varies from state to state, and RLAs require more oversight than most other post-election audits due to possible escalation if discrepancies are discovered while conducting the audit.

C. Voting System
The type of voting systems used in jurisdictions varies from state to state. Some states use a single-vendor voting system and most states allow their local jurisdictions to use a state-approved voting system. RLAs are adaptable and can be used with any type of voting system as long as a paper trail exists.

D. Ballot Marking Errors
Some potential causes of ballot errors are: misinterpreting voter intent, ballot duplication errors such as duplicating a ballot using a pencil, and uncalibrated voting equipment. In addition to training, these are all errors that can be mitigated by having a state-specific voter intent guide, only allowing black or blue pens to be used in the ballot duplication area, and having voting equipment maintenance at least once per year.

E. Software and Other Utilities
Additional software and utilities may need to be developed or purchased to assist with conducting an RLA. How much will it cost to obtain the license or to be developed? What are the specifications? Who will write the specifications? Who will conduct testing? How long will testing take? Will it be used on the internet? Who will host it?

F. Consultants/Contractors

Consultants or contractors may need to be employed to assist with developing and implementing an RLA. How much will they cost? What are their deliverables? What duties will they perform? How long will they be employed?

G. Testing and Certification
While considering the voting system(s) that will be used to conduct an RLA, determine if voting system requirements will need to be drafted or revised to meet RLA requirements. If voting system requirements are changed, the next step will be to have the voting system(s) tested and certified to the requirements.

H. Project Management Planning
Assign a project manager and give him/her the necessary authority to lead in developing and executing the project management plan.

I. Risks
Identify risks and mitigations throughout planning and conducting the RLA. Some risks and mitigations to consider are:

- Ballots
  Risk: Losing or misplacing ballots, or not maintaining chain-of-custody of ballots poses the greatest risk to properly conducting an RLA.
  Mitigation: Establish procedures for maintaining chain-of-custody. For mail ballots, consider using a mail ballot tracking system.

- Personnel
  Risk: Staffing shortage (election judge no-show, sickness, etc.)
  Mitigation: Even though RLAs can be more efficient and less resource heavy, ensure that there are sufficient personnel to conduct the audit. Data should be maintained and analyzed to understand where and when resources should be allocated to the process.

- Training
  Risk: Lack of training or not attending trainings can cause unforeseen issues with successfully completing the audit.
  Mitigation: Training on how to conduct an RLA and how to use RLA software should be conducted by state and local election officials. Provide in-person hands-on training and online training and resources.

- Full manual recount
  Risk: The audit may result in a full manual recount if an audited contest is very close or enough discrepancies are discovered that could potentially change the outcome of the election.
  Mitigation: Anticipate and plan for this when developing the budget and allocation of personnel, as it could lead to extending the audit timeframe and require additional resources.

- **Software**
  Risk: The software is compromised or fails during the RLA.
  Mitigation: Thoroughly test (load, beta, penetration) any software that will be used in conducting an RLA, and create a contingency plan if the software is unusable.

- **Internet**
  Risk: Any use of the internet comes with inherent security risks (hacking, denial of service, malware, etc.).
  Mitigation: Develop internet-independent software, or develop a contingency plan that allows users to conduct the RLA offline.

- **Weather**
  Risk: Weather (snow, wildfires, tornadoes, etc.) can pose a risk for power outages and could impede travel for personnel who are required to conduct the audit.
  Mitigation: Develop a contingency plan to address natural disasters and other emergencies.

J. Types of ballot records:
  - Voter marked paper ballot
  - Voter marked electronic ballot that produces a paper record of the vote selections
  - Voter marked electronic ballot with vote selections recorded on VVPAT and memory card
  - Voter marked electronic ballot with votes recorded on memory card
  - Uniformed and Overseas Citizens Absentee Voting Act (UOCAVA) ballots, which are duplicated at the local election official's office

K. Tabulation environment
  Vote tabulation occurs at a central count location (usually a local election official's office/facility), polling place, or vote center.

  Central Count Environment – All ballots are sent or transported to a central location to be tabulated. The central count environment is the optimal location for conducting RLAs since it allows local election officials to organize and store ballots in a uniform manner. In turn, that makes the ballot selection during an RLA go more smoothly.

  Precinct Count/Vote Center Environment – Ballots are tabulated on DREs or scanning devices.

L. Schedule
  Knowing the exact schedule (start time, time frame for each phase, deadline) of RLA activities helps with setting expectations for stakeholders and allows local election officials to properly plan for conducting an RLA.

M. Security
  Establishing and maintaining ballot chain-of-custody and ensuring that all files used for an RLA are secure is essential for conducting an RLA and for public confidence in the RLA process.

N. Beware of:
- Parallel scanning: Parallel scanning is re-scanning ballots using non EAC-certified scanner and software to obtain a CVR. This method is also called a transitive audit. Using this method might make sense for conducting an RLA pilot, but it is not a good long-term solution for conducting RLAs.
- RLA modules: Some voting system manufacturers claim to have an RLA module that can be used to conduct RLAs. Any type of vendor-supplied module defeats the purpose of conducting an independent audit.

O. Intangibles
Understanding cultural dynamics within jurisdictions is an essential element to implementing RLAs. State and local election officials should be on the same page when it comes to understanding terminology, planning pilots, and communicating to the public. A good relationship between state and local election officials will pay dividends in implementing a successful RLA.

Requirements

There is not a one-size-fits-all solution for conducting RLAs. However, there are basic requirements that are applicable to all RLA methods.

Paper Trail
A paper trail must exist. A key difference between an RLA and other audits is that RLAs always refer back to the original ballot cast, or to the batch of ballots in a batch-level comparison audit.

Chain-of-custody
Chain-of-custody for ballots is essential for assuring that the ballots are secure and have not been tampered with. Ballot containers (bags, boxes, tubs, etc.) should be properly identified, signed, and sealed by at least two election officials. A chain-of-custody log should accompany every container, and seal numbers should be confirmed by local election officials at the ballot storage facility. The ballot manifest may be derived from the chain-of-custody logs. Elements of the chain-of-custody log may include the precinct ID, number of ballots in each container, and the tabulator ID. The same principles apply to voting equipment chain-of-custody.

Ballot Traceability
For comparison audits, an auditor must be able to trace a ballot to its CVR. If legally allowed, using a voting system that can imprint ballots with a unique identifier is the optimal solution. Another solution is to Bates stamp, hand mark ballots in the order in which they were cast, or add a removable sticker label with a unique ballot identifier to each ballot. The least optimal method is to keep ballots in the same order in which they are scanned.

Path Forward

Future Implementations

Colorado will conduct RLAs with statewide and cross-jurisdictional contests for the 2018 Primary Election and 2018 General Election. The Commonwealth of Virginia must implement a risk-limiting audit by the 2018 General Election. Rhode Island must implement a risk-limiting audit by the 2020 General Election. Washington is drafting rules to conduct RLAs. Other states are considering RLAs.

Additional Pilots

Colorado conducted their 2017 Coordinated Election RLA in a hybrid environment. Most of the ballots were tabulated in a central count environment, but some counties used DREs at vote centers. Additional pilots will need to be conducted in other environments – central count, polling place, and vote center – and with other methods, such as the Bayesian audit that has been developed by Dr. Ron Rivest.

Acknowledgements

References

Although not specifically sited throughout this paper, the following references have served as my guide while conducting pilot RLAs and RLA trainings.

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- Colorado risk-limiting audit legislation
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  C.R.S. 1-7-515: https://www.sos.state.co.us/pubs/info_center/laws/Title1/Title1Article7.html

- Colorado risk-limiting audit webpage
  https://www.sos.state.co.us/pubs/elections/RLA/2017RLABackground.html

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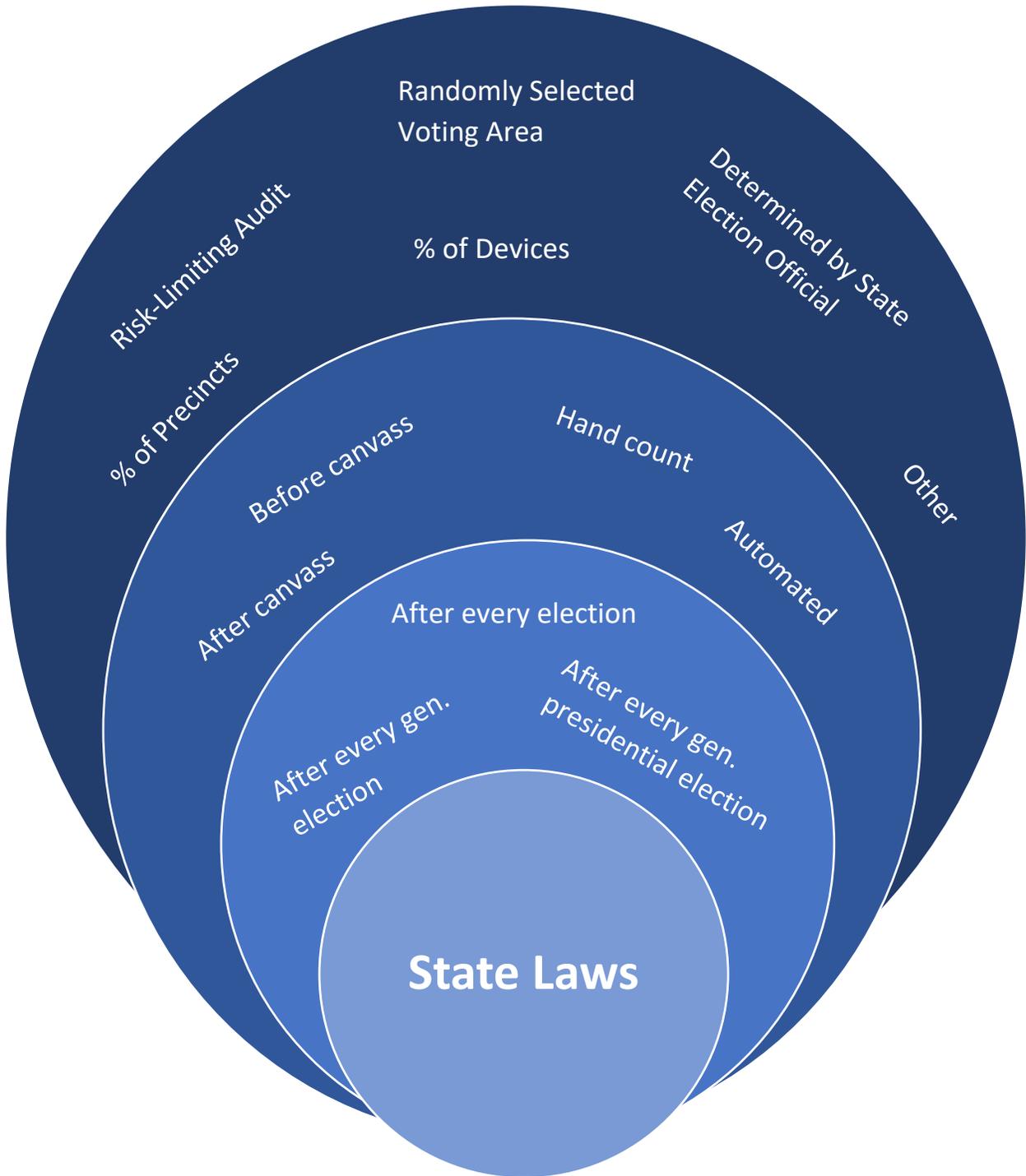# Appendix A – How States Audit

Figure 8 - Many Ways to Audit

Figure 8 displays many of the ways that post-election audits are conducted. Included in each element are variables such as:
- Exact timeframe that the audit can be conducted
- Exact quantity of ballots to audit
- The number of contests
- The types of contests and voting equipment
- Escalation factors
- Who conducts the audit
- Contingent policies
- Varying definitions of "random"
- Reporting requirements

Table 4, described below, attempts to capture the essence of how each state conducts post-election audits.
- "Audit Required" refers to post-election audits that are automatically required. **Indiana** has requirements for conducting a post-election audit if requested by a county chairman.

- "Audit Type" refers to the basic structure of the post-election audit and is generalized as follows:
  o Automatic Comparison (**South Carolina**) – From the South Carolina Election Commission's website: The audit process compares the tabulated results of the election with the raw data collected in the electronic audit files by each iVotronic voting machine on a flash card.
  o Determined by State Election Official (**Indiana, Iowa, Utah**) – The State Election Official is responsible for determining the audit procedures.
  o Percentage of Precincts (**Hawaii**) – Ballots are audited from a percentage of precincts.
  o Percentage of Voting Devices (**Wyoming**) – Ballots are audited from a percentage of voting devices.
  o Percentage/Number of Precincts and Early Ballots (**Arizona**) – A percentage or minimum number of precincts and early ballots are selected for the audit.
  o Random Sample of Ballots (**Pennsylvania**) – The county board of elections audits a random sample of ballots.
  o Randomly Selected Devices (**Nevada, New Mexico, New York**) – Voting devices are randomly selected to audit.
  o Randomly Selected Polling Place Results (**Vermont**) – The Secretary of State randomly selects the polling place results to audit.
  o Randomly Selected Precincts (**Alaska, California, Florida, Kentucky, Maryland, Massachusetts, Minnesota, Missouri, North Carolina, Tennessee, Texas, West Virginia**) – A certain number or percentage of precincts are randomly selected by the state election official or local election official. Some states randomly select other voting areas, which have been included as a sub-bullet. Along with the random selection of precincts, some states require that additional ballot records or devices be audited.
    ▪ Randomly Selected Election Districts (**New Jersey**)
    ▪ Randomly Selected Precincts and Centrally Tabulated Voter-Verifiable Paper Records (**Washington, D.C.**)
    ▪ Randomly Selected Precincts and Contests (**Michigan, Montana**)

- Randomly Selected Precincts and Early Voting Devices (**Illinois**)
- Randomly Selected Precincts or Ballot Count Batches (**Oregon**)
- Randomly Selected Reporting Units (**Wisconsin**) – A reporting unit is a ward, combination of wards, or other district by which votes are tallied.
- Randomly Selected Units (**Ohio**) – A unit is precinct, polling location, or voting device.
- Randomly Selected Voting Districts (**Connecticut**)
  - o Risk-Limiting (**Colorado, Rhode Island, Virginia, Washington**) – Risk-limiting audit methodology varies by state.

- "Reference" provides legal references and links to gain a more comprehensive understanding about how each state conducts post-election audits.

Table 4 - How States Conduct Post-Election Audits

State	Audit Required	Audit Type	Reference
Alabama	No	N/A	N/A
Alaska	Yes	Randomly Selected Precincts	Alaska Statute 2017 §15.15.420 - §15.15.450
Arizona	Yes	Percentage/ Number of Precincts and Early Ballots	Election Procedures Manual Section 4.3.3
Arkansas	No	N/A	N/A
California	Yes	Randomly Selected Precincts	California Election Code §15360
Colorado	Yes	Risk-Limiting	Colorado Revised Statute §1-7-515
Connecticut	Yes	Randomly Selected Voting Districts	Connecticut General Statute §9-320f
Delaware	No	N/A	N/A

State	Audit Required	Audit Type	Reference
District of Columbia	Yes	Randomly Selected Precincts and Centrally Tabulated Voter-Verifiable Paper Records	[D.C. Official Code §1-1001.09a](#)
Florida	Yes	Randomly Selected Precincts	[Florida Statute §101.591](#)
Georgia	No	N/A	N/A
Hawaii	Yes	Percentage of Precincts	[Hawaii Administrative Rules §3-172-102](#)
Idaho	No	N/A	N/A
Illinois	Yes	Randomly Selected Precincts and Early Voting Devices	[10 Illinois Compiled Statute 5/24C-15](#)
Indiana	Yes*	Determined by State Election Official	Indiana Code [§3-11-13-37](#) and [§3-12-3.5-8](#)
Iowa	Yes	Determined by State Election Official	[Iowa Administrative Code §50.51](#)
Kansas	No	N/A	N/A
Kentucky	Yes	Randomly Selected Precincts	[Kentucky Revised Statutes §117.383](#)
Louisiana	No	N/A	N/A
Maine	No	N/A	N/A

State	Audit Required	Audit Type	Reference
Maryland	Yes	Randomly Selected Precincts	Code of Maryland Regulations §33.08.05
Massachusetts	Yes	Randomly Selected Precincts	Massachusetts General Law Chapter 54 §109A
Michigan	Yes	Randomly Selected Precincts and Contests	Michigan Compiled Law §168.31a
Minnesota	Yes	Randomly Selected Precincts	Minnesota Statute §206.89
Mississippi	No	N/A	N/A
Missouri	Yes	Randomly Selected Precincts	Missouri Code of State Regulations 15 §30-10.090
Montana	Yes	Randomly Selected Precincts and Contests	Montana Election Rule 44.3.1719
Nebraska	No	N/A	N/A
Nevada	Yes	Randomly Selected Devices	Nevada Administrative Code 293.255
New Hampshire	No	N/A	N/A
New Jersey	Yes	Randomly Selected Election Districts	New Jersey Statutes Annotated §19:61-9
New Mexico	Yes	Randomly Selected Devices	New Mexico Administrative Code 1.10.23
New York	Yes	Randomly Selected Devices	New York Compilation of Rules & Regulations 6210.18

State	Audit Required	Audit Type	Reference
North Carolina	Yes	Randomly Selected Precincts	North Carolina General Statutes §163A-1169(b)(2)
North Dakota	No	N/A	N/A
Ohio	Yes	Randomly Selected Units	Secretary of State Directive 2017-14
Oklahoma	No	N/A	N/A
Oregon	Yes	Randomly Selected Precincts or Ballot Count Batches	Secretary of State Rule 165-007-0290
Pennsylvania	Yes	Random Sample of Ballots	Pennsylvania Election Code Article XI Section 1117-A
Rhode Island	Yes	Risk-Limiting	Rhode Island General Laws §17-19-37.4
South Carolina	Yes	Automatic Comparison	South Carolina Election Commission Description
South Dakota	No	N/A	N/A
Tennessee	Yes	Randomly Selected Precincts	Tennessee Code Annotated §2-20-103
Texas	Yes	Randomly Selected Precincts	Election Advisory No. 2012-03
Utah	Yes	Determined by State Election Official	Utah Code Annotated 20A-3-302
Vermont	Yes	Randomly Selected Polling Place Results	17 Vermont Statute Annotated §2493

State	Audit Required	Audit Type	Reference
Virginia	Yes	Risk-Limiting	Virginia Code §24.2-671.1
Washington	Yes	Risk-Limiting	Washington Revised Code Annotated §29A.60.185
West Virginia	Yes	Randomly Selected Precincts	West Virginia Code §3-4A-28
Wisconsin	Yes	Randomly Selected Reporting Units	Wisconsin Statutes Annotated §7.08(6)
Wyoming	Yes	Percentage of Voting Devices	Wyoming Statutes 22-11-104

* If requested

Appendix B – Colorado Post-Election Audit Form (Random Machine Audit)

2015 Coordinated Election Post-Election Audit
Delta County

			Delta County			Machine Count	Manual/Hand Count
Type	Make / Model	Serial #	Contest Name to Audit	Candidate			
DRE	Hart / eSlate	A0ACD8	Town of Hothckiss Ballot Question 2C	Yes/For		2	2
DRE	Hart / eSlate	A0ACD8	Town of Hothckiss Ballot Question 2C	No/Against		0	0
DRE	Hart / eSlate	A0ACD8	Proposition BB (STATUTORY)	No/Against		1	1
DRE	Hart / eSlate	A0ACD8	Proposition BB (STATUTORY)	Yes/For		4	4
CCOS	Hart / Canon DR-X10C	ED301609	Delta County Ballot Question 1A	Yes/For		374	374
CCOS	Hart / Canon DR-X10C	ED301609	Delta County Ballot Question 1A	No/Against		116	116
CCOS	Hart / Canon DR-X10C	ED301609	Delta County School District 50J Board of Education Director - District Three	Pete Blair		379	379
CCOS	Hart / Canon DR-X10C	ED301609	Town of Cedaredge Referred Ballot Question 2F	Yes/For		68	68
CCOS	Hart / Canon DR-X10C	ED301609	Town of Cedaredge Referred Ballot Question 2F	No/Against		22	22

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Appendix C – Risk-Limiting Audit History in Colorado

2009

Colorado House Bill 09-1335 created Colorado Revised Statute 1-7-515 which required counties to conduct risk-limiting audits commencing with the 2014 General Election. Also included in C.R.S. 1-7-515 was authorization for the Colorado Secretary of State to conduct a pilot program beginning with the 2010 Primary Election, and to promulgate rules to implement and administer risk-limiting audits.

2010

The Colorado Secretary of State in conjunction with Douglas County and members of the Colorado State University Department of Statistics conducted a pilot of study of risk-limiting audits. The purpose of the study was to see if it was feasible for Colorado to conduct a comparison RLA with the current voting systems in use. A comparison RLA would not have been possible with the current voting systems because they could not produce a CVR that would provide a one-to-one match.

Concurrently, Boulder County was conducting a batch-level RLA. Boulder scanned ballots in batches of 250. Each batch was recorded on a memory card. To conduct the audit, Boulder randomly selected memory cards and hand-counted the ballots. The hand-counted tally sheet was compared to the batch report from the memory card.

2011

Colorado applied for, and received, an EAC grant to conduct research on risk-limiting audits. Five counties were selected to participate in the pilot project: Arapahoe, Boulder, Eagle, Jefferson, and Pitkin. The pilot project had three goals:

1. To develop, test, and implement a risk-limiting audit system for Colorado that meets the letter and spirit of Colorado Revised Statute 1-7-515.
2. To create a user-friendly, transparent, post-election audit process that can be performed successfully in a variety of jurisdictions, using a variety of voting methodologies, technologies, and vendors.
3. To widely disseminate the project's research results to encourage appropriate replication in jurisdictions across the state and the nation.

At the onset of the RLA pilot project, Colorado had to resolve the issue of ballots being subject to open records requests. This eventually was resolved and it was determined that ballots are subject to open records requests subject to the conditions detailed in Colorado Revised Statute 24-72-205.5.

2012

Dr. Stark and Dr. Mark Lindeman published "A Gentle Introduction to Risk-Limiting Audits" which provides an explanation of RLAs with the supporting algorithms.

2013

Colorado conducted its first RLA pilot at Arapahoe County for the 2013 Coordinated Election by scanning ballots in parallel. Arapahoe County scanned ballots using their Sequoia 400C scanners and then re-scanned the ballots using Clear Ballot scanners and software to obtain CVRs for the parallel-scanned

ballots. Colorado House Bill 13-1303 revised the RLA date from 2014 General Election to the 2017 Coordinated Election.

2014

Arapahoe County used OpenCount Software with COTS scanners to conduct a comparison RLA pilot.

2015

Comparison RLA pilots were conducted in Denver, Adams, Garfield, and Jefferson counties as part of the Pilot Election Review Committee review.

2016

Comparison RLA pilots were conducted in Pitkin and Arapahoe counties using the Dominion Democracy Suite voting system.

2017

Ballot-polling RLA pilot at Garfield using the Hart InterCivic 6.2.1 voting system.

Appendix D – Ballot-Polling Audit Spreadsheets

Figure 94 - Ballot-Polling Audit Selection Sheet

Figure 50 - Ballot-Polling Audit Tally Sheet