obtained in individual cultures suggests that crossing-over can occur in spermatogonial divisions.

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NUMBER OF THE ABELIAN GROUPS OF A GIVEN ORDER

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The identity is the only group of order 1 and this may be regarded as an abelian group even if the law of combination of its operators cannot present itself since there is only one operator in it. There is one and only one group of order 2 and there are two groups of order 4. It will be proved in what follows that these are the only three orders for which the number of abelian groups is as large as half the order of these groups. On the contrary, the number of non-abelian groups of a given order may exceed this order. The lowest order for which this is the case is 32. In fact, 44 of the 51 groups of this order are known to be non-abelian. An Indian mathematician, S. Ramannjan (1887–1920), found an asymptotic formula from which it follows that the number of the abelian groups of order p^{200}, p being a prime number, is nearly four million millions.

Since every abelian group whose order is not a power of some prime number is the direct product of its Sylow subgroups, the number of the abelian groups of order $g = p_1^{\alpha_1} p_2^{\alpha_2}, \ldots, p_\lambda^{\alpha_\lambda}$ ($p_1, p_2, \ldots, p_\lambda$ being distinct prime numbers) is equal to the product of the numbers of the abelian groups of order $p_1^{\alpha_1}, p_2^{\alpha_2}, \ldots, p_\lambda^{\alpha_\lambda}$, respectively. Since the number of the abelian groups of order p^m, p being a prime number, is independent of the value of p, we proceed to prove that the number of the abelian groups of order 2^m, $m > 2$, is less than 2^{m-1}.

To prove this fact it is desirable to bear in mind that the number of the abelian groups of order 2^m is equal to the number of ways in which m

can be separated into positive integral parts so that the sum of these parts is equal to m. When m has been thus separated in every possible manner a partial number of the possible separations of $m + 1$ into such addends can be obtained by adjoining the number 1 to each of the given sets of numbers whose sum is equal to m. It may be assumed that these sets of numbers are so arranged that a number in a set is never followed by a larger number in the same set and that the first n numbers of a given set are separately at least as large as the corresponding numbers of the following set.

To complete the sets of such addends for $m + 1$ we form new sets by replacing the last two numbers of each one of the given sets by their sum whenever this sum does not exceed the third number from the end in this set, and insert the set thus obtained just before the set thus affected. In this manner we obtain at most two sets of addends for $m + 1$ from a given set for m. When $m > 1$ we always obtain two such sets for $m + 1$ from the first set for m but we never obtain more than one set for $m + 1$ from the last set for m. Hence the number of sets of such addends for $m + 1$ is always larger than the number of the sets for m but it is never twice as large, $m > 1$. This proves the theorem that when $m > 2$ the number of the abelian groups of order 2^m is less than 2^{m-1}. Hence there results the following theorem: *The number of the abelian groups of a given order greater than unity cannot exceed a power of* 2 *whose index is the sum of the largest exponents of the different primes which divide this order diminished by the number of such primes.*

A useful corollary of this theorem is that the number of the abelian groups of a given order greater than unity does not exceed half this order. In fact, it is equal to half this order only when the order is 2 or 4. When all the prime factors of the order of such a group are distinct the given theorem shows correctly that there is only one such abelian group, but when prime factors of the order are repeated a large number of times this theorem will give a number which varies more and more from the number of existing abelian groups as the factors are repeated a larger number of times.