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On the Welfare Costs of Perceptions Biases

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Abstract

Are households significantly harmed by inaccurate beliefs about inflation? This paper analyzes two established inflation perceptions biases and evaluates their welfare effects. The first bias is the frequency bias, where households overweight goods that they purchase frequently but are a small share of their consumption basket. In my French sample, I find that households fixate on bread prices. The second bias is that households consistently overestimate the current inflation rate, which I call the level bias in this paper. I estimate the magnitude of these biases using a confidential French household survey. To evaluate the welfare losses of the two biases, I incorporate biased inflation perceptions into a partial-equilibrium model where households save in a single nominal bond subject to inflation risk. The level bias significantly reduces welfare and asset accumulation, while the frequency bias has negligible effects. The welfare loss shrinks if I remove the perceptions bias while keeping the expectations bias, which suggest that inaccurate perceptions can harm households beyond the effect on forecasts.

JEL Codes: E31, E70, G51

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1 Introduction

To make spending and saving decisions, households must assess the current price level and form expectations about future prices. A large literature based on survey evidence shows that these perceptions and expectations often deviate from full rationality due to a variety of possible causes, such as noisy information, asymmetric loss functions, and personal experiences.[1] This paper studies whether deviating from rational beliefs about inflation and the price level affects household welfare significantly.

This paper examines two established biases in household perceptions and constructs a model to evaluate their welfare implications. I focus on the effect of biases on inflation perceptions, which are the beliefs that households have about the current inflation rate. The first bias is the *frequency bias*, which describes how households overweight the prices of goods that they purchase frequently when they think about inflation.[2] For example, if food prices in France rise suddenly, households may raise their perceptions of the inflation rate significantly, even though food is a modest component of their consumption basket. The second bias is that households consistently overestimate the current inflation rate.[3] I refer to this phenomenon as the *level bias*. These biases may seem surprising, given that the cost of obtaining accurate information about the current inflation rate is low.[4] If biases about the inflation rate were detrimental to households, then would be relatively simple for households to correct them. This paper contributes to the literature by examining whether these welfare losses are in fact small.

I measure the perception biases and evaluate welfare losses in five steps. First, I use a confidential French household survey to establish that perceptions about the current inflation rate are correlated with expectations and savings behavior, which suggests that perceptions aren't irrele-

[1]See Woodford (2003), Capistran and Timmermann (2009), and Malmendier and Nagel (2015), respectively.

[2]Tversky and Kahneman (1973) originally postulated the frequency bias for general contexts, while Georganas et al. (2014) applied it to price inflation.

[3]Detmeister et al. (2016) and Abildgren and Kuchler (2021) similarly find that U.S. and Danish households overestimate the current inflation rate and offer several explanations for this behavior.

[4]The national statistical agencies of many developed countries post inflation statistics publicly on their websites with a short lag, which means that the monetary cost of obtaining accurate information about current inflation is almost zero.

vant for household actions.[5] Second, I provide causal evidence that French households suffer from the frequency bias by overweighting bread inflation when reporting their perceptions.[6] Third, I estimate the level bias and find that the average French household overestimates the inflation rate by 2.5 percentage points. The size of this effect varies by income and education status.[7] Fourth, I develop a partial equilibrium model in which households use their biased inflation perceptions to make savings decisions in order to self-insure against idiosyncratic income shocks. I use my estimates of the frequency bias and the level bias to parameterize the model. I find that the consumption equivalent welfare loss of the level bias is a significant 0.17% of consumption each period. However, the consumption equivalent loss of the frequency bias is minuscule, less than 0.043% of consumption each period. Finally, I conduct counterfactual analyses and find that a) reducing the correlation between bread and the overall inflation rate increases welfare loss, b) low-income households suffer a greater welfare loss than high-income households, and c) removing households' perceptions bias about the inflation rate while keeping their biased expectations of future inflation decreases their welfare loss, which suggests that biased inflation perceptions can affect households' decision beyond their impact on forecasts of future inflation.

In my empirical work I use France's Monthly Consumer Confidence survey (CAMME) to estimate the magnitude of the inflation perceptions biases. In contrast to many existing surveys, the CAMME inquires about perceptions of the current inflation rate. The survey is conducted on a monthly basis from January 2004 to December 2017. Each household is re-surveyed for up to three consecutive months. I combine the households' beliefs with nationally aggregated inflation statistics to measure the accuracy of their perceptions.

My three main empirical findings are as follows. First, in Section 2, I find that perceptions of the current inflation rate are linked to households' expected inflation and their economic decisions.[8]

[5]This paper uses the French Monthly Consumer Confidence survey (CAMME), which is collected by the Institut national de la statistique et des études économiques (INSEE). I applied for permission to access the confidential CAMME survey microdata through Réseau Quetelet.

[6]See Brachinger (2008), Shwayder (2012), Armantier et al. (2016a), D'Acunto et al. (2019b) for other papers measuring the frequency bias.

[7]Bryan and Venkatu (2001b), Bruine de Bruin et al. (2010), Ehrmann et al. (2017), and D'Acunto et al. (2019a) have also documented demographic differences in household inflation expectations.

[8]Jonung (1981) finds that Swedish household expectations are highly correlated with perceptions.

To provide evidence that households are acting on their perceptions, I conduct a logistic regression self-reported declarations about savings on inflation perceptions. French households that report higher inflation perceptions in the survey believe that it is a worse time to save money. This result supports the hypothesis that households use inflation beliefs to make savings decisions and that biased inflation perceptions can lead to suboptimal economic decisions.

Second, French households overweight bread inflation when forming perceptions about the current inflation rate. To test this hypothesis, I regress household inflation perceptions on the overall inflation rate and the bread inflation rate in Section 3. Under the framework of perfect information and rationality, households should assign no weight towards the bread inflation rate on average, because any relevant information it contains has already been incorporated into the overall inflation rate. However, I find that households put more weight on bread inflation than they do on the overall inflation rate.[9] The regression may suffer from an endogeneity problem, such as a confounding variable or reverse causation. To resolve this concern, I rerun my regression analysis with the lagged world wheat inflation rate as an instrument for the bread inflation rate.[10] I find that bread inflation remains a statistically significant predictor of inflation perceptions, which indicates that it affects French inflation perceptions.

My third empirical contribution is to measure the level bias, document heterogeneity by demographics, and examine several possible causes in Section 4. Households overestimate the actual inflation rate by 2.5 percentage points on average. This overestimation is relatively constant over my sample period from 2004 to 2017. I find that low-income households and low-education households overestimate the current inflation rate by more than high-income and high-education households. I propose and test several possible explanations for the level bias and the difference by income. I examine whether households asymmetrically weight price increases relative to price decreases, but find that this weighting does not explain the magnitude of the level bias. The income difference in the level bias cannot be explained by consumption baskets heterogeneity by

[9]Conducting this analysis on other types of frequently purchased goods indicates that households do not overweight them as heavily as they do bread.

[10]Shwayder (2012) and Coibion and Gorodnichenko (2015) have used world oil prices as a proxy for gasoline prices when evaluating the effect of gasoline on inflation expectations in the United States.

income.[11][12]

Following my empirical work, I turn to the task of evaluating how these biases affect household welfare in Section 5. *A priori*, if households in the sample could easily correct the welfare losses by obtaining accurate inflation perceptions, then we would expect that the losses are minuscule. I develop a partial equilibrium heterogeneous agent model that can incorporate these two perception biases. The purpose of this model is to evaluate the welfare losses and savings behavior associated with both biases.

Households have a risky, idiosyncratic labor income that they partially smooth by purchasing a nominal bond, which is subject to inflation risk and a no-borrowing constraint. Households use their biased inflation perceptions to forecast future inflation, which can result in inaccurate inflation expectations. When households solve their savings problem each period, they act as if their biased perceptions and expectations are correct.[13] However, from the perspective of an outside observer with full information rational expectations, these households make sub-optimal savings decisions.[14]

Biased perceptions influence households through two channels: biased expectations and biased wealth perceptions. Households forecast future inflation using their perception biases, which results in distorted expectations. As a result, households may underestimate the expected real return to savings, disincentivizing them from savings. Regarding wealth perceptions, if households know their nominal wealth accurately but overestimate the current price level, then they will believe that their real wealth is lower than it actually is.[15] As a result, households may reduce their planned savings.

[11]The frequency bias does not explain the level bias in my dataset. The average overall inflation rate in France during my sample is 1.6%, while the average bread inflation rate is 1.7%. Even if households only paid attention to the bread inflation rate they would still perceive the correct inflation rate on average.

[12]Zekaite (2020) suggests that households have a different consumption basket in mind when they think of inflation, which overweights some goods sectors relative to CPI.

[13]If a household was aware that it had a level bias, the household could correct for the bias and obtain rational expectations.

[14]Throughout this paper I use the term bias to primarily refer to departures from full information rational expectations. These biases may in fact be cognitive biases or may reflect optimization under information constraints. In either case, an observer with full information rational expectations would view the biased households as making inefficient savings plans.

[15]Households in my model do not know the current inflation rate or price level.

According to the model, both the level bias and the frequency bias generate surprisingly large welfare losses for the households that suffer from them. The level bias is around an order of magnitude more damaging than the frequency bias. These welfare differences are driven by bond-holding behavior: in the steady state, households that suffer from the level bias hold significantly fewer bonds compared households with rational beliefs. Households with the level bias consistently expect that future inflation will be high, which reduces the real return to nominal bonds and discourages them from saving. Lower bond holdings means that households are less able to smooth out consumption in response to shocks in their labor income, reducing their overall welfare.

I conduct three extensions to the model in order to explore the intuition behind these results, which I present in Section 7. First, I examine the relationship between bread inflation and overall inflation. I weaken the correlation between the inflation rate of bread and the overall inflation rate, which degrades the signal quality of bread. The welfare loss associated with the frequency bias increases by an order of magnitude compared to the baseline parameters. Increasing the volatility of bread price changes to match U.S. gasoline volatility increases welfare loss twenty-fold. Fixating on volatile or unrepresentative goods can harm households more than simply overestimating inflation each period.

Second, I investigate the role of the biased wealth perceptions channel on welfare and savings. I consider a version of the model where households know the current inflation rate perfectly, but still have biases when forecasting inflation.[16] After solving this model, I find the welfare loss associated with the level bias decreases by about 20% compared to the version of the model where households have biased perceptions and expectations. Households that consistently overestimate the inflation rate each period believe that their real wealth is lower than it really is, which affects their savings decisions. As a result, I find that households with biased inflation perceptions also hold less savings in the steady state.

Finally, I examine how variation in the level bias by household income affects welfare loss

[16] If household only have the level bias for inflation expectations, then every period they believe that the annualized future inflation rate will be 2.5 percentage points higher than they would if they had rational expectations. If households have the frequency bias when forecasting, then they correctly weight bread when estimating current inflation, but overweight it when forecasting inflation.

and aggregate bond-holding. High income households have a smaller level bias than low-income households, which means that the welfare loss for low-income households is significantly greater than high-income. However, it is unclear if aggregate bond-holdings are significantly affected: if high-income households hold a disproportionate share of bonds and suffer from a small level bias, then the total number of bonds held in steady state may only deviate slightly from rational expectations. In my model, I find that the aggregate bonds held is still lower than the rational expectations amount, because high-income households still suffer from a moderate level bias, which indicates that the level bias can have macroeconomic effects.

In summary, this paper makes three significant contributions to the existing literature on household beliefs. First, it demonstrates that biased perceptions about the inflation rate can be extremely damaging to households. Second, it shows that perceptions of the current inflation rate can affect households, not just expectations of future inflation. Section 8 concludes and discusses future work.

Related literature

This paper is draws upon several areas of research on inflation beliefs, and contributes to them by studying the welfare effects of inaccurate beliefs.[17] The first subfield studies how agents form beliefs about economic variables under imperfect information. While this literature is too vast to cover here, one particularly relevant area is rational inattention. This paradigm, which was pioneered by Sims (2003) and Sims (2010), studies how agents allocate a limited amount of attention between different macroeconomic variables. Mackowiak and Wiederholt (2015) incorporates inflation misperceptions into a rational inattention business cycle model and find that optimally households would pay little attention to inflation relative to other macroeconomic variables.[18] Kamdar

[17]Throughout this paper, I use the term inflation beliefs as an umbrella term for perceptions of the current inflation rate, expectations of future inflation rates, and beliefs about other characteristics of the inflation process, such as the average inflation rate. Although inflation perception biases are the focus of this paper, I refer to beliefs to place it in the broader literature, which often studies inflation expectations.

[18]Rational inattention has been applied to how firms perceive inflation and set prices, as in Kumar et al. (2015), Cavallo et al. (2017), and Coibion et al. (2018).

(2019) finds that households choose to focus on real wages rather than the overall price level. My paper deviates by not assuming that these perceptions biases arise from an optimal household decision. Rozsypal and Schlafmann (2019), which analyzes how overestimating the persistence of income affects households' savings decisions, bears the most similarity to this paper.

Numerous papers in the last decade have studied how households use inflation expectations to make savings decisions.[19] My paper finds further support that inflation perceptions affect savings behavior, and then examines the welfare implications of this for households. Several papers assert that inflation expectations affect readiness to spend or self-reported spending, but they disagree about the sign of the effect. Armantier et al. (2015), D'Acunto et al. (2016) and D'Acunto et al. (2019a) find that households increase spending if their inflation expectations rise, while Burke and Ozdagli (2014) and Bachmann et al. (2015) find the opposite effect. Andrade et al. (2020) argue that households react to changes in the broad inflation regime when making durable purchases, rather than fluctuations to the inflation rate. Vellekoop and Wiederholt (2019) links a household opinion survey to their actual financial data and finds that households do reduce their savings when their inflation expectations are higher, which supports the mechanism in my paper.

My analysis contributes to this frequency bias literature by establishing the frequency bias for a single good (e.g. bread in France) and by evaluating the effect of the frequency bias on households' welfare. Georganas et al. (2014) conducts a laboratory experiment and demonstrates that consumers overweight frequently encountered prices when evaluating inflation. Shwayder (2012) and Coibion and Gorodnichenko (2015) provide evidence that U.S. households overweight gasoline when forming expectations, although Binder (2018) suggests that gasoline prices do not affect inflation perceptions. Both Cavallo et al. (2017) and D'Acunto et al. (2019b) study the inflation perceptions of consumers in relation to their supermarket purchases and find that people react to price increases in goods they purchase. D'Acunto et al. (2019b) constructs a frequency-weighted consumer price index, which is similar to the approach of Brachinger (2008).

Lastly, this paper is related to a body of work that investigates the heterogeneity of inflation

[19] The effect of beliefs on savings is an oft-studied question, because several long-running household surveys, such as the Michigan Survey of Consumers, inquire about households' savings behavior and attitudes.

beliefs, particularly by demographic status such as income or education. I contribute to this literature not only by evaluating the welfare effect of the level bias, but by studying how the welfare effect varies by household income. Arioli et al. (2017) and Abildgren and Kuchler (2019) provide a good overview of how low-income and low-education households consistently overestimate past inflation rates relative to high-income and high-education households, as well as different possible causes for this effect.[20] The literature suggests a number of possible explanations for this phenomenon. First, households could be purchasing different consumption baskets and therefore face different inflation rates.[21] However, Arion et al. (2018) constructs income group-specific consumption baskets and find that the inflation rates do not diverge significantly.[22] Another possible explanation is that low-income households have low financial literacy and are unfamiliar with concepts like inflation. A third possibility is that households asymmetrically view price increases compared to price decreases.[23] I test several of these explanations, but ultimately remain agnostic about the cause of the level bias.

2 Inflation Perceptions

In this section I describe the household survey methodology and establish the relevance of inflation perceptions. I introduce the household survey, study the link between expectations and perceptions, and conclude by investigating savings beliefs.

[20]Armantier et al. (2016a) conducts an experimental survey where they provide households with accurate inflation statistics. They find that low-income and low-education households update their beliefs less in response to the new information. This result suggests that low-income households not only have less accurate beliefs, but more difficulty processing new information.

[21]Johannsen (2014) uses the Bureau of Labor Statistics Consumption Expenditures Survey to examine how consumption baskets varies by demographic status and finds that the consumption baskets can explain some of the dispersion in responses.

[22]Other papers cast doubt on the role of differing consumption baskets. Diamond et al. (2020) constructs household-specific inflation rates and conclude that differences in experienced inflation does not explain differences in expectations. Bryan and Venkatu (2001a) find that men and women differ not only in their beliefs about the overall inflation rate, but also the inflation rates of food and clothing.

[23]See Capistran and Timmermann (2009), where forecasters form their expectation of inflation using an asymmetric loss function.

2.1 Survey Data

In order to study how household inflation perceptions are formed, I require a long-running survey that includes a question about current inflation. This paper incorporates data from the Monthly Consumer Confidence Survey (CAMME), which is collected by the National Institute of Statistics and Economic Studies (INSEE) in France. The CAMME is a monthly survey of 1,800 French households that inquires about consumer behavior and expectations. My sample is limited to the period from 2004-2017 when INSEE asked detailed questions about inflation beliefs.[24] This paper focuses on survey questions on inflation beliefs and willingness to save. Inflation beliefs refers to how households believe prices have changed over the previous 12 months and how they believe prices will change over the next 12 months. I also use demographic variables, such age, education, family status, gender, and income. Altogether the dataset includes 171,606 households that answer the question about quantitative inflation perceptions. Appendix A describes the survey wording in more detail.

When INSEE conducts the CAMME survey, they resurvey each household for up to three consecutive months, i.e. February, March, and April.[25] The dataset is a revolving panel, which I use to study how households update their perceptions in response to changes in inflation.[26] I use internal survey variables[27] to construct unique household identifiers,[28] which track households across all three waves to which they respond.[29] I can only track households across waves between

[24]INSEE did not conduct the CAMME survey in the month of August until 2008. Aggregated statistics were reported as interpolations of the responses in July and September.

[25]Before 2008, INSEE would skip August and resurvey some households in September. For example, a household may be surveyed in July, September and October.

[26]For example, the first time a household is surveyed is called its first wave, the second time is its second wave, etc.

[27]I specifically use the NUMFA variable in the CAMME. After repeated inquiries to INSEE, I believe that this is a semi-unique variable related to where households are stored in the internal database. For a given date, say March 2008, every household that is sampled for the first time is assigned a NUMFA value that is unique among all households that were first surveyed at the given date. If two households have the same NUMFA value, then they were first surveyed in two different months.

[28]The CAMME survey does not an identification variable that uniquely links to a specific household. Andrade et al. (2020) uses time-invariant household demographic variables to link them across time.

[29]INSEE cannot be certain that they contact the same person within a household each time they survey it. It is possible that they are surveying the spouses of the original respondents in subsequent waves.

2004 and 2014.[30] Approximately 40% of households that answer the inflation perceptions question are present for three consecutive months.

The CAMME survey allows me to examine the understudied topic of inflation perceptions. Many surveys of households beliefs ask about forecasts of future inflation rather than perceptions of current inflation. For example, two of the most commonly used household surveys are the Michigan Survey of Consumers and the Survey of Consumer Expectations in the United States. While both surveys have a rich set of questions overall, they restrict their questions to expectations about future inflation.[31]

This paper employs definitions of the words perceptions and beliefs that differs from other fields of study, such as philosophy. In the economics literature regarding inflation beliefs,[32] the term inflation perceptions is synonymous with beliefs about the current inflation rate; households that believe that the current inflation rate is high are said to perceive a high inflation rate.[33] Inflation beliefs are an umbrella term that refers to the collection of beliefs that individuals have about the current inflation rate, future inflation rates, and other characteristics about inflation, such as the average inflation rate. For brevity's sake, I will follow this convention throughout the paper. However, in the philosophical literature, the word perceptions refers to direct sensory experience.[34] Under this definition, the inflation rate cannot be directly perceived by people, as it is the aggregation of numerous prices across the entire economy. In philosophical terms, people hold beliefs, statements that they think are true, about the current inflation rate. As philosophers such as Hanson (1958) note, previously held beliefs can affect how observers perceive the world around them. People who believe that the inflation rate is consistently high may perceive individual prices that are increasing rapidly, even if the prices are actually steady or rising slowly. Unfortunately, inves-

[30]INSEE changed the household identification system in 2015 and 2016. I restrict my revolving panel analysis to the pre-2015 subsample.

[31]Axelrod et al. (2018); Detmeister et al. (2016) add a supplement to the Michigan Survey of Consumers that inquires about inflation perceptions and expectations. They find that U.S. households overestimate current inflation over a short horizon, but have more accurate beliefs about inflation over a longer horizon. However, their sample of households begins in 2016, which presents a narrower time frame compared to the CAMME dataset.

[32]See papers such as Jonung (1981) and Branch (2004) for further uses of the term

[33]The term inflation expectations is synonymous with beliefs of future inflation rates.

[34]For example, if I see a red box with my eyes, then I have perceived a box that is red.

tigating the link between individuals' beliefs about long-run inflation and how they form beliefs about the current inflation rate on a day-to-day basis is beyond the scope of this paper.

I use two nationally aggregated statistics from INSEE to complement the CAMME survey. I calculate the inflation rates of specific goods categories and construct inflation rates for specific household demographics. I obtain the pricing data from the Harmonized Consumer Price Index (HICP), which I use to calculate the overall inflation rate. The HICP contains aggregated price indices for different categories of goods, such as food, gasoline, and rent. In order to obtain the consumption weights for households, I draw on the Household Budget Survey (HBS), which records the fraction of their income households spend on different goods categories. I combine the consumption weights with the pricing data in HICP to create inflation rates specific to household demographic groups, i.e. low-income.

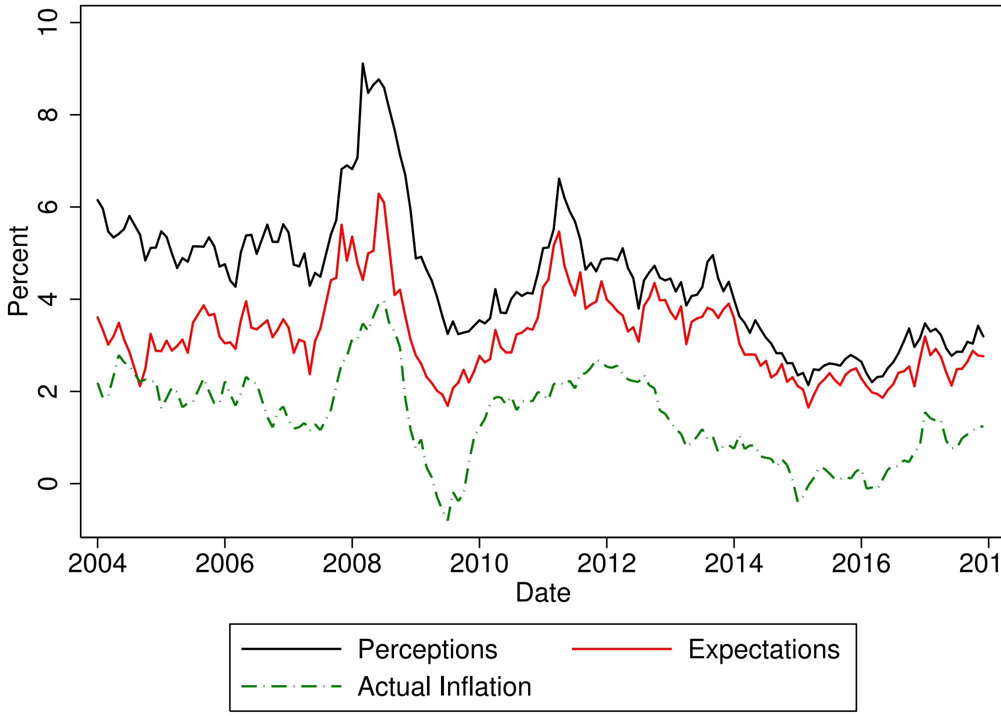
2.2 Inflation Perceptions and Expectations

Although households may hold inaccurate perceptions of inflation, these biases are only harmful if they affect other household beliefs and result in sub-optimal household decisions. For example, if a household has an accurate forecast of future inflation but overestimates the current inflation rate, then biased perceptions could have little economic significance. The following two subsections link inflation perceptions to expectations and savings behavior, which supports the economic significance of perceptions.

Households in the CAMME survey use their perceptions of the current inflation rate to forecast future inflation. Figure 1 presents the average inflation perception and the average inflation expectation for households over the sample period.[35] As a comparison, the graph contains the actual inflation rate over the past 12 months. Households have inaccurate inflation perceptions, and aggregated inflation perceptions are correlated with aggregated inflation expectations. On average, households have inaccurate perceptions and in fact overestimate the inflation rate for each period

[35] I censor any responses that are larger than 20% as outliers. Roughly 5% of observations in any given month lie outside of this window; incorporating them in the analysis does not meaningfully affect results.

Figure 1: Inflation Expectations and Perceptions



Notes: The figure depicts the mean inflation perception and the mean inflation expectation for households in the CAMME survey over time. The actual inflation rate is calculated as the year-on-year percentage change in the HICP. Households with inflation perceptions above 20% are dropped when calculating the mean beliefs.

in the sample. Average perceptions and expectations in Figure 1 co-move closely, particularly after 2007 when the actual inflation rate begins to fluctuate. The correlation between the two time series is 0.822, which indicates that expectations are linked to perceptions in aggregate.

The relationship between perceptions and expectations holds on the household level. I regress inflation expectations on inflation perceptions using the CAMME survey microdata by following standard approach of assuming that households believe that the inflation rate follows an auto-regressive process with one lag.[36] Let the operator $E_{i,t}$ denote household i's expectation of a variable at time t, where t is defined on a monthly frequency. Household i's perception of inflation over the past 12 months is $E_{i,t}\pi_t$,[37] and their expectation of inflation over the next twelve months

[36]While true inflation process may be quite complicated, papers in the learning literature, such as Evans (2001) and Malmendier and Nagel (2015), often model perceived inflation process as being AR(1).

[37]That is, inflation from period $t-12$ to period t.

is $E_{i,t}\pi_{t+12}$. The variable D_i denote demographic controls for individual *i,* including income, education, age, and sex.[38] The regression specification is

$$E_{i,t}\pi_{t+12} = \alpha + \beta E_{i,t}\pi_t + D_i + \varepsilon_{i,t}. \tag{1}$$

Table 1 displays the relationship between household expectations and perceptions of inflation. In all specifications, inflation perceptions are a statistically significant predictor of inflation expectations. In the baseline analysis of column (1), inflation perceptions explain 39% of the variation in expectations. A 1 percentage point increase in inflation perceptions translates to a 0.57 percentage point increase in expectations.

Table 1: Inflation Expectations

	(1) $E_t\pi_{t,t+12}$	(2) $E_t\pi_{t,t+12}$	(3) $E_t\pi_{t,t+12}$	(4) $E_t\pi_{t,t+12}$	(5) $E_t\pi_{t,t+12}$	(6) $\Delta E_t\pi_{t,t+12}$
Inflation Perceptions	0.570***	0.561***	0.648***	0.588***	0.561***	
	(0.009)	(0.009)	(0.008)	(0.012)	(0.012)	
Change in Inflation Perceptions						0.485***
						(0.014)
Constant	0.592***	0.788***	1.510***	0.529***	0.583***	0.067
	(0.020)	(0.092)	(0.110)	(0.037)	(0.026)	(0.043)
Demographic Controls	No	Yes	Yes	No	No	No
Sample	Full	Full	06/2010-	High-Income	Low-Income	Full
R^2	0.392	0.384	0.471	0.369	0.412	0.241
Observation	136660	119550	58140	24650	43055	23589

Notes: This table shows the relationship between inflation expectations and inflation perceptions on a household level. Columns (1), (2), (4), and (5) are run the full sample from 2004-2017. Column (6) is run on the subsample of households between 2004-2014 that I am able to link across survey waves. High income households refer to households reporting an income above the top quartile of the sample. Standard errors are in parentheses. Errors are clustered by the month of the survey. ***,**, and * denote the 1, 5 and 10 % significance levels, respectively.

Perceptions predict expectations even when controlling for other variables. I rerun the analysis with demographic controls and find that the relationship between perceptions and expectations

[38] Although INSEE contacts residences as part of the CAMME survey, they record the inflation beliefs of only one respondent within the household. The survey microdata contains individual demographic information, such as age, as well as whether the respondent is the head of household. As noted in 29, INSEE cannot be certain that they are contacting the same individual in a household each time they resample it.

remains unchanged. Inflation expectations cannot be explained solely by between-demographic group effects. Limiting the analysis to the period after the financial crisis in column (3), when the inflation rate is less volatile, does not weaken this relationship. Lastly, I limit my regression analysis to households with income below the 1st quartile in column (4) and above the 3rd quartile in column (5). The coefficient on inflation perceptions is similar for both analyses, which indicates that the relationship between perceptions and expectations is not driven by an unusual subsample of the population.

Household fixed effects may still play a large role in forming inflation perceptions and expectations. I use the revolving panel component of the survey to test this hypothesis. In column (6) I analyze how households update their expectations between their first wave and their third wave in response to updates to their inflation perceptions. The new regression specification is a differenced version of equation (1):

$$\Delta E_{i,t}\pi_{t+12} = \alpha + \beta \Delta E_{i,t}\pi_t + \varepsilon_{i,t}. \tag{2}$$

The regression results in column (6) of Table 1 indicate that if a household raises its perceived inflation rate by one percentage point, then it also updates its expected future inflation by about 0.48 percentage points. By differencing out household fixed effects, this regression illustrates the role of time variation in the regressions. The R^2 score of 0.28 is relatively high, given that this regression seeks to explain all variation in household updating of expectations. However, the true relationship between perceptions and expectations may be even stronger. In Appendix B I explore the possibility that some households may be incorrectly coded as having expectations of 0%. In summary, I find that there is a strong link between inflation perceptions and inflation expectations, which is consistent across time and demographics. Biased inflation perceptions lead to biased inflation expectations.

2.3 Inflation Perceptions and Savings

Do biased inflation perceptions affect household plans to save? According to the household Euler equation, households should decrease their savings if they expect that the future inflation rate will be higher, because a higher price level in the future reduces the purchasing power of their nominal assets. A number of papers have addressed the role of inflation expectations on savings, such as Vellekoop and Wiederholt (2019), which finds that Dutch households with higher inflation expectations do reduce their savings. This subsection focuses on the effect of inflation perceptions specifically.

The CAMME survey includes several questions about savings beliefs and planned savings. I focus two questions: whether it is a good time for people to save and whether households are actually planning to save. The former measures households' beliefs about the macroeconomic climate and reflects expectations about the interest rate and future inflation. The latter reflects households' intention to save, which may be affected by idiosyncratic variables like household income. Households have multiple qualitative responses, as described in Appendix A. For each variable I create a binary response that corresponds to "yes" or "no."

I conduct a logistic analysis to study the effect of perceptions on plans to save and attitudes to savings in general. This takes the form of a regression of the logarithm of the odds that a household believes that it is a good time to save on a linear combination of explanatory variables.[39] The predicted result of the regression is that a higher inflation perception is associated with being less willing to save. However, the logistic regression requires additional controls. Beliefs about savings may be correlated with other characteristics, such as household income or their beliefs about the general economic situation. I control for other demographic statuses $D_{i,t}$, including education and income. The variable $F_{i,t}$ is an indicator variable corresponding to the households' beliefs about

[39] Another possible technique is an ordered logit, where the dependent variable is a categorical variable with more than two values that have some sort of ordering. The CAMME survey does allow multiple options for household beliefs. However, an ordered multinomial regression violates the proportional odds assumption, which states that explanatory variables have a linear effect on the odds of moving from one category to another. Therefore, an ordered regression is inappropriate in this case.

the future economic outcome.[40] I also incorporate year fixed effects and cluster the errors by the monthly date to capture the effect of the macroeconomic variables.

The regression setup is

$$\text{logit}(P(\text{Savings = Yes})_{i,t}) = \alpha + \beta E_{i,t}\pi_t + D_{i,t} + F_{i,t} + \varepsilon_{i,t}. \tag{3}$$

Table 2: Savings Responses

	(1) Good Time	(2) Good Time	(3) Good Time	(4) Plan Save	(5) Plan Save	(6) Plan Save
Inflation Perceptions	-0.014***	-0.023***	-0.016***	-0.037***	-0.035***	-0.021***
	(0.002)	(0.001)	(0.002)	(0.001)	(0.001)	(0.002)
Inflation Expectations			-0.004*			-0.010***
			(0.002)			(0.002)
Constant	0.440***	0.213***	0.197**	0.138***	-0.760***	-0.274***
	(0.026)	(0.031)	(0.073)	(0.008)	(0.023)	(0.083)
Demographic Controls	No	Yes	Yes	No	Yes	Yes
Year Controls	No	Yes	Yes	No	Yes	Yes
Economic Opinion	No	No	Yes	No	No	Yes
Pseudo R^2	0.001	0.024	0.028	0.006	0.048	0.063
Observations	158381	138960	117045	158381	138960	117045

Notes: Columns 1-3 table shows the logarithm of the odds-probability of a household stating that "now is a good time to save" conditional on inflation perceptions and other variables. Column 4-6 reports the logarithm of the odds-probability of a household stating that they "plan to save." Standard errors are in parentheses. Errors are clustered by date. ***,**, and * denote the 1, 5 and 10 % significance levels, respectively. See subsection 2.3 and equation (3) for more details.

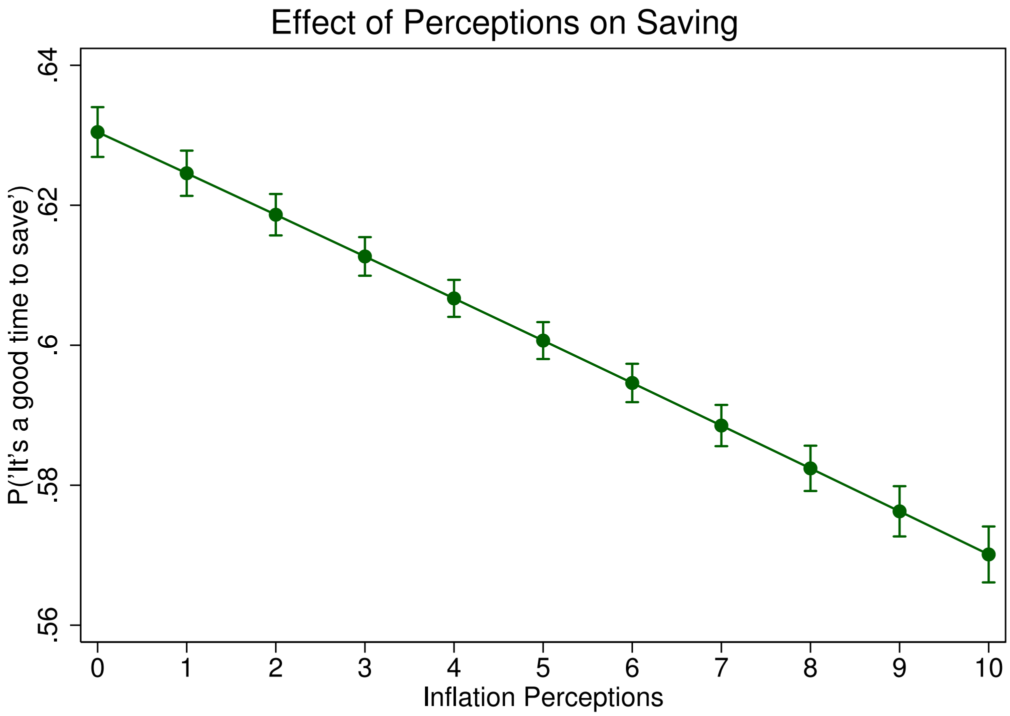
I present the results of the logit regression in Table 2. Households with higher inflation perceptions are less likely to believe that it is a good time to save. Incorporating demographic controls and year fixed effects in equation (2) does not change the results. Surprisingly, inflation perceptions negatively affect beliefs about the savings climate even controlling for inflation expectations and other economic beliefs.[41] One possible explanation is that households that have high inflation perceptions in addition to high inflation expectations are more likely to believe that the average

[40] See Appendix A for a description of the wording for this question and the possible answers.

[41] Inflation expectations are statistically insignificant in determining household saving beliefs.

inflation rate is high, as opposed to inflation being temporarily high before return to a low level. Columns (4), (5) and (6) repeat the analysis with the dependent variable being whether households personally plan to save and find similar results.[42]

Figure 2: Savings Response



Notes: This figure depicts the probability that a household agrees with the statement that "it is a good time to save," conditional on their inflation perception. The figure only displays results for integer responses between 0% and 10%. Other explanatory variables, including demographic variables and monthly date, are evaluated at their means.

Logistic regressions are notoriously difficult to interpret, particularly in terms of economic significance, because they are non-linear, so a 1 percentage point increase does not result in the dependent variable changing by a constant percentage. Instead, I calculate the marginal effect of inflation perceptions evaluated at the mean of the other covariates. Figure 2 shows how, as inflation perceptions rise from 0% to 10%, the probability of agreeing with the statement that it is a good time to save drops from 64% to 56%.[43]

[42]Conducting this analysis using a linear probability model instead of a logit model does not alter the statistical significance of the result.

[43]Note that these marginal effects are not perfectly linear.

3 Frequency Bias

In this section I estimate frequency bias in inflation perceptions. First I describe the frequency bias in more detail and test whether bread prices are salient for households. Next, I consider that my results could be suffering from two empirical issues: endogeneity and serial correlation. I conduct an instrumental variable analysis and run a differenced regression to address these problems. I will use these results in the second half of the paper when I construct the theoretical model and estimate the welfare loss associated with the frequency bias.

3.1 Fixating on Bread

The frequency bias has a long history in both psychology and economics. Tversky and Kahneman (1973) pioneered the concept the frequency bias by asserting that people judge the probability of events using the availability of evidence that comes to mind. Economists such as Ranyard et al. (2008) argue that when considering beliefs about inflation, availability should be seen as the frequency with which people purchase certain goods. Georganas et al. (2014) provides experimental evidence for the presence of a frequency bias by giving study participants a list of prices and asking them the about inflation rate. They find that people fixate on the prices of goods they encounter frequently when forming beliefs.

Which goods would consumers with a frequency bias overweight? A good that people purchase often and disproportionately focus on when evaluating changes in prices would be a strong candidate. This product should be something that households purchase regularly, but that represents a small component of the household's budget. For example, gasoline may be a good candidate for the frequency bias in the U.S., because many people purchase gasoline on a weekly basis and view posted gasoline prices throughout the day. On the other hand, automobiles are not a good candidate, because people tend to buy a new automobile once every couple of years.

I propose that French households overweight bread prices when evaluating inflation rates. Bread can be purchased on a daily or weekly basis and is a minor share of the annual household budget. According to the Household Budget Survey (HBS), which is a European survey of household purchases for the purpose of constructing expenditure weights, the typical French household spends 0.4% of its income on bread. Although I do not observe how the share of bread in the household budget varies by income in France, bread does not vary as a share of consumption across income in Belgian version of the HBS. Finally, bread can be purchased from a specialized store, i.e. a bakery. Unlike a supermarket, where households may view dozens of prices over the course of one visit, people can view a small number of prices and buy only a handful of items at a bakery, making it easier for people to notice and keep track of prices.

Anecdotal evidence supports the assertion that French people purchase bread regularly. According to the industry group Observatoire du Pain, French people buy bread fairly often and consume it in meals.[44] Their web brochure claims that twelve million households visit a bakery to purchase bread on any given day, which is roughly a fifth of the population of France. This count should be interpreted as a lower bound on the salience of bread in France, because the Bread Observatory organization represents bakeries only, and does not count supermarkets where people can purchase bread.

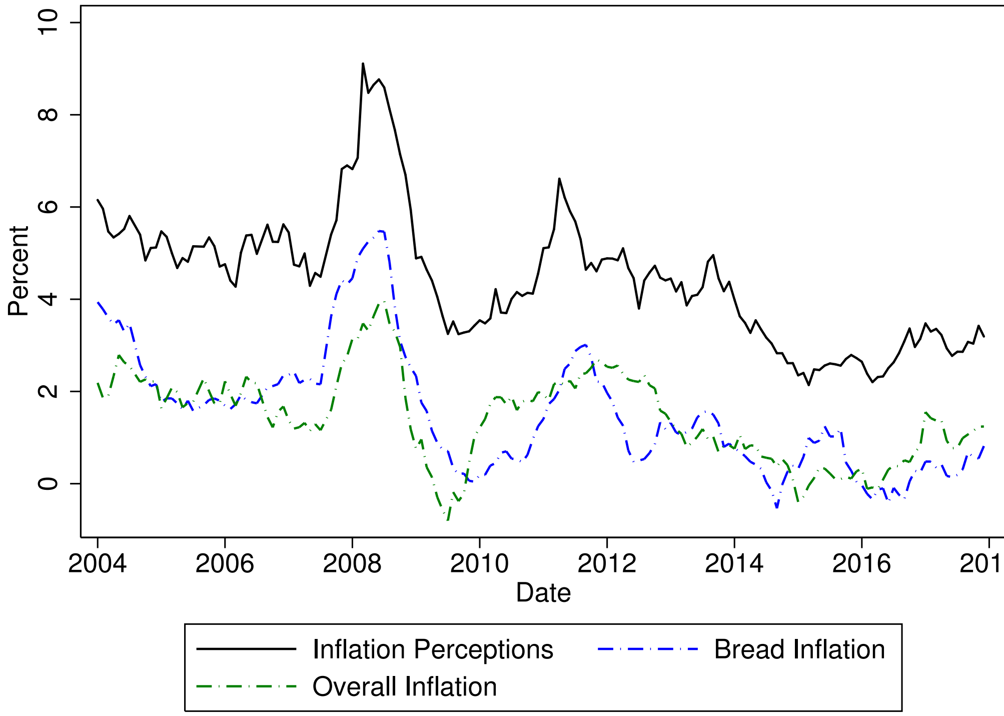
Bread inflation could serve as an excellent, but not perfect, signal for the overall inflation rate. Figure 3 plots the aggregated bread inflation rate against the overall inflation rate at a monthly frequency.[45] Two relevant facts stand out. First, the year-on-year bread inflation rate over the sample is roughly the same as the average overall inflation rate, about 1.7%. The bread inflation rate is the same as the overall inflation rate. Second, the bread inflation rate has a high, but not perfect correlation with the overall inflation rate. The inflation rate for bread does diverge from the overall inflation rate at times.[46]

[44] See Opinion Valley (2016).

[45] Nationally aggregated bread prices come from INSEE's consumer price index statistics. Berardi et al. (2017) tracks the prices of supermarket goods purchased through an online vendor throughout France.

[46] It's possible that households do not suffer from the frequency bias here, but instead are estimating the overall inflation rate from a frequent, noisy signal of bread inflation. The problem is not psychological, but signal extraction.

Figure 3: Bread Inflation Rate



Notes: This figure displays the actual overall inflation rate and the bread inflation rate. The overall inflation rate refers to the inflation rate over the entire consumption basket. These inflation rates are calculated using consumer price index data taken from HICP. Bread refers to the CP0113 category of goods. Inflation rates are calculate as the year-on-year percentage change of the price index. The inflation perception time series is the mean inflation perception for French households in the CAMME survey. Inflation perceptions above 20% are dropped when calculating the mean.

3.2 Frequency Bias Reduced Form

As a first step, I study the effect of changes in the price of bread on how French households perceive overall inflation using a ordinary least squares regression. I regress household inflation perceptions on the actual overall inflation rate, with the addition of the actual bread inflation rate. Let π_t be the overall inflation rate for the household consumption basket and π_t^{Bread} be the inflation rate of aggregate bread prices over the past twelve months. The regression specification is:

$$E_{i,t}\pi_t = \alpha + \beta\pi_t + \gamma\pi_t^{Bread} + \varepsilon_{i,t} \tag{4}$$

Full-information rational expectations, imperfect information, and the frequency bias all make different predictions about the result of this regression. Under full-information rational expectations, the coefficients should be $\beta = 1$ and $\gamma = 0$, because all relevant information contained in bread prices has already been incorporated in the overall inflation rate, π_t.[47] Households should not have any perception errors for the average month, which results in $\alpha = 0$. If households suffer from noisy information, then $\beta < 1$ as they only view past prices through the distorted lens of noisy or sticky information. However, sign of γ is a priori unclear. In contrast, if households overweight bread prices due to the frequency bias, then the coefficient $\gamma > 0$.

Table 3 presents the results and confirms that households put a high importance on bread prices. Column (1) is the baseline regression of inflation perceptions on inflation and demonstrates that inflation affects perceptions. Column (2) compares the effect of the actual overall inflation rate and the overall bread inflation rate. I find that these coefficients are almost identical. The coefficient on bread inflation is statistically significantly larger than 0, which provides evidence against the households having rational expectations. I cannot reject the null hypothesis that the coefficient on bread inflation and overall inflation are the same; households pay attention to both time series.[48]

Is bread special? Households purchase a wide variety of goods on a regular basis and bread does not appear to be an unusual good *a priori*.[49] If changes in bread prices are correlated with changes in the overall food price index, then the results in column (2) would be misleading. This effect would still be an example of the frequency bias, but households would fixate on overall food prices instead of bread specifically. Column (3) introduces a control for the overall food inflation rate. Table 4 reruns the specification of column (2) on other grocery items and goods that are commonly purchased, in order to examine whether other frequently-purchased goods have the same effects. Although households appear to overweight over consumption goods, such as meat, none of the other goods offer same goodness-of-fit and statistical significance as bread.

[47] The high correlation between bread and overall inflation poses a challenge. If the correlation between the two is high, then my analysis may suffer from multi-collinearity, as there is not enough variation in bread inflation that is orthogonal to the overall inflation rate. If this concern is true, then the standard errors on γ should be large and the effect of bread inflation will be statistically insignificant. My results are not biased upwards by a high correlation.

[48] The vertical coefficient in the regression, which I will call "level bias," will be explored in greater detail in the following section.

[49] Armantier et al. (2016b) find that grocery prices in general affect household beliefs about inflation, which suggests that people are looking at food prices in general.

Table 3: Frequency Bias Regressions

	(1)	(2)	(3)	(4)
Overall Inflation	1.271***	0.572***	0.553***	0.640***
	(0.073)	(0.073)	(0.077)	(0.085)
Bread Inflation		0.659***	0.646***	0.699***
		(0.046)	(0.046)	(0.053)
Food Inflation			0.031	-0.023
			(0.034)	(0.041)
Constant	2.601***	2.591***	2.599***	1.958***
	(0.111)	(0.078)	(0.079)	(0.085)
Sample	Full	Full	Full	High Income HH
T-test: Bread Infl. = Overall Infl.		0.437	0.408	0.640
R^2	0.061	0.074	0.074	0.098
Observations	162243	162243	162243	49944

Notes: This table shows the results of regressing household inflation perceptions on the overall inflation rate, the bread inflation rate, and other variables. The sample is French households in the CAMME survey from 2004-2017. High income households refers to households reporting an income above the top quartile of the sample. Standard errors are in parentheses. Errors are clustered by date. ***,**, and * denote the 1, 5 and 10 % significance levels, respectively.

The existing literature on inflation expectations suggests that low-income and low-education individuals tend to have more inaccurate beliefs and are more affected by cognitive biases than high-income and high-education individuals.[50] I test this hypothesis by rerunning my frequency bias regression on the high-income households. Column (4) displays the regression results for only the top quartile of income. I find that the coefficients are mostly unchanged from the overall sample of households: households in my sample focus.

[50] See papers such as Armantier et al. (2016a); Blanchflower and MacCoille (2009); Bruine de Bruin et al. (2010) for examples of income and education effects.

Table 4: Frequently Purchased Goods

	(1)	(2)	(3)	(4)	(5)
Overall Inflation	0.572***	0.805***	1.232***	1.286***	1.865***
	(0.073)	(0.066)	(0.060)	(0.080)	(0.080)
Bread Inflation	0.659***				
	(0.046)				
Meat Inflation		0.538***			
		(0.055)			
Liquor Inflation			0.227***		
			(0.053)		
Tobacco Inflation				-0.006	
				(0.010)	
Gasoline Inflation					-0.084***
					(0.008)
Constant	2.591***	2.330***	2.305***	2.607***	1.944***
	(0.078)	(0.077)	(0.106)	(0.110)	(0.100)
R^2	0.074	0.069	0.064	0.061	0.069
Observation	162243	162243	162243	162243	162243

Notes: This table shows the results of regressing household inflation perceptions on bread and other frequently purchased items. The sample is French households in the CAMME survey from 2004-2017. Standard errors are in parentheses. Errors are clustered by date. ***,**, and * denote the 1, 5 and 10 % significance levels, respectively.

3.3 Endogeneity and Serial Correlation

In this subsection I address two challenges, endogeneity and serial correlation. First, perceptions might not be affected by bread inflation, but rather be driven by some confounding factor. Second, my analysis incorporates time series and may suffer from

3.3.1 Endogeneity

The reduced form results may be affected by two types of endogeneity. First, there may be a confounding variable that affects both French household inflation perceptions and bread prices.[51] Second, these results may reflect reverse causality. If French people believe that prices are rising quickly and substitute towards relatively cheap, low-quality bread, then firms may view this as a demand shock and respond by raising their own prices. In both cases bread prices would not actually affect perceptions.

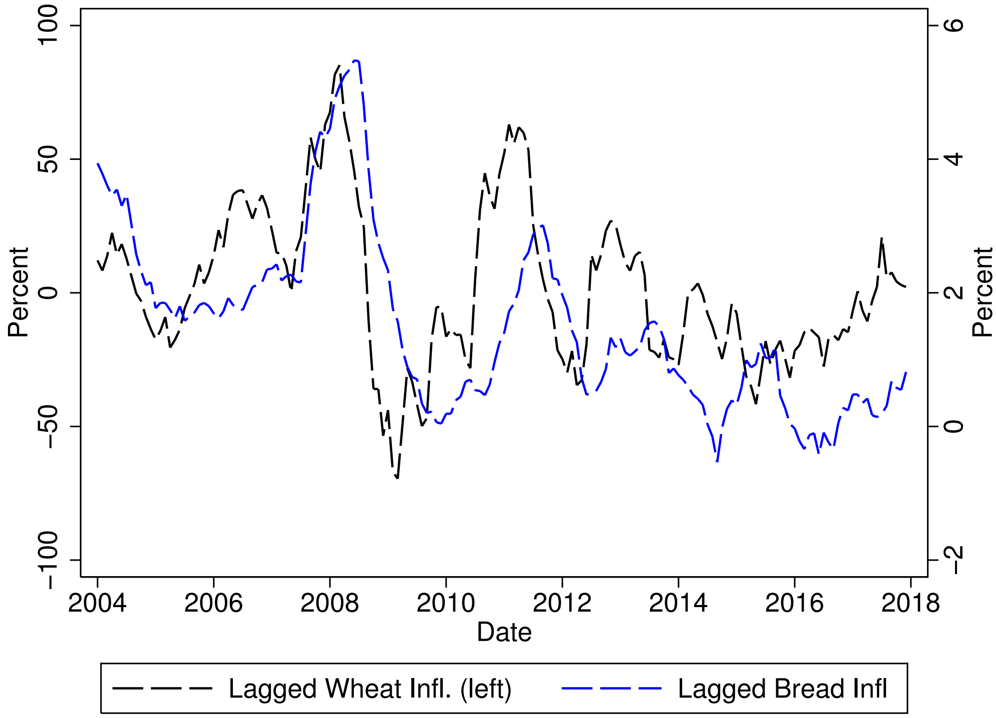
I obtain an instrument in order to solve this endogeneity problem, specifically world wheat prices. These commodity prices are both exogenous to local confounding factors in the French economy and highly correlated with French bread prices, as agricultural ingredients are a key input for making bread. I use the year-on-year percentage change in world wheat prices as a instrument for the year-on-year inflation rate for bread in France. I obtain the monthly world wheat prices[52] from the Chicago Board of Mercantile Exchange. As the Figure 4 shows, the correlation between lagged wheat prices and French bread prices is 0.62, which suggests that global wheat prices affect bread prices.[53] The technique of using commodities prices as exogenous shocks to domestic economic variables is not unique to this paper. Crude oil prices are often used as a proxy, because the pass-through between the input of crude oil to gasoline is quite high (see Shwayder (2012)).

[51] For example, gasoline prices could affect the cost of transporting inputs for businesses and household costs. If gasoline prices rise, then both household perceptions and bread prices would rise as well.

[52] The specific wheat variety that I measure is hard red winter wheat.

[53] The correlation between wheat inflation and French bread inflation is highest when year-on-year wheat inflation is lagged by six months. Throughout the instrumental variable analysis lagged wheat inflation will be used.

Figure 4: Wheat and Bread Inflation



Notes: This figure displays inflation rate for bread in France and the inflation rate for world wheat prices. See the notes of Figure 3 for a description of how bread inflation is calculated. Wheat prices are defined as the price of Hard Red Winter Wheat on the Chicago Mercantile Exchange. Inflation rates are calculated as the year-on-year percentage change in the consumer price index.

Using world wheat prices as an instrument should solve the endogeneity concern, particularly the reverse causation problem. French households are too small to significantly affect world wheat prices. However, commodity prices could be affected by global business cycles, and so could affect inflation perceptions through some real output channel. In this scenario, other macroeconomic variables could confound the analysis and violate the exclusion restriction of instrumental variables. To address this concerns, my instrumental variable analysis incorporates unemployment and real GDP growth statistics.

I conduct a two-stage least-squares regression. The macroeconomic variables unemployment and real GDP growth are denoted by M_t. The second- and first-stages of the regression are, respectively,

$$E_{i,t}\pi_t = \alpha + \beta\pi_t + \gamma\hat{\pi}_t^{Bread} + M_t + \varepsilon_{i,t}. \quad (5)$$

$$\pi_t^{Bread} = \alpha_1 + \beta_1\pi_{t-6}^{Wheat} + u_{i,t}. \quad (6)$$

Table 5 display the results of this instrumental variable regression. Columns (1) and (2) both run the instrumental variable approach. Bread inflation remains a significant determinant of French inflation inflation perceptions, even when I use world wheat prices as an instrument to partial out any confounding factors. The F-statistic for the first stage of the regression is large, with a p-value below 0.001, which indicates that the instrument predicts bread inflation.[54] The coefficient on bread rises from the 0.66 in the ordinary least-squares regression to 0.85 in the instrumental variable approach.[55] French households are actually reacting to changes in the bread inflation rate as opposed to some confounding factor; this relationship is more than a simple correlation.

3.3.2 Serial Correlation

Both the overall inflation rate and the bread inflation rate time series have a high degree of auto-correlation, which could cause the regressions above to suffer from spurious correlation.[56] An Augmented Dickey-Fuller test, as reported in Table 6, fails to reject the null hypotheses that the overall French inflation rate or the bread inflation rate are a random walks. If both of these time series are random walks, then regressing the former on the latter could result in erroneously significant regression coefficients, even if the there is no true relationship between bread inflation and

[54]**?** argue that the F-statistic for the first stage of instrumental variable regressions must be larger than 100 in order for the t-statistic to be accurate. Given that the F-statistic for the first stage is around 31 in column (1), the critical value for the t-statistic at the 5% is level 3.51. Therefore, I can reject the null hypothesis that households place no excess weight on bread inflation when controlling for overall inflation, despite the presence of a possibly weak instrument. I thank Dominic Smith for mentioning this issue.

[55]The increase of the bread coefficient could reflect measurement error in the bread inflation rate. If aggregated bread prices are measured with an error when reported to the HICP dataset, then using an instrument of world commodities prices would help eliminate the error and gives more accurate regression results. The coefficient on bread in the instrumental variable analysis should be seen as more accurate than the result in the ordinary least squares analysis.

[56]Recall that the inflation rates are year-on-year at a monthly frequency. Atkeson and Ohanian (2001) argues that for forecasting purposes, it is best to treat the quarter-on-quarter inflation rate as a random walk.

Table 5: Endogeneity and Serial Correlation Regressions

	(1) $E_t\pi_t$	(2) $E_t\pi_t$	(3) $E_t\pi_t$	(4) $\Delta E_t\pi_t$
Bread Infl.	0.852***	0.861***		
	(0.126)	(0.158)		
Overall Infl.	0.366*	0.413***		
	(0.145)	(0.095)		
Chg. in Infl.			0.405***	0.332***
			(0.103)	(0.091)
Chg. in Bread Inf.			0.495***	0.418***
			(0.087)	(0.100)
Constant	2.588***	1.784	-0.718***	-0.732***
	(0.079)	(1.220)	(0.043)	(0.054)
Macroeconomic Controls	No	Yes	No	Yes
Regression Type	IV	IV	OLS	OLS
First Stage				
F-statistic	31.32	23.21		
R^2	0.073	0.075	0.005	0.005
Observation	162243	162243	29969	29969

Notes: Errors are clustered by date. Standard errors are in parentheses. ***,**, and * denote the 1, 5 and 10 % significance levels, respectively.

the overall inflation rate. As a result, bread prices could appear to influence perceptions when there is no actual effect.

A second problem is that inflation perceptions may also suffer from serial correlation due to the way the survey data is collected. If one household faces a given year-on-year inflation rate π_{2012m1}, then a household in the next month views the year-on-year variable π_{2012m2}. These two inflation rates overlap for 11 months, specifically February 2011 until December 2012, so there is significant correlation between the two. If households view the month-on-month inflation rate with an error, then any perception error will show up over multiple months. As a result, the assumption of uncorrelated perception errors in the ordinary least squares regression is violated and the standard errors are estimated incorrectly.

I resolve the spurious correlation and serial correlation issues by differencing the regression. I use the revolving panel structure of the CAMME survey to study how households update their

Table 6: Dickey-Fuller Test

Inflation Rate	Frequency	Number of Obs	Test Statistic	1% Critical Value	5% Critical Value	10% Critical Value
Bread	Monthly	204	-1.567	-3.475	-2.883	-2.573
Overall	Monthly	205	-2.178	-3.475	-2.883	-2.573
Bread	Quarterly	68	-1.812	-3.555	-2.916	-2.593
Overall	Quarterly	68	-2.282	-3.555	-2.916	-2.593

Notes: This table shows the results of running a Dickey-Fuller test with a trend and one lag on the overall inflation rate and bread inflation rate separately. The Dickey-Fuller test is run on a monthly frequency and a quarterly frequency for comparison. The critical values for the test are displayed in the right-most three columns.

perceptions between their first and third wave responses. The new analysis regresses this perceptions update on changes in the year-on-year inflation rate for overall goods and the inflation rate of bread. In brief, if the inflation rate of bread rises over the course of two months, do we see households raising their inflation perceptions as well? If my previous results were driven by a serial or spurious correlation, then bread inflation should become statistically insignificant. The regression specification is

$$E_{i,t+2}\pi_{t+2} - E_{i,t}\pi_t = \alpha + \beta(\pi_{t+2} - \pi_t) + \gamma(\pi_{t+2}^{Bread} - \pi_t^{Bread}) + (M_{t+2} - M_t) + \varepsilon_{i,t}. \qquad (7)$$

Columns (3) and (4) of Table 5 report the regression results. Bread prices remain a statistically significant determinant of perceptions: an increase in the bread inflation rate by 1 percentage point results in perceptions rising by a bit less than 0.5 percentage points. Incorporating the macroeconomic controls slightly reduces the effect of overall and bread inflation, but does not eliminate it. Bread remains weakly more important that overall inflation, supporting the frequency bias hypothesis. Changes in the bread inflation rate appear to be a larger driver than the overall inflation rate of perceptions, but I cannot reject that null hypothesis that overall inflation and bread inflation have the same effect.[57]

[57]The explanatory power of my analysis is low, because there is considerable heterogeneity in how households update their perceptions, as shown by the low R-square values of the regressions. Households may either form beliefs form their own idiosyncratic experiences (as in Madeira and Zafar (2015); Mankiw and Reis (2002)) or have other

In summary, I have established the presence of a frequency bias related to bread in this section. I have obtained a parameter estimate of 0.861 for the effect of actual bread inflation on inflation perceptions, which I use to parameterize my theoretical model and evaluate how the frequency bias distorts saving and spending decisions for households. Additionally, I have also conducted several regressions to check robustness, which allow me to establish some causality from bread inflation to inflation perceptions.

4 Level Bias

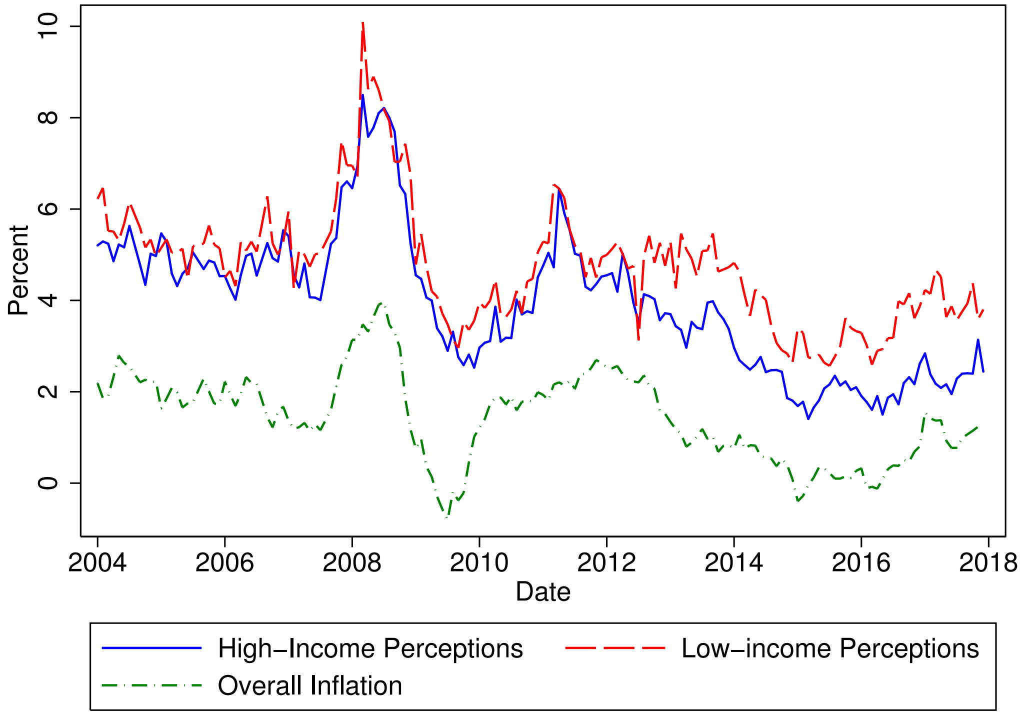
This section begins by measuring the magnitude of the level bias and how it varies by household income. I find that the typical household overestimates the current inflation rate by 2.5 percentage points, and that low-income households overestimate inflation more than high-income. In the second half of the paper, I use these results to parameterize the theoretical model. Next, I study three possible causes for the level bias and how it varies by household income: different consumption baskets, asymmetric loss, and the distribution of households perceptions. My results are: low-income households have similar inflation rates to high-income households; even if households are overweighting price increases relative to decreases, it does not explain the size of the level bias; and median household perceptions are more accurate than mean perceptions, but still overestimate the inflation rate. Ultimately, I am unable to explain the size and cause of the level bias.

4.1 Level Bias Measurement

As Figure 1 in section 2 displays, French households believe that the current inflation rate is significantly higher than it actually is. However, there is significant heterogeneity in how households view inflation. In order to determine the welfare and macroeconomic effects of the level bias, I will estimate the level bias and how it varies by household.

private sources of information I cannot capture. If I run the regression on levels, my R-squared is normally around 0.05, an order of magnitude larger.

Figure 5: Level Bias by Income



Notes: This figure displays the mean inflation perception for low-income households and high-income households in the CAMME survey. Low-income households are defined as those reporting a monthly household income below the bottom quartile of the sample. High-income households are those reporting an income above the top quartile. The overall inflation rate is the year-on-year percentage change in the consumer price index.

Figure 5 depicts the average inflation perception for high-income households compared to low-income households over time, plotted against the actual overall inflation rate. Unlike the level bias, the difference in the level bias by household income varies over time. Prior to the financial crisis both high-income and low-income households drastically overestimate inflation. After the financial crisis high-income households become more accurate and their level bias declines.[58]

Some households may receive more information about changes to the inflation rate than others. As noted in section 2, the average household updates their inflation perception at a roughly one-for-one basis with changes in the actual inflation rate. If high-income households allocate more

[58]It's possible that high-income households have been learning the long-run average inflation rate as time goes on. The introduction of the Euro, which is controlled by the European Central Bank, may have functioned as a regime shock or a sudden change to the monetary policy parameters. Since 2002, French households have had relearn the stochastic process of inflation under this new ECB regime. A European consumer survey conducted over a longer period of time would allow me to study these low-frequency changes.

attention towards inflation or are able to process signals more efficiently, then they would be able to react to changes in the inflation rate more readily. Low income households, in contrast, may receive fewer or noisier signals about the aggregate inflation rate and therefore be more insensitive to any shifts in the inflation rate.

Beliefs about economic variables are associated with demographics beyond income in the literature. Low-education respondents, women, and elderly respondents may consistently over-perceive current inflation or overpredict future inflation.[59] I incorporate these demographic variables in some specifications of the regression analysis in order to examine how household income specifically affects beliefs. The regression specification is

$$E_{i,t}\pi_t = \alpha_{Income} + \beta_{Income}\pi_t + D_i + \varepsilon_{i,t}, \tag{8}$$

where the subscript *Income* denotes the income class of individual i, and D_i contains non-income demographic controls.[60] Table 7 presents the results.

Both low- and high-income households overestimate the current inflation rate and react to it. Column (1) of Table 7 reports the baseline regression without differentiation by income and finds that households overestimate the current inflation rate. Column (2) reports the regression specification above controlling for income without any additional demographic controls. Low-income households over-perceive the current inflation rate by roughly 1.2 percentage points more compared to high-income households: these differences are statistically significant according to the reported t-tests. High-income households are more sensitive than low-income households to past inflation. However, low-income households do respond to the current inflation rate. This parallels the graph above, which shows that low-income households react to changes in the inflation rate, particularly when inflation spikes in 2008.

[59] See papers such as Blanchflower and MacCoille (2009); Bruine de Bruin et al. (2010); Bryan and Venkatu (2001a); D'Acunto et al. (2019a); Johannsen (2014); Jonung and Laidler (1988) for details on how education, gender, and age are correlated with inflation beliefs.

[60] In this regression I limit the sample to households below the first quartile of income and above the 3rd quartile, with the intention of highlighting the role of household income. If I rerun my analysis on the full sample and control for household income quartile I obtain similar results.

Table 7: Level Bias

	(1)	(2)	(3)
Inflation	1.251***		
	(0.017)		
Low-Income		3.152***	3.994***
		(0.046)	(0.092)
High-Income		1.988***	3.294***
		(0.037)	(0.081)
Low-Income X Infl.		1.111***	1.057***
		(0.028)	(0.030)
High-Income X Infl.		1.359***	1.212***
		(0.022)	(0.024)
Constant	2.452***		
	(0.029)		
Demographic Controls	No	No	Yes
T-test:			
Poor constant = Rich constant		0.001>	0.001>
Observation	80409	80409	70369

Notes: This table shows the results of regressing household inflation perceptions on the overall inflation rate. The regressions include fixed effects for income quartile and allow the coefficient on the overall inflation rate to vary by income. High income and low-income households refers to households reporting an income above the top quartile and below the bottom quartile of the sample, respectively. Standard errors are in parentheses. Errors are clustered by date. ***,**, and * denote the 1, 5 and 10 % significance levels, respectively.

The income results remain statistically significant after controlling for other demographics, including demographic variables that could be correlated with income such as education. The level bias for low-income households drops to 2.5%, shrinking the income differential. The rationale for this results is that households that are low-income tend to be low-education as well: the explanatory variables are correlated. Column (2) therefore reports the effect of income status on perceptions; the column combines income with education, to the extent that education is correlated with income.[61]

[61] In column (3), I set the base level for education to be "college-educated," so the results in this column should be interpreted more accurately as the income differential for college-educated households.

4.2 Possible Explanations

In this subsection I posit three possible explanations for the level bias. First, I examine whether income differences in the level bias are due to different consumption baskets for high income households compared to low-income. I find that low-income households have roughly the same inflation rate as high-income households. Second, I study whether households could be reacting more strongly to price increases than decreases. The evidence available suggests that this mechanism is not large enough to explain the level bias. Third, I investigate the role of heterogeneity in beliefs. Although the a substantial minority of households dramatically overestimate inflation, I find that the median households still suffers from the level bias. In summary, I remain agnostic about the cause of the level bias.

4.2.1 Consumption Baskets

If different demographic groups consume different consumption baskets, then they may disagree about the current inflation rate because they actually face different inflation rates in reality. Low-income households might purchase goods from noncompetitive stores with high markups and volatile costs relative to high-income households. Arion et al. (2018) at INSEE investigate this hypothesis by constructing consumer price indices for different income deciles of households. The authors obtain consumption basket weights for different households using the Household Budget Survey (HBS) and use the aggregated price data by consumption good type to construct two consumer price indices that differ by household income of the representative household.[62] The INSEE researchers find that although the inflation rates for low-income and high-income households differ for some months, on average the low-income inflation rate is only 0.1 percentage points higher than the high-income inflation rate. I duplicate their approach and show the results in the first graph of Figure 6.[63] The inflation rate for low-income households rarely deviates significantly from the

[62]Both consumer price indices use the same prices, but differ in the expenditure weight that they attribute to each consumption good or service. Goods that low-income households purchase more than high-income receive a heavier weight for the low-income consumer price index.

[63]My expenditure data is coarser than the data used by the INSEE researchers. They have consumption data for households down to the 4-digit COICOP level, i.e. bread. I have prices at the 3-digit level, i.e. bread and cereals.

inflation rate for high-income households, which suggests that households aren't responding to wildly different inflation rates.[64]

Another possible explanation is that households are reacting to the dispersion of price changes that they face, not the average price change. If households only recall the price of goods that have seen large changes recently, then a rise in inflation dispersion could lead to a higher perceived inflation rate. The second graph of Figure 6 depicts the cross-good volatility of the low-income and high-income households,[65] using the same weights derived from the HBS, where I define the cross-good volatility as

$$V_\pi = \sum_j \omega_j (\pi_j - \bar{\pi})^2,$$

where ω_j is the expenditure weight on good j, $\bar{\pi}$ is the overall inflation rate, and π_j is the inflation rate for good j. The graph indicates that although the low-income households' consumption basket displays a greater dispersion of inflation rates than high-income's, the difference for most periods is negligible. Low-income households are not significantly more likely to encounter large price changes for individual goods than high-income households.[66]

4.2.2 Asymmetric Weighting

Households could be biased towards remember price increases more than price decreases.[67] This would be an example of loss aversion, where the marginal disutility of a negative event is larger than the marginal utility of an equivalent positive event.[68] Using the nationally aggregated data of

[64] It's possible that households are purchasing different goods within each product category or shopping at different retail outlets. Although low-income and high-income households would have the same expenditure shares for different goods categories, they would be facing different inflation rate. Hottman and Monarch (2020) analyzes import prices at a fine level in the U.S. and finds that low-income households do face a higher inflation rate than high-income.

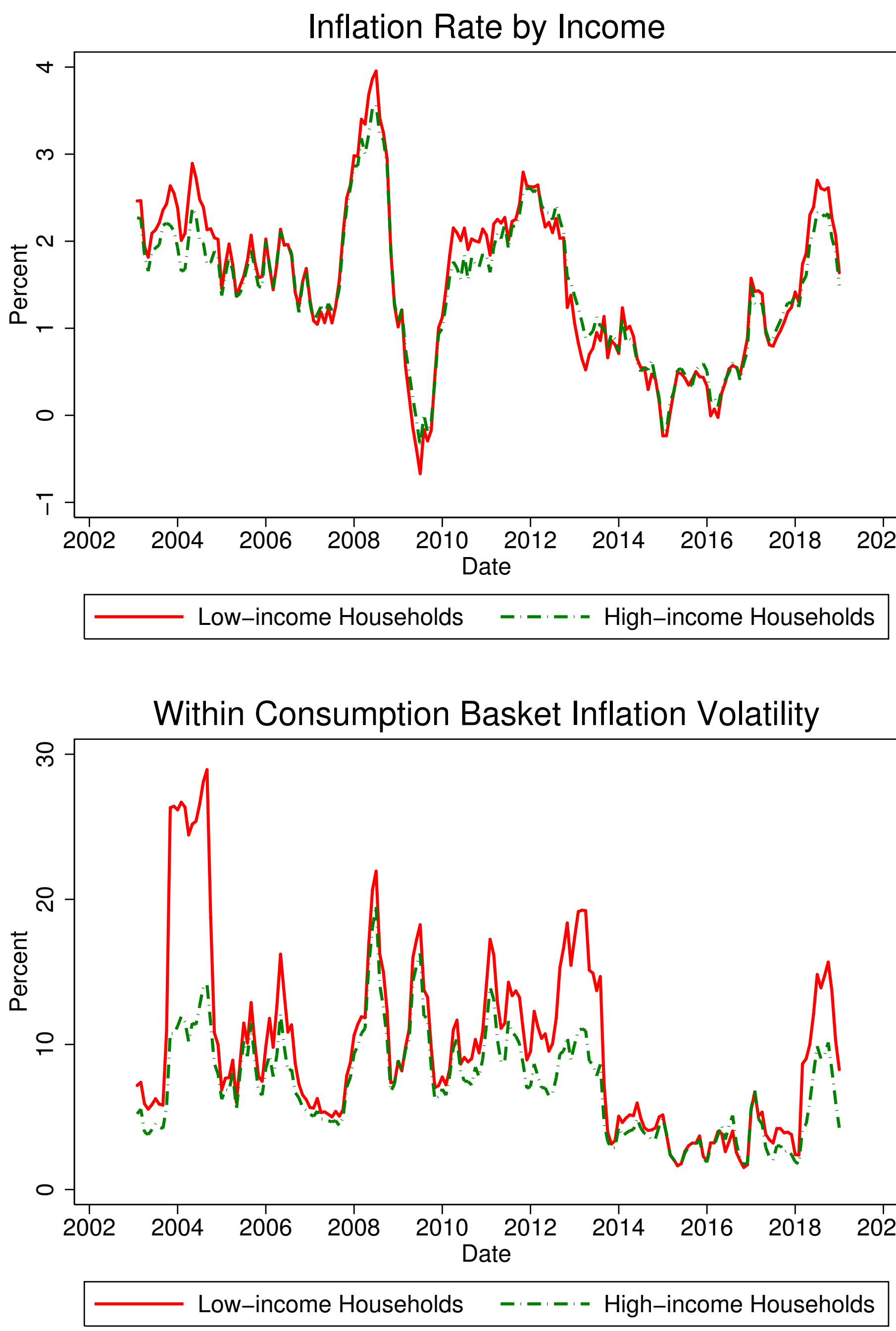
[65] I define volatility across goods instead of across time. For example, if a household only purchases apples and bikes, and the year-on-year inflation rate for both goods is 2%, then the household's cross-good inflation volatility is 0.

[66] A third possible explanation is unlike statistical agencies, households are not performing quality adjustments when perceiving price changes. If high-income households are more savvy at these calculations than low-income households, then these quality adjustments may be driving differences in the level bias.

[67] Stanisławska (2019) examines whether Polish households react more to increases in the inflation rate than decreases. She finds that asymmetric responses to aggregate inflation does not explain their beliefs.

[68] See Kahneman and Tversky (1979) for a description of loss aversion.

Figure 6: Inflation by Income Quartile



Notes: This figure displays the inflation rate and inflation volatility by household income quartile. All calculations use the same inflation rate data from the HICP. Consumption weights on different goods categories vary by household income. Inflation volatility is the cross-sectional volatility of inflation rates across consumption goods categories.

pricing by consumption good category, I construct an extreme version of this loss-averse perceived inflation rate:

$$\pi^{loss} = \frac{1}{\sum_j 1[\pi_j > 0]\omega_j} \sum_j 1[\pi_j > 0]\omega_j \pi_j. \tag{9}$$

π^{loss} is the inflation rate that households would obtain if they only considered prices that increased, and disregarded any prices that stayed the same or decreased. This is equivalent to saying that price increases are infinitely more important, at least psychologically, to households than price decreases, in terms of relative weights. Figure 7 compares π^{loss} to the actual inflation rate and the overall mean inflation perception. The asymmetric inflation rate isn't that much larger than the overall inflation rate: it's rare for specific goods to see price decreases. Since my analysis provides an upper bound for the size of the asymmetric perception, this graph rules out loss aversion as an explanation for the level bias.

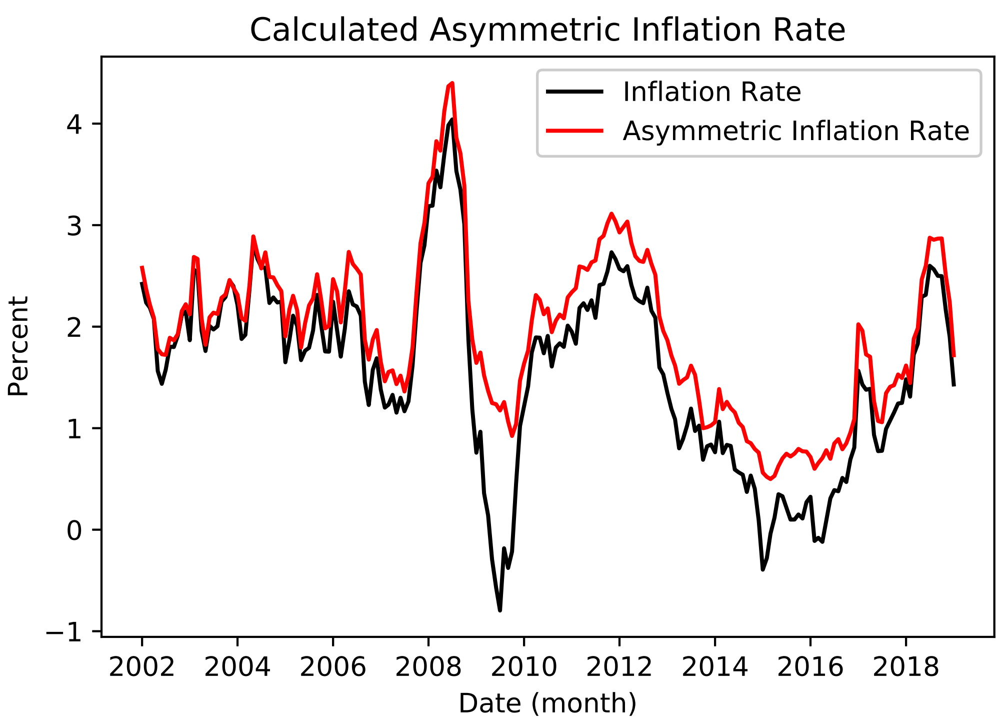
4.2.3 Perceptions Heterogeneity

Households don't have identical inflation perceptions, even within demographic groups. In practice, there is significant disagreement about the current inflation rate, and some households dramatically overestimate the current inflation rate. As a result, the inflation perception of the median household tends to be lower than the inflation perception of the average inflation, as Figure 8 depicts. I infer two results from this figure. First, the level bias is only half as large when evaluating median beliefs, which suggests that households in the right tail are an important component in explaining why the average household overestimates inflation. Second, the median household does overestimate the inflation rate by 1.5 percentage points on average. My results are not being driven by eccentric outliers alone.

To examine household heterogeneity further, I display two histograms of inflation perceptions in Figure 9 of responses.[69] In both histograms a significant fraction of households either believe that prices are stable or that they have changed very little over the past year. However, there is

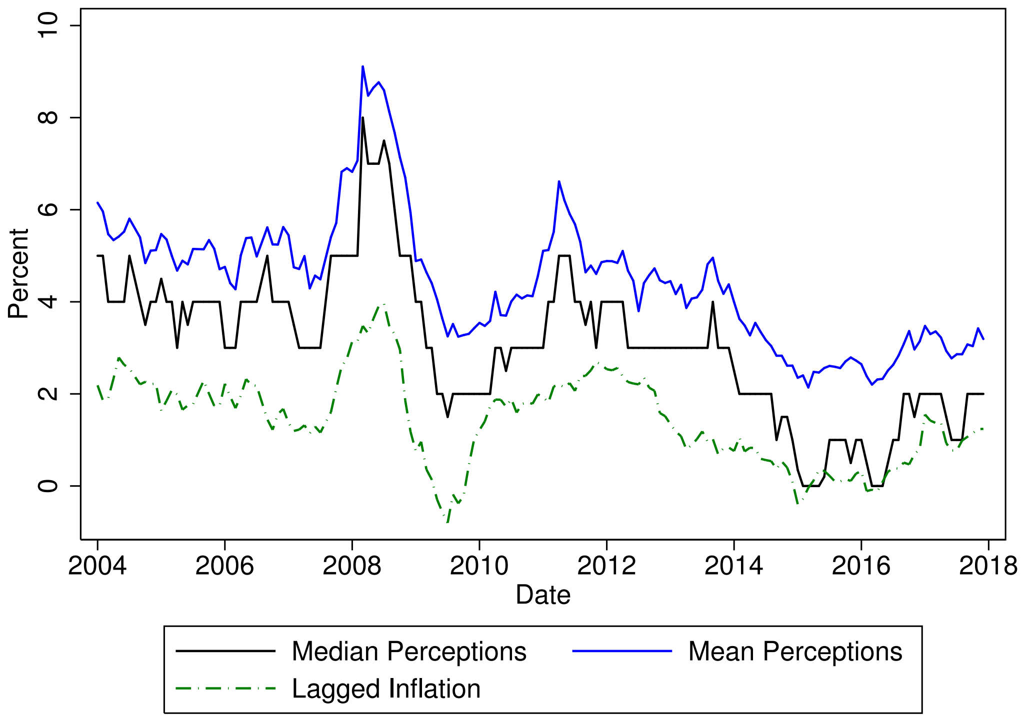
[69] I depict the responses for March 2010 and February 2015. These two dates provide representative cross-sections of household beliefs at a given point in time.

Figure 7: Asymmetric Inflation Rate



Notes: This figure displays the overall inflation rate for French consumers and a weighted, asymmetric inflation rate. The asymmetric inflation rate is the average inflation rate for all goods categories where prices increase, weighted by household consumption. Household consumption weights come from the HBS and the inflation rates of different goods categories come from the HICP.

Figure 8: Median Inflation Perceptions

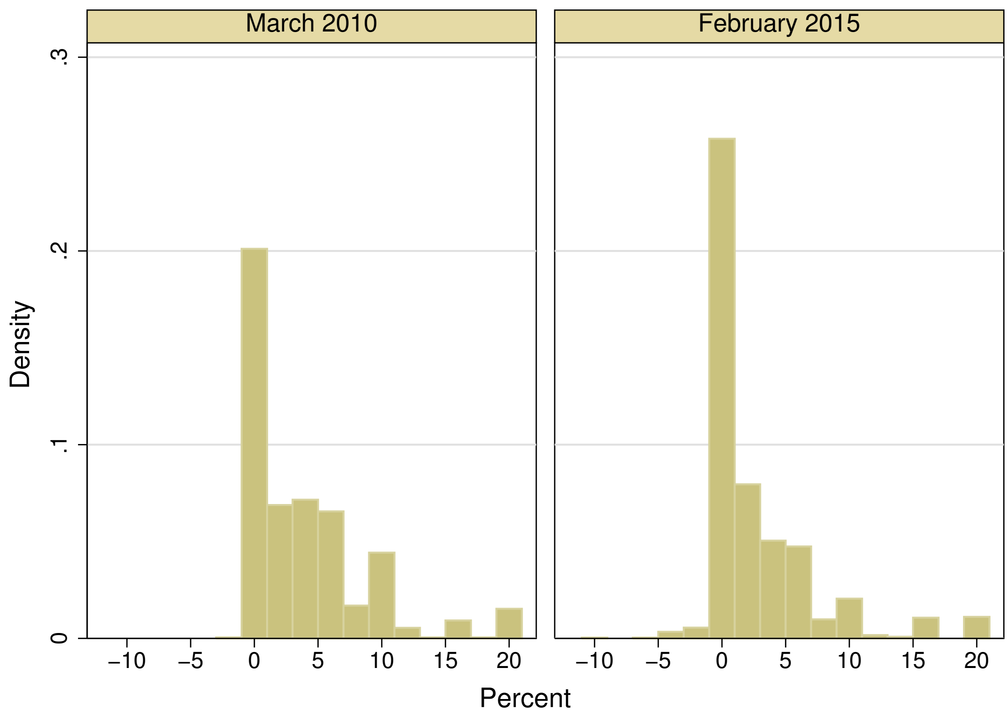


Notes: The figure depicts the mean inflation perception and the median perception expectation for households in the CAMME survey at a monthly frequency. The actual inflation rate is calculated as the year-on-year percentage change in the HICP consumer price index. Inflation perceptions above 20% are dropped when calculating the mean and median.

substantial disagreement on the current inflation rate, and very few households list the correct answer. Both distributions have a large positive skew thick right tail, which drives a lot of the result. Further research should be conducted into how households in the right tail form beliefs and whether they act on them in reality.

In summary, households consistently overperceive the inflation rate over the past twelve months, regardless of economic circumstances. This level bias varies by the income status of the respondent. Low-income households overperceive past inflation to a greater degree than high-income households, even when controlling for other demographic characteristics. The average size of this level bias is 2.45 for households overall. For low-income households the level bias is 3.15, while for high-income households it is 1.99. In the following sections I use these results to estimate the welfare implications in following sections.

Figure 9: Histogram of Responses



Notes: The figure depicts two histograms of perceived inflation responses from the CAMME survey for two dates, March 2010 and February 2015. The bins have a width of 2 percentage points and the endpoints are even integers. The density of all bins sum to 1 in each histogram. All observations larger than 20% are dropped.

5 Model

In this section, I build a partial equilibrium household that can incorporate biased inflation perceptions. The purpose of the model is to evaluate the welfare loss of the two perceptions biases. The model answers three questions. First, do the frequency bias and the level bias have significant negative effects on households? Second, which is worse? Third, what is the intuition for my results?

The model's key feature is that households do not know the current inflation rate due to their perceptions biases. Households also use their biased perceptions to forecast future inflation, which results in non-rational inflation expectations. Biased households choose their level of savings as if their perception of inflation are correct, but are subject to the actual inflation rate each period.

This section begins by describing the basic components of the household problem. I then set up the households' nominal and real budget constraints. The biased inflation beliefs play a crucial rule in how households view their budget constraint. Next, I describe the stochastic process for inflation and idiosyncratic income. I then write the recursive formulation for households with rational expectations and for households with biased inflation perceptions. I conclude by discussing how I model the frequency bias and the level bias.

5.1 Model Setup

I use a partial equilibrium model with a continuum of infinitely-lived households. Households receive an idiosyncratic stochastic endowment Y. Households maximize expected utility, which is time-separable and discounted at a constant rate β. The households have a per-period utility function

$$U(C_t) = \frac{C_t^{1-\sigma}}{1-\sigma}.$$

Households purchase consumption goods from two sectors, $C_{1,t}$ and $C_{2,t}$. I incorporate the two sectors to model the frequency bias. In the context of this paper, one can think of the two sectors as the "bread" sector and the "not-bread" sector. Households aggregate consumption goods using a Cobb-Douglas function to obtain total consumption,

$$C_t = C_{1,t}^{\gamma} C_{2,t}^{1-\gamma}.$$

Households face a two-step problem each period. First, households decide how much of their nominal wealth and income to spend on consumption goods and how to much to save for future periods. Seconds, households allocate their consumption spending between the two sectors. Prices for the consumption goods in both sectors are exogenous to households and the goods are supplied elastically. The household consumption allocation problem results in a general price index, $P_t = P_{1,t}^{\gamma} P_{2,t}^{1-\gamma}$, which is the cost of purchasing one unit of composite consumption C_t.[70] The inflation rate between period $t-1$ and period t is

$$\pi_t = \frac{P_t}{P_{t-1}} = \frac{P_{1,t}^{\gamma} P_{2,t}^{1-\gamma}}{P_{1,t-1}^{\gamma} P_{2,t-1}^{1-\gamma}} = \pi_{1,t}^{\gamma} \pi_{2,t}^{1-\gamma}.$$

5.2 Asset Decision

At the start of each period households know their nominal income and their nominal assets perfectly. Households choose how much of their nominal wealth to allocate towards purchasing consumption goods and how much to save. Households save by purchasing nominal bonds A_t^N each period, which pay a constant exogenous nominal interest rate $(1+i)$ at the start of the next period. I assume that agents cannot issue their own nominal bonds, so that they are subject to a no-borrowing constraint. Let C_t^N denote the household's total expenditure on consumption goods and let Y_t^N denote the household's nominal idiosyncratic income. The household's budget constraint in nominal terms at time t is

[70]Throughout the rest of the section I will focus on the composite consumption good C_t and the first stage of the household problem where they choose how much to spend on consumption goods.

$$
\begin{aligned}
C_t^N + A_t^N =& (1+i)A_{t-1}^N + Y_t^N \\
\text{s.t. } A_t^N \geq& 0.
\end{aligned} \tag{10}
$$

Although households' budget constraint and choice variables are nominal, they seek to maximize utility through real consumption. Households choose how much to spend on consumption goods C_t^N, even though they may not know P_t accurately. Using the general price index P_t, I rewrite the nominal budget constraint in real terms and define the variable $A_t = \frac{A_t^N}{P_t}$ as the household's purchase of price deflated bonds. The real return to price deflated bonds that mature in period t is $\frac{(1+i)}{\pi_t}$. Households' real consumption is therefore $C_t = \frac{C_t^N}{P_t}$, which means that households are concerned about the true price level.

Households may have non-rational expectations. I denote the expectations operator for households as E_t^* and assume that all households have the same biases and homogeneous beliefs. For example, the term $E_t^*[\pi_t]$ denotes the households' belief about the current inflation rate at time t. If households have rational expectations, then E_t^* corresponds to the objective expectations operator E_t.[71]

A household at time t perceives its real budget constraint as

$$
\begin{aligned}
C_t + A_t =& \frac{(1+i)A_{t-1}}{E_t^*[\pi_t]} + Y_t \frac{\pi}{E^*[\pi_t]} \\
\text{s.t. } A_t \geq& 0.
\end{aligned} \tag{11}
$$

This budget constraint depends on the perception of inflation $E_t^*[\pi_t]$. It may not be the same as the households' actual budget constraint. For example, a household believes that prices have fallen by half over the past period, when in fact the inflation rate has been 0%. In this case, $E_t^*[\pi_t] = 0.5$,

[71] Here, I use rational expectations to refer to households that correctly perceive the current inflation rates and forecast future inflation without any biases.

but $\pi_t = 1$. If the household had saved a large amount of assets A_{t-1}, then it would incorrectly believe that its financial wealth could purchase more real consumption goods C_t than it actually could.

The actual real budget constraint for households each period is

$$\begin{aligned} C_t + A_t =& \frac{(1+i)A_{t-1}}{\pi_t} + Y_t \\ \text{s.t. } A_t \geq& 0. \end{aligned} \tag{12}$$

I refer the households perceived budget constraint (as in equation 11) as its "subjective" budget constraint. In contrast, I refer to the actual real budget constraint as its "objective" budget constraint. The objective budget constraint is the budget constraint that an outside econometrician or a household with rational expectations would observe. This terminology will carry over to the households' policy and value functions. Note that when households purchase consumption goods they are governed by the objective price index, which means that they receive utility from their objective consumption $C_t = \frac{C_t^N}{P_t}$ rather than their subjective consumption $\tilde{C}_t = \frac{C_t^N}{E_t^*[P_t]}$. For the remainder of the paper, I will denote any subjective variables with a tilde.[72]

Let the set $S = (\pi_1, \pi_2, Y)$ denote the exogenous states, where (π_1, π_2, Y) follow a stochastic process to be described in subsection 5.3.[73] Households forecast the stochastic process using their biased perceptions, which is also described in subsection 5.3. From household's perspective and beliefs E_t^*, the recursive formulation of its problem is

[72]The purpose of constructing the model in this fashion is to ensure that households never violate their budget constraint accidentally. For example, if households incorrectly believe that there has been significant deflation and that their real wealth is quite large, then will allocate more of their nominal wealth towards consumption, but still fulfill their nominal budget constraint. If households chose real consumption in the model instead, then they might purchase more consumption goods then they could afford, which would violate the budget constraint.

The derivations above depend on several subtle assumptions about households, particularly regarding how they view uncertainty. Appendix D contains a more complete description of how to derive the subjective real budget constraint in equation 11 from the nominal budget constraint in equation 10.

[73]The inflation variables π_1 and π_2 are aggregate economic variables, while Y is household specific.

$$
\begin{aligned}
\tilde{V}(S,A) =& \max_{\tilde{A}',\tilde{C}} u(\tilde{C}) + \beta E^*[\tilde{V}(S',A')|S] \\
\text{s.t. } \tilde{C} + \tilde{A}' =& \frac{(1+i)}{E^*[\pi]} A + Y \frac{\pi}{E^*[\pi]}.
\end{aligned} \tag{13}
$$

5.3 Inflation and Income Processes

The aggregate inflation rate is

$$
\pi_t = \pi_{1,t}^{\gamma} \pi_{2,t}^{1-\gamma} = \frac{P_{1,t}^{\gamma} P_{2,t}^{1-\gamma}}{P_{1,t-1}^{\gamma} P_{2,t-1}^{1-\gamma}}, \tag{14}
$$

where $\pi_{i,t}$ is the inflation rate for goods sector i in period t. π_1, π_2 follow a joint vector autoregressive process with one lag. Let $\overrightarrow{\pi} = (\pi_1, \pi_2)$. Then

$$
\overrightarrow{\pi}_t = \alpha + A \overrightarrow{\pi}_{t-1} + \varepsilon_t,\, \varepsilon_t \sim N(0,V). \tag{15}
$$

If the two inflation sectors have correlated shocks, then the variance-covariance matrix V is not diagonal. I rewrite the time series process for inflation rates as terms of deviations from means. I assume that households are aware of the vector autoregressive structure of the inflation rates and use it to make forecasts of future inflation. For example, if a household believes that the inflation rates for the two sectors at time t is $E_t^*[\overrightarrow{\pi}_t]$, then its forecast for the two inflation rates for time $t+1$ is

$$
E_t^*[\overrightarrow{\pi}_{t+1}] = \alpha + A E_t^*[\overrightarrow{\pi}_t].
$$

The logarithm of household labor income is the sum of a persistent component and a temporary component, such that $\ln Y_{i,t} = \ln Y_{i,t}^P + \ln Y_{i,t}^T$. The temporary income component is i.i.d. white noise following a normal distribution $N(0, \sigma_T^2)$. The permanent income component follows an AR(1) process, such that

$$\ln(Y^P_{i,t}) = \rho_Y \ln(Y^P_{i,t-1}) + \varepsilon_{i,t}. \tag{16}$$

5.4 Recursive Formulation

I write the recursive formulation households' problem under two specifications. First, I describe the recursive problem if the households have rational expectations and accurate inflation perceptions. I then describe the problem under a generic inflation perception bias E^*_t.

5.4.1 Rational Expectations Setup

In the rational expectations households' problem, they know the true overall inflation rate from the previous period and have rational expectations about future inflation. All of the households subjective variables, i.e. their planned real consumption, are the same as the objective variables. Agents know the purchasing power of their nominal wealth each period.

The households' problem is

$$\begin{aligned} V^{RE}(S,A) =& \max_{A',C} u(C) + \beta E[V^{RE}(S',A')|S] \qquad (17)\\ s.t.\, C + A' =& Y + \frac{(1+i)}{\pi} A,\, A' \geq 0 \\ \pi =& \pi_1^{\gamma} \pi_2^{1-\gamma}. \end{aligned}$$

Inflation only enters the households' optimization problem to the extent that it affects the expected ex-ante real return to savings next period, $E_t\left[\frac{(1+i)}{\pi_{t+1}}\right]$. I define the policy function for households' with rational expectations as their optimal level of savings,

$$g_{RE}(S,A) \equiv argmax_{A'} u(C) + \beta E[V^{RE}(S',A')|S], \tag{18}$$

such that the budget constraint holds.

5.4.2 Biased Belief Setup

Households with biased perceptions optimize a misspecified model of the economy. The households do not realize that they have incorrect perceptions about inflation, so they act as if their beliefs are correct.[74] For a generic belief operator E^*, the biased households' subjective problem is

$$\tilde{V}(S,A) = \max_{\tilde{A}',\tilde{C}} u(\tilde{C}) + \beta E^*[\tilde{V}(S',\tilde{A}')|S] \tag{19}$$
$$s.t. \tilde{C} + \tilde{A}' = Y + \frac{(1+i)}{E^*[\pi]} A, \tilde{A}' \geq 0.$$

If households misperceive inflation, then they may purchase more or fewer composite consumption goods C than planned. In the two-stage problem households choose their nominal expenditure on consumption first. Once a household decides how to allocate its nominal wealth between consumption goods C^N and A^N, it goes to the consumption market and purchase goods using the true, objective price level P. If the household underestimated the price level, then it receives fewer consumption goods then it planned and $C < \tilde{C}$. In general, the expression relating subjective, planned consumption to objective, actual consumption is

$$C = \frac{E^*[\pi]}{\pi} \tilde{C}. \tag{20}$$

This intuition also applies towards purchases of price deflated bonds A. If a household's inflation perceptions are biased, then $A' \neq \tilde{A}'$. The relationship between the two is

$$A' = \frac{E^*[\pi]}{\pi} \tilde{A}'.$$

The biased households solve their subjective optimization problem to obtain a policy function

[74] If households were aware that they had biased beliefs, then presumably they could correct for the biases and return to rational expectations.

for optimal bonds savings A'. The policy function is

$$\tilde{g}(S,A) = argmax_{\tilde{A}'} u(\tilde{C}) + \beta E^*[\tilde{V}(S', \tilde{A}')|S], \, s.t. B.C. \text{hold.} \tag{21}$$

In terms of the biased household's policy function, the relationship between the objective policy function and the subjective policy function is

$$g(S,A) = \frac{E^*[\pi]}{\pi} \tilde{g}(S,A). \tag{22}$$

In summary, the key components of the model are the households planned savings $\tilde{g}(S,A)$, their actual savings policy $g(S,A)$, and their subjective value function $\tilde{v}(S,A)$.

5.5 Perception Biases

This framework is general enough to allow comparisons of different belief systems, specifically rational expectations, the frequency bias, and the level bias. The subsections below discuses the setup of the perceptions biases.

5.5.1 Frequency Bias Beliefs

Households with the frequency bias overweight price movements in one sector relative to its importance. Let sector 1 be the sector that household fixate on with the frequency bias, so that γ is objectively small. If households have rational expectations, then they pay little attention to the inflation rate π_1 when perceiving or forecasting inflation.

Households suffering from the frequency bias correctly measure π_1 and π_2. However, they place too much weight on the first sector and mis-aggregate these inflation rates according to the process

$$E_t^{FR}[\pi_t] = \tilde{\pi}_t = \pi_{1,t}^{\tilde{\gamma}} \pi_{2,t}^{1-\tilde{\gamma}}, \, \tilde{\gamma} > \gamma. \tag{23}$$

The perceived inflation rate can deviate from the actual current inflation rate. If inflation in the frequency biased sector is higher (lower), i.e. $\pi_1 > \pi_2$ ($\pi_1 < \pi_2$), then $\tilde{\pi} > \pi$ ($\tilde{\pi} < \pi$). Although the frequency bias distorts the belief of the inflation rate between periods, households still solve their intra-period consumption allocation sub-problem optimally.[75] Let $\tilde{V}^{FR}(S,A)$ be the subjective value function for households with the frequency bias, i.e. the value function they would have if their beliefs were correct.[76] Households with the frequency bias choose an optimal savings policy to solve

$$\begin{aligned} \tilde{g}^{FR}(S,A) =& argmax_{\tilde{A}'} u(\tilde{C}) + \beta E^{FR}[\tilde{V}^{FR}(S',\tilde{A}')|S] \\ \text{s.t. } \tilde{C}+\tilde{A}' =& Y + \frac{(1+i)A}{\tilde{\pi}}, \tilde{A}' \geq 0. \end{aligned} \tag{24}$$

5.5.2 Level Bias Beliefs

Households suffering from the level bias misperceive the inflation rates of both sectors. They both overestimate the current inflation rate of both sectors by some constant c and believe that the average, long-run inflation rate is higher than it actually is. The households understand the other parameters of the vector autoregressive process governing inflation to the extent that they accurately know the persistence and volatility of inflation in the two sectors. They believe that the current inflation rates of the two sectors are

$$E_t^{LB}[\pi_{i,t}] = \pi_{i,t} + c. \tag{25}$$

Households with the level bias choose a subjectively optimal savings policy to solve

[75]Households suffering from the frequency bias do not consume vast quantities of bread because they erroneously believe that it's a huge component of their consumption basket. The frequency bias only affects the first step of their two-step problem, where households choose how much wealth to expend on consumption. In the second step, where households allocate consumption between the two sectors, they observe the current prices and inflation rates accurately.

[76]V^{FR} is a special case of the value function V^* defined above.

$$\tilde{g}^{LB}(S,A) = argmax_{\tilde{A}'} u(\tilde{C}) + \beta E^{LB}[\tilde{V}^{LB}(S',\tilde{A})|S] \tag{26}$$
$$\text{s.t. } \tilde{C} + \tilde{A}' = Y + \frac{(1+i)A}{E^{LB}[\pi]}, \tilde{A}' \geq 0.$$

6 Model Parameterization

6.1 Preferences

The model period is quarterly. I set the parameter of risk aversion to be $\sigma = 2$, which I obtain from Rozsypal and Schlafmann (2019). I set $(1+i) = 1.015$ to match the quarterly household bank deposit rate over the 2007-2017 time period according to Banque Du France. I calibrate the discount rate β in order to match the household wealth to household income ratio in France of 4.125.[77] The Household Budget Survey suggests that the typical French household spends about 0.4% of their income on bread, so I set the parameter for aggregating the sectors as $\gamma = 0.01$. The level bias parameter c is set such that households overestimate the inflation rate of each sector by 2.5% annually.

The logarithm of idiosyncratic labor income has a temporary component and a persistent component, such that $\ln Y_t = \ln Y_t^P + \ln Y_t^T$. The persistent component follows a logarithmic AR(1) process, $\ln Y_t^P = \rho_Y \ln Y_{t-1}^P + \varepsilon_{Y,t}$, $\varepsilon_{Y,t} \sim N(0, \sigma_P^2)$. I set the $\sigma_P = 0.042$, $\sigma_T = 0.1$, and $\rho = 0.9774$, which I take from Rozsypal and Schlafmann (2019).

6.2 Inflation and Biases

The inflation rates for the two sectors π_1 and π_2 follow a vector autoregression. The idiosyncratic labor income is uncorrelated with the aggregate inflation rates. I estimate the inflation process described in subsection 5.3 using quarterly inflation data from France for bread inflation π^{Bread}

[77]This ratio comes from the OECD's Better Life Initiative and represents households in 2017.

and overall inflation π at a quarterly frequency. From this analysis I obtain both the persistence of the inflation rates and the correlation of their shocks.

I derive the parameters for the frequency bias and level bias from the empirical regressions in sections 3 and 4. I set the parameter governing the frequency bias to be $\tilde{\gamma} = 0.676$, which is the size of the coefficient of bread inflation relative overall inflation in my instrumental variable analysis.[78] The parameter governing the level bias on a quarterly frequency is $c = 2.5/4 = 0.625$, which is the level bias I measure in the regressions.

7 Results

Using the model setup of section 5 and the parameterization of section 6, I calculate the objective policy functions for households with rational expectations, the frequency bias, and the level bias, which are denoted as $g(S,A)$, $g^{FR}(S,A)$, and $g^{LB}(S,A)$ respectively.[79] In the following subsection, I discuss deriving the value function of households following a suboptimal policy and estimating the welfare loss of following an objectively suboptimal policy. Next, I present the main welfare results for the paper. The section concludes by presenting three extensions in order to explore the intuition and the macroeconomic implications.

7.1 Welfare Loss Calculation

Households with perception biases believe that their value function for a given state is $\tilde{V}(S,A)$, as depicted in equation 19. However, these households misperceive the actual inflation rate and as a result consume more or fewer composite goods than they plan. The value function $V(S,A)$ that households actually obtain, governed by their income and the true inflation rate, is

$$V(S,A) = u\left(\frac{(1+i)A}{\pi} + Y - \frac{E[\pi]}{\pi}\tilde{g}(S,A)\right) + \beta E[V(S', \frac{E[\pi]}{\pi}\tilde{g}(S,A))|S]. \tag{27}$$

[78] If consumers are overweighting a variety sectors, of which bread is the most prominent, then this parameter may be viewed as how much households are overweighting non-representative goods when considering the inflation rate.

[79] See Appendix C for a discussion of the solution methodology.

From the perspective of an outside observer or someone with rational expectations, these biased households are implementing objectively suboptimal saving policies $\frac{E[\pi]}{\pi}\tilde{g}(S,A)$ instead of $g^{RE}(S,A)$. For any state and asset combination (S,A),

$$V^{RE}(S,A) > V(S,A).$$

Because value functions and utilities are ordinal, I cannot simply divide one value function by another to obtain a reliable measure of welfare loss. I apply the standard consumption equivalence method of Lucas (1987).[80] In my setup, this approach looks like

$$E\sum_{t=0}^{\infty}\beta^t\frac{C_t^{*1-\sigma}}{1-\sigma} = E\sum_{t=0}^{\infty}\beta^t\frac{(C_t^{RE}(1-\lambda))^{1-\sigma}}{1-\sigma},$$

where the consumption C_t^* on the left-hand side is the expected future discounted consumption under a perception bias and the consumption C_t^{RE} is consumption under the savings policy formed using rational expectations. For a given combination (S,A) of exogenous states and asset holdings, the fraction of consumption that households would be willing to pay to have rational expectations is

$$\lambda = 1 - \left[\frac{V^*(S,A)}{V^{RE}(S,A)}\right]^{\frac{1}{1-\sigma}}, \tag{28}$$

where $V^*(S,A)$ corresponds to the objective value function for households with the frequency bias or the level bias. The amount of consumption that households would be willing to forgo depends on the state of the households (S,A), which raises the question of how to evaluate welfare loss for households overall.

To facilitate comparison, I analyze the welfare loss at the rational expectations steady state where the inflation rates for both sectors are at their long-run averages.[81] Individual households

[80] Lucas (1987) uses the CRRA utility function to measure what fraction of consumption households would pay each period to avoid business cycles.

[81] The stationary distribution of assets held by households depends on their inflation perceptions. In the long-run, biased inflation perceptions cause households to accumulate the incorrect level of assets: for example, if agents persistently overestimate the level of inflation, then they will tend to save less than households with rational expectations

may still have differing idiosyncratic incomes in the steady state. In brief, I ask "if agents wake up one day and transition from rational expectations to a bias (either the frequency or the level bias), how much consumption would they be willing to sacrifice to avert this?"

The stationary distribution of assets under rational expectations can be calculated by setting an initial distribution of assets A_0, and then iterating forward using the policy rule $g^{RE}(S,A)$ until convergence towards a distribution $\overrightarrow{A}_{SS}$. Because agents are subject to idiosyncratic income shocks each period, this distribution is nondegenerate.

In addition to welfare loss, I will estimate two additional metrics to provide intuition for the results. First, I sum up the distribution to find the total assets held in the economy in the scenario that all households have the level bias or the scenario that all households have the frequency bias. The asset holdings result will provide intuition for the welfare analysis and help inform the policy implications. Second, I will find the mean squared error for forecasts of future inflation for households with the level bias and frequency bias, which will provide context for how inaccurate the households' beliefs are.

7.2 Main Result

Table 8 displays the primary results of this paper. The welfare loss for the frequency bias is relatively small, roughly 0.04% of consumption. For comparison, Otrok (2001) estimate that the welfare loss attributable consumption volatility in the U.S. equivalent to 0.0044% of consumption per year. The magnitude of the welfare loss indicates that the frequency bias is not very damaging to households in the economy, and that overweighting bread can serve as a decent second-best approach. In contrast, the welfare loss of the level bias is an order of magnitude larger, almost 0.1% of consumption each period. The difference in welfare results may be driven by the steady state asset-holding behavior. Agents with rational expectations and the frequency bias hold roughly the same level of assets in the steady state, despite the fact that they are subject to different subjective

and will converge to a lower level of asset holdings. In this case, the distribution of asset holdings will be first-order stochastically dominated by the asset distribution under rational expectations.

Table 8: Perception Biases Welfare Results

	Welfare Loss	Assets Held	Mean-Squared Forecast Error
Rational Expectations		4.125	0.0045
Frequency Bias	0.0433%	4.979	0.0057
Level Bias	0.2379%	0.356	0.0254

Notes: This table shows the welfare effects for households suffering from the frequency bias only or the level bias only. Assets held refers to the total assets held by households in the steady state. The mean-squared forecast error is the forecast error for households in the model that are suffering from none or some of the biases. See the text for more details.

inflation stochastic processes. There are two reasons for this: first, the overall volatility of the inflation rate is similar for both types of agents, and they both seek to hedge against the same labor income risk.[82] Another reason that the welfare loss is so small is that the frequency bias is really damaging if the bread inflation rate substantially differs from the overall inflation rate. In practice, bread is highly correlated with the overall inflation rate, so this event rarely occurs.[83]

In contrast to the frequency bias, households with the level bias act quite differently. Households with the level bias save a tenth as many assets as households with rational expectations or the frequency bias. Because agents with the level bias persistently estimate that inflation will be 2.5 percentage points higher than rational expectations agents on an annual basis, they believe the real rate of return is low and save dramatically less. Due to the no-borrowing constraint and the risky labor income, households with the level bias still wish to smooth out consumption volatility and accumulate precautionary savings against a large, persistent, negative shock to labor income. Because they sub-optimally undersave, the welfare effects of the level bias are large.

Households with the frequency bias have a relatively accurate forecast of future inflation, especially compared to the level bias.[84] The frequency bias only affects forecasts whenever the current inflation rate of bread deviates significantly from the overall inflation rate, which rarely occurs. Additionally, because inflation follows an auto-regressive process, any shocks to inflation in the

[82] Households with the frequency bias may increase their savings when the inflation rate for bread (the overweighted sector) is low, believing incorrectly that the future real interest rate will be high.

[83] The welfare loss of for households with both biases is less than the sum of the welfare losses of each bias individually. Households suffering from the frequency bias save slightly more in the steady state than agents with rational expectations, which counteracts the dissavings effect of the level bias slightly.

[84] Rather than derive the analytical expressions for the mean-squared error, I run a Monte Carlo simulation of each expectations method assuming that inflation does follow the VAR(1) process described in the Model section.

current period diminish by the next period. The level bias results in the largest forecast error, because households consistently overestimate the inflation rate each period, regardless of the state of the world.

7.3 Extensions

The welfare and savings results above raise a series of questions about their cause and impact. How much does it matter that bread inflation is highly correlated with the overall inflation rate? What is the role of inflation perceptions as opposed to expectations? Does the level bias affect the economy if high-income households have more accurate beliefs?

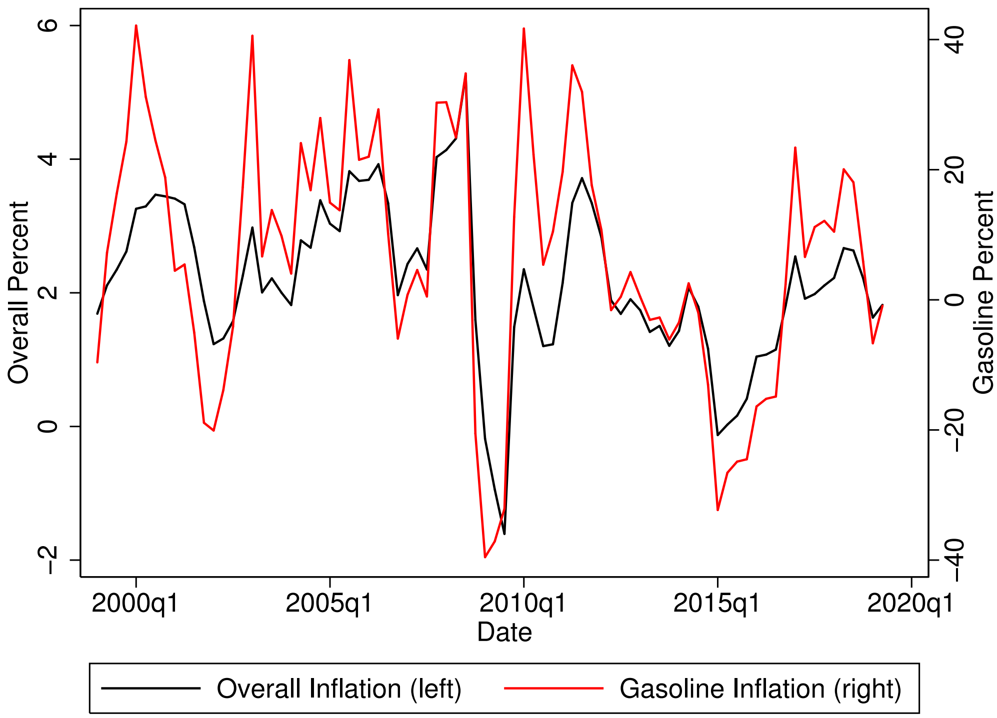
To answer these questions, I conduct three extensions of the model in order to find how sensitive the results are to the calibration. First, I rerun the frequency bias under different calibrations of the bread inflation rate, either by making it less correlated or more volatile. Second, I examine a version of the model where households have accurate inflation perceptions, but biased inflation expectations. Third, I examine how the welfare effects of the level bias vary by income.

7.3.1 Frequency Bias Sensitivity

I examine the frequency bias through two approaches: the correlation of the good that the frequency bias applies to, and the volatility of its inflation rate. Not only is the correlation between bread and the overall inflation rate high, the standard deviation of bread inflation is also quite small. As a result, the probability that bread inflation will be substantially higher than the overall inflation rate is low. I recalibrate my model with the same vector autoregressive process for the two inflation rates, but I impose a negative correlation between the shocks for bread and the overall inflation rate. Additionally, I recalibrate the discount rate β to hit the wealth to income ratio in the rational expectations scenario.

A second exercise is to model the frequency bias for a good has that a volatile inflation rate. For example, in the United States gasoline prices affect people's beliefs about the future inflation rate. Unlike bread prices in France, gasoline prices are extremely volatile and only weakly correlated with the overall inflation rate in the U.S., as Figure 10 shows. I estimate my model using U.S. gasoline inflation by running a vector auto-regression on the gasoline inflation rate and the U.S. overall inflation rate and insert that calibration into my model. I use the Kumar et al. (2015) to obtain estimates for the size of the frequency bias on gasoline.[85] I assume that gasoline is only 5% of a household's consumption basket, but they perceive it to be much more, about 20% of their budget. I use these estimates to set $\gamma_{US} = 0.05$ and $\tilde{\gamma}_{US} = 0.2$, where sector 1 in my model for the US represents gasoline.

Figure 10: U.S. Gasoline Extension



Notes: This figure displays overall inflation rate in the United States and the inflation rate of gasoline. Both rates are defined as year-on-year percentage increases in the price indices. Inflation data is taken from the Bureau of Labor Statistics. The frequency of the statistics is quarterly.

Table 9 below displays the results for all of my frequency bias counterfactuals and compares them to rational expectations and the baseline frequency bias. For each of these exercises, I am also supposing that the objective inflation rate in these counterfactuals also changes, and that house-

[85] Binder (2018) suggests that despite the appearance of gasoline overweighting, U.S. households actually suffer from no frequency bias and use the negative serial correlation of gas inflation in their beliefs.

holds with rational expectations recognize this.[86] Flipping the correlation between bread and overall inflation increases the welfare loss by an order of magnitude and greatly decreases the amount of assets held. The frequency bias dwarfs the effect the level bias, despite the fact that households with the household bias hold substantially fewer assets. The intuition is that bread is worse than random noise, to the extent that when bread inflation declines the overall inflation rate typically increases. Therefore, households misperceive the true inflation rate quite often, which affects their savings decisions. In periods where the bread inflation rate is low (high), households will oversave (undersave).

Frequency biased households hold significantly fewer assets than optimal in the steady state if bread is negatively correlated with the overall inflation rate. This result stems from the two-sector nature of the model, where $\pi_t = \pi_{1,t}\pi_{2,t}$. If the inflation rates for the two sectors are negatively correlated, $Corr(\pi_1, \pi_2) < 0$, then the overall inflation rate will have a lower variance as shocks to the sectors partially cancel each other out.

Table 9: Frequency Bias Extensions

	Welfare Loss	Assets Held
Rational Expectations		4.125
Baseline Frequency Bias	0.0433%	4.979
Level Bias	0.2379%	0.356
Negative Correlation	0.2978%	3.317
U.S. Gasoline	0.4333%	5.247

Notes: This table shows the welfare effects for households suffering from different calibrations of the frequency bias. The negative correlation row refers to the calibration where bread inflation has a negative correlation with the overall inflation rate. The U.S. gasoline row refers to the calibration where the good with the frequency bias has the volatility and correlation with overall inflation of gasoline in the United States. The first three rows are reprinted from table 7 to facilitate comparison. See the text for more details.

The frequency bias in with gasoline in the U.S. generates a much larger welfare loss than the frequency bias with greater salience on bread in France. I believe that two factors drive this result. First, the average gasoline inflation rate is higher than the average overall inflation rate in the United States. Households that overweight gasoline will, on average, overestimate the current

[86]For the exercise with U.S. households, I compare U.S. households suffering from the frequency bias to U.S. households with rational expectations. Households with the frequency bias on gasoline in the United States should not be compared to households that fixate on bread in France.

inflation rate. Second, the U.S. gasoline inflation rate is much more volatile than bread in France, which means that it is more likely that the gasoline inflation rate will significantly diverge from the overall inflation rate, even if gasoline is correlated with the overall inflation rate. Informally, I find that if I increase the weight that households with the frequency bias put on gasoline to $\tilde{\gamma}_{US} = 0.5$, the magnitude of the welfare loss is similar to that associated with the French level bias.

7.3.2 Effect of Perceptions Bias

Biased inflation perceptions affect savings through two channels: a misperception of the real value of the households' wealth and by biased expectations affecting the expected return to savings. To disentangle these effects, I set up a version of the model where households have perfect inflation perceptions of the current period, but have biased expectations about future inflation. Households may misjudge the real value of their wealth, even if they know their nominal wealth.

Households with the frequency bias now obtain their subjective policy function by solving:

$$\tilde{g}^{FR}(S,A) = \text{argmax}_{A'} u(Y + \frac{(1+i)A}{\pi} - A') + \beta E^{FR}[\tilde{V}^{FR}(S',A')|S]. \tag{29}$$

For households with the level bias, they correctly assess the current inflation rate, but overestimate the inflation rate for the next period using the level bias. They obtain their subjective policy function by solving:

$$\tilde{g}^{LB}(S,A) = \text{argmax}_{A'} u(Y + \frac{(1+i)A}{\pi} - A') + \beta E^{LB}[\tilde{V}^{LB}(S',A')|S]. \tag{30}$$

I take these subjective policy functions and plug them into the objective budget constraint as above. From this calculation, I obtain two objective value functions, $V^{FR}(S,A)$ and $V^{LB}(S,A)$, which represent the utility households obtain from following the suboptimal savings policies. Using the same consumption equivalence method as above, I calculate the amount of consumption households would be willing to sacrifice each period to avoid these expectations-only biases. Table

10 displays the results.

Table 10: Expectations Channel

	Baseline		Expectations	
	Welfare Loss	Assets Held	Welfare Loss	Assets Held
Rational Expectations		4.125		4.125
Frequency Bias	0.0433%	4.979	0.0246%	6.228
Level Bias	0.2379%	0.356	0.182%	0.486

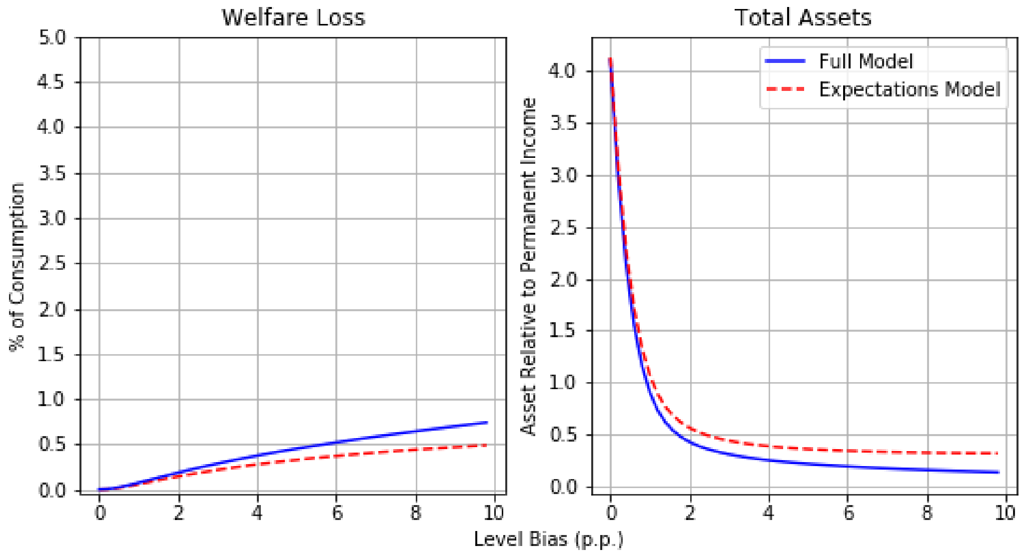
Notes: This table shows the welfare effects for households suffering from the frequency bias only or the level bias only in the baseline model and the model with only biased expectations. The baseline model refers to the model where households have biased inflation perceptions and biased inflation expectations. The expectations model refers to the model where households have accurate inflation perceptions and biased inflation expectations. See the text for more details.

Under biased inflation expectations, both the frequency bias and the level bias have weaker effects on welfare than under the baseline scenario where households have biased expectations and perceptions. Households accurately perceive their current real wealth, which means that they are able to purchase the planned amount of real consumption and allocate their nominal wealth effectively. As a result, the welfare loss diminishes and households are closer the rational expectations policy.

Households with accurate perceptions but biased expectations save more than households with biased perceptions and expectations. For households with the level bias in expectations, they believe that the future inflation rate will be high and that their future real income will be lower. The households react by increasing their savings in the steady state. Likewise, households with the frequency bias in expectations may believe that future inflation is more volatile that it actually is and increase current savings to smooth out those shocks.

I solve the model for different magnitudes of the level bias and graph the results in Figure 11. I find that as Akerlof and Yellen (1985) predicts, in the neighborhood of 0% the level bias generates negligible, second-order welfare losses, even though it generates a first-order reduction in assets held. As the level bias increases, households hold fewer and fewer assets in steady state, and the welfare loss grows. Regardless of the magnitude of the level bias, Figure 11 indicates that the biased perceptions magnify effect of the biased expectations by decreasing savings and increasing the welfare loss. In all cases, households with the accurate perceptions are better off than households without.

Figure 11: Effect by Size of Level Bias



Notes: The left panel shows the welfare loss that households face under different magnitudes of the level bias. The right panel depicts the steady state asset holdings of households. The full model refers to the version where households have biased inflation perceptions and expectations. The expectations model refers to the model where households have perfect inflation perceptions, but biased inflation expectations. See the text for more details.

7.3.3 Level Bias Extension

The level bias generates larger welfare losses and changes in savings behavior than the frequency bias. However, unlike the frequency bias, the magnitude of the level bias can vary significantly by household demographics.[87] Low-income households have a worse level bias than the high-income households: their perception of the current inflation rate is a full percentage point larger than the perception of high-income households. Given that high-income households suffer from a milder version of the level bias, does the level bias affect aggregate savings in the economy? If high-income households own most of the assets in the economy, then the macroeconomic implications of the level bias might be muted. In an extreme scenario, low-income households are consistently

[87]In the empirical work on the frequency bias, I found that high-income as well as low-income households overweight bread inflation.

liquidity constrained and unable to increase borrowing even if they did have accurate inflation perceptions. As a result, even though the level bias appears important in the household surveys, it has no effect on the aggregate economy.

To study this effect, I rerun the model on two different types of households, differing by their average permanent income and the level bias. The difference in permanent income is fixed: households never change their average permanent income. I simplify the analysis by assuming that each household has the same belief process over all horizons and never reduces its level bias as it obtains more income.[88] The income process for households is:

$$\log(Y^P_{i,t}) = \bar{y}_i + \rho \log(Y^P_{i,t-1}) + \varepsilon_{i,t}. \tag{31}$$

I refer to the French Household Budget Survey to obtain the average income for households in the bottom and top quartile, which I use to pin down $\bar{y}_{low}$ and $\bar{y}_{high}$. In the model, both types of household have the same type of budget constraint, can purchase the same nominal bond, and are otherwise the same. The nominal interest rate remains exogenous in this partial equilibrium model.

Table 11 reports the results of the welfare loss for low-income and high-income separately, and then for the aggregated economy. Because low-income households have a larger level bias, their welfare loss is higher and asset holdings lower than their high-income counterparts. The intuition is that the more that households underestimate the real returns to savings, the less they save and the worse off they are.

The last row reports the aggregate asset holdings of the income-decomposed model. Overall, households in the economy hold 0.405 nominal bonds in the steady state, relative to the rational expectations holdings of 2.603. High-income households overestimate the inflation rate by 2 percentage points on an annual basis, which is still significant enough to reduce their savings. Even

[88]If the households' permanent income can fluctuate between two states, then I can rewrite the problem with two value functions (v_{low} and v_{high}), two policy functions, and solve the problem. However, in this framework low-income households are aware that they have a level bias and are aware that it will be weaker if they become high-income households, yet they don't correct their level bias. If the setup was that high-income households had more information or better signals then this approach wold be plausible.

Table 11: Level Bias by Income

	Welfare Loss	Assets Held
Rational Expectations		2.603
Unweighted Level Bias	0.238%	0.356
Low-Income Level Bias	0.295%	0.302
High-Income Level Bias	0.186%	0.431
Income-weighted Average	0.241%	0.405

Notes: This table shows the welfare effects for different calibrations of the level bias. The low-income level bias row refers to the welfare loss for households with the level bias of low-income households in the sample.The high-income level bias row refers to the welfare loss for households with the level bias of high-income households. The income-weighted average row combines these results: the welfare loss is the unweighted average of low-income and high-income households, while the assets held is the average weighted by household income. See the text for more details.

though decomposing the level bias reduces the aggregate effect, it does not eliminate it.

8 Conclusion

In this paper I document two biases in inflation perceptions and evaluate their effects on welfare and savings. Using a French household survey, I establish that households overweight bread inflation and consistently overestimate the current inflation rate. The former is an example of the well-known frequency bias, while I refer to the latter as the level bias. Because reliable inflation statistics are provided by national statistical agencies, one may believe that the cost of obtaining accurate inflation information should be low. The fact that households have flawed perceptions of current inflation would suggest that households choose to allocate little attention to it, because inflation has a relatively small effect on households.

The principal contribution of this paper is to construct a model that can incorporate the perception biases for the purposes of estimating the welfare loss. Using my empirical measures of the size of the biases to discipline the model, I find that households with the level bias in fact suffer large welfare losses and significantly decrease their savings. In contrast, despite the prominence of the frequency bias in the survey responses, it has little effect on households. Because the level bias is so deleterious to households, my results indicate that they are not be processing information rationally under some constraint. If policymakers could correct the level bias in households, for example by conveying the historical average inflation rate succinctly, then they could significantly increase household savings.

This paper highlights the importance of economic perceptions for researchers. In the model, I find that biased inflation perceptions can affect welfare, albeit to a lesser extent than biased inflation expectations. Inflation perceptions not only influence forecasts for future inflation, but also affect households' assessment of their real wealth and their savings decisions. Future research could investigate whether households have accurate perceptions of other economic variables, such as unemployment, and how any systematic perception errors could influence their decision-making.

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A Survey Methodology

The CAMME survey questionnaire follows the guidelines set out by the EU Harmonized Consumer Survey. The wording of the survey is standardized to facilitate cross-country comparisons, although the method by which statistical institutes conduct the survey varies. The inflation perceptions and expectations questions are slightly more complicated than the other survey questions. For example, the interviewers ask households about perceived inflation as a two part question. First, the interviewers ask respondents if they believe that prices have gone up, stayed the same, or gone down. If the respondents give any answer other than stayed the same, the interviewers follow up by asking for a precise number for inflation perceptions. If the respondent states they believe that prices stayed the same, they are automatically coded as having an inflation perception of 0%, and the interviewer skips the quantitative question. The inflation expectations component follows a similar two part structure.

The CAMME does not record the exact value of household income. Rather than reporting households income, INSEE records which quartile they fall in, i.e. between the 1st and 2nd income quartile.[89] Each month INSEE takes all of the household respondents, orders them by household income, and divides the sample into fourths. I assume that these sample income quartiles more or

[89]Even the CAMME microdata does not contain the actual monthly income of households.

less correspond to the national income quartiles when calculating the effect of biased inflation perceptions by households' income. I define "low-income" to indicate households that report income below the 1st quartile, and "high-income" to indicate households that report income above the 3rd quartile.

The relevant questions and possible answers from the EU Guidelines are displayed below.

Inflation Perceptions:

Q5: How do you think that consumer prices have developed over the last 12 months? They have. . .
+ + risen a lot
+ risen moderately
= risen slightly
– stayed about the same
– – fallen
N don't know.

Q51: If question 5 was answered by 1, 2, 3 or 5: By how many per cent do you think that consumer prices have gone up/down over the past 12 months? (Please give a single figure estimate).
Consumer prices have increased by , % / decreased by , %.

Inflation Expectations:

Q6: By comparison with the past 12 months, how do you expect that consumer prices will develop in the next 12 months? They will. . .
+ + increase more rapidly
+ increase at the same rate
= increase at a slower rate
– stay about the same
– – fall
N don't know.

Q61: If question 6 was answered by 1, 2, 3 or 5: By how many per cent do you expect consumer prices to go up/down change in the next 12 months? (Please give a single figure estimate).

Consumer prices will increase by , % / decrease by , %.

Saving:

Q10: In view of the general economic situation, do you think that now is...?

+ + a very good moment to save

+ a fairly good moment to save

– not a good moment to save

– – a very bad moment to save

N don't know.

Economic Situation

Q4: How do you expect the general economic situation in this country to develop over the next 12 months? It will...

+ + get a lot better

+ get a little better

= stay the same

– get a little worse

– – get a lot worse

N don't know.

B Inflation Expectations Correction

According to the user's guide for the EU Harmonized Consumer Survey, the survey interviewer asks about the expected inflation rate in a two step process. First, the interviewer asks a qualitative question about whether the respondent believes that prices will go up, stay the same, or go down. If the respondent states that they believe that prices will go up or down, then the interviewer follows up with the quantitative question. If the respondent states that they expect prices to "stay the same," then the interviewer skips the quantitative portion and codes a response of 0%. In a typical month, about 30% of respondents state that they expect "prices to stay the same" and are assigned a 0% response in this fashion. However, in September 2009 INSEE ran a short followup survey for these 0% households about survey wording. They found that about half of the households that stated that they expected prices to "stay the same" really meant "the inflation rate will stay the same." The households were erroneously coded as 0%, when in fact their actual inflation expectations were higher.

I can not identify whether a specific household that states that they believe prices will "stay the same" actually holds that belief.[90] As an imperfect solution, I create an indicator variable to identify any household that believes that prices have risen by more than 5% over the past twelve months, but that prices will "stay the same" over the next 12 months. The rationale is that households that believe that the current inflation rate is high would be unlikely to actually expect inflation to drop to 0% in the future. For the rest of this appendix, I refer to these expectations as dubious 0% expectations.

I drop any households that give dubious 0% responses as defined above, and I rerun my analysis linking inflation expectations to inflation perceptions. Figure 12 displays the mean inflation perceptions and corrected inflation expectations time series. The discrepancy between inflation perceptions and expectations is small, particularly after 2011. Households appear to believe that inflation over the next 12 months will be closer to how it behaves over the past 12 months.

[90]Half of the respondents in the September 2009 supplemental survey actually did believe that the inflation rate would be 0%, so assigning them a non-zero belief would be incorrect.

The link between inflation perceptions and inflation expectations is much higher on the household level, as Table 12 shows. The R^2 statistic is around 0.6, which indicates that inflation perceptions explain most of the variation in household inflation expectations. The large coefficient on inflation perceptions and the low constant term indicate that the best-fit line is close to 45 degrees and anchored near the origin. In summary, miscoded household expectations dampened the effect of perceptions in my original analysis.

Figure 12: Modified Expectations

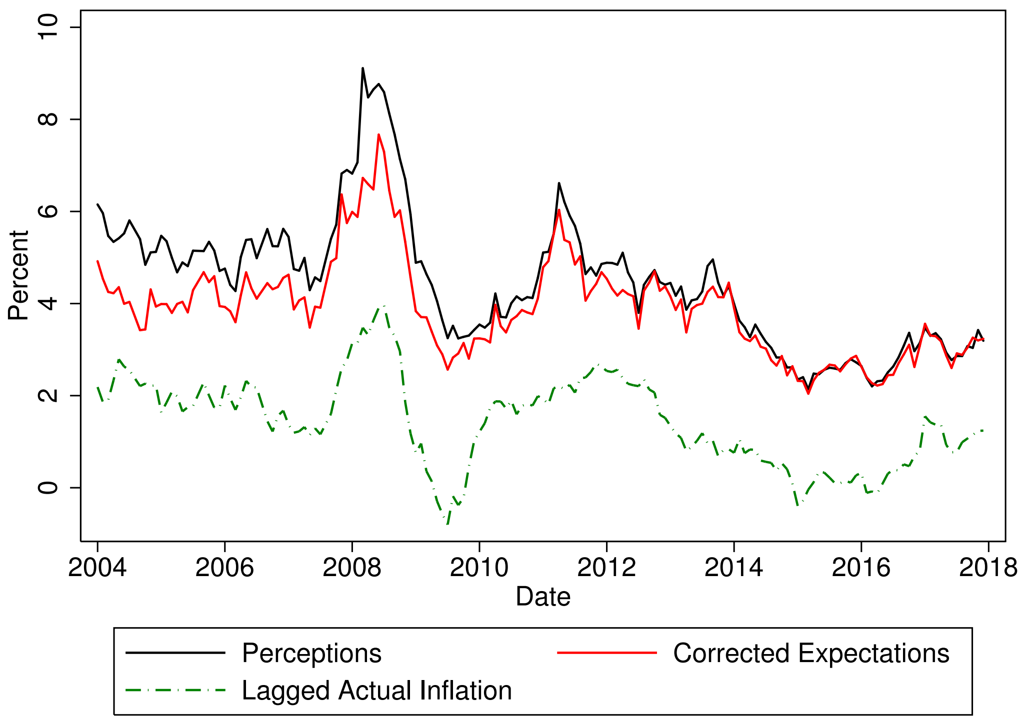

Table 12: Expectations on Savings without Miscoding

	(1)	(2)	(3)	(4)	(5)	(6)
	$E_t\pi_{t+12}$	$E_t\pi_{t+12}$	$E_t\pi_{t+12}$	$E_t\pi_{t+12}$	$E_t\pi_{t+12}$	$\Delta E_t\pi_{t+12}$
Inflation Perceptions	0.774***	0.769***	0.813***	0.805***	0.748***	
	(0.006)	(0.006)	(0.005)	(0.010)	(0.008)	
Change in Inflation Perceptions						0.556***
						(0.013)
Constant	0.200***	0.345***	1.026***	0.104**	0.241***	0.363***
	(0.018)	(0.082)	(0.088)	(0.038)	(0.020)	(0.042)
Demographic Controls	No	Yes	Yes	No	No	No
Sample	Full	Full	06/2010-	High-Income	Low-Income	Full
R^2	0.601	0.599	0.639	0.576	0.609	0.305
Observation	129336	112661	56018	23109	41214	22515

C Solution Methodology

The model is solved using the endogenous grid point method of Carroll (2006). For the rational expectations version of the model, I set up the problem in real terms. I initialize a guess for the utility of consumption for each gridpoint in the combination of assets and exogenous states and iterate backwards using the Euler equation and budget constraint until I find the optimal policy solution. This technique returns the optimal policy function under rational expectations.

For households with biased perceptions, I apply the same technique as above. However, I set the exogenous states and transition matrix as if the households were correct.[91] I use the endogenous gridpoints method to find the subjectively optimal policy functions by iterating until the solution converges. Next, I evaluate the objective value function under suboptimal policy by starting at each asset gridpoint and exogenous state and iterating forward with the policy function. Both the real return to savings and the households real income are reweighted by the households' subjective beliefs about inflation.

The last type of policy function to solve for is for households with accurate inflation perceptions, but biased inflation expectations. First, I set the exogenous states and transition matrix as if the households' biased expectations were correct. I iterate backward with the endogenous grid point method until the policy functions converge. Next, I the policy function backwards one more time, using the objectively correct exogenous state and transition matrix. This grants me the policy function for households with accurate perceptions, but biased expectations. Finally, find the value function that a household would get by applying this suboptimal policy function.

I discretize the exogenous states using a multi-variate Tauchen method. This method involves setting up the correlated VAR process for π_1, π_2, Y^P, andY^T, taking an orthogonal transformation of the variables to generate four orthogonal variables and then discretizing that process. The orthogonal transformation of the exogenous state variables π_1, π_2, and Y^P have 5 states, while Y^T has 3 states, meaning that overall there are 375 distinct states. Because π_1 and π_2 are correlated,

[91] Specifically, I adjust the inflation rates in the current period to match the households' beliefs. Additionally, I multiply the real income in the current period by $\frac{\pi}{E[\pi]}$ to reflect the fact that households in the model know their nominal income perfectly, but not necessarily their real income.

the discretized states for π_1 and π_2 are not evenly distributed in a square.

D Nominal Budget Constraint

In this appendix I will set up the sequential markets version of the households' problem in nominal terms, where households consider the price level P_t. I will then describe the steps and assumptions that are required to rewrite the households' problem in a stationary, recursive form. Because households do not necessarily know the price level P_t each period and P_t may not be stationary, this derivation is nontrivial.

I assume that households know their nominal income and nominal bond wealth at the start of the period, but may not know the current price level P_t perfectly. The households choose how much of their nominal wealth to allocate towards consumption expenditures and how much to save, given their beliefs about the current price level. The households also form expectations about the real value of their future income and the inflation rate next period. The household's problem at time t is:

$$\max_{\{C_t^N\}} E_0^* \sum_{t=0}^{\infty} \beta^t u(C_t) \tag{32}$$

$$\text{s.t. } C_t^N + A_t^N = (1+i)A_{t-1}^N + Y^N \tag{33}$$

$$C_t = \frac{C_t^N}{P_t}. \tag{34}$$

At time t, households need to decide how much to spend on consumption given that they might not know the price of composite goods P_t and decide how much to save given that they do not know the real rate of return $\frac{(1+i)}{\pi_{t+1}}$. I make two assumptions to simplify the analysis. First, I assume that at time t households know P_{t-1}, the price level last period. This analysis allows me to divide the nominal budget constraint by P_{t-1} to express the households budget constraint in real, stationary terms.[92] One of the households state variables is price deflated bonds A_{t-1}, which is the amount

[92]Making this assumption also prevents incorrect inflation perceptions from compounding over multiple periods. If households overestimated the inflation rate by 2 percentage points each period and never learns the true price level,

of nominal bonds purchased deflated by the price index, $A_t = \frac{A_t^N}{P_t}$. Any uncertainty that households have about the current inflation rate π_t only reflects uncertainty that they have about the current price level π_t.

My second assumption is that households have certainty equivalence with regards to the inflation rate and the current price level. Households only care about their average perceptions of the current inflation rates, rather than the full distribution of π_t. I make this assumption for two reasons. First, under full information rational expectations households know π_t exactly, so they have a degenerate distribution of subjective beliefs with all the mass put on π_t. To facilitate comparisons between full information rational expectations and biased perceptions, I assume that households with biased perceptions also believe that the inflation rate was $E^*[\pi_t]$ with certainty. Second, certainty equivalence is more analytically tractable in my setup.[93] If households have certainty equivalence over current inflation $E^*[\pi_t]$ and $E^*[P_t]$, then I can set up the computationally problem to find the optimal policy for each subjective $E^*[\pi_t]$. In contrast, if households have a non-degenerate distribution of beliefs of π_t, then I would have to evaluate their optimal policy over each possible subjective distribution of π_t.

If households in period t believe that the current price level is $E_t^*[P_t] = P_t^*$ with a degenerate distribution, then they believe that their real budget constraint is

$$\begin{aligned} \frac{C_t^N}{P_t^*} + \frac{A_t^N}{P_t^*} &= \frac{(1+i)A_t^N}{P_t^*} + \frac{Y_t^N}{P_t^*} \\ \tilde{C}_t + \tilde{A}_t &= \frac{(1+i)}{E_t^*[\pi_t]} A_t^N + \tilde{Y}_t. \end{aligned} \tag{35}$$

The right-hand side of equation 35 is a combination of the household's nominal budget constraint, which they know accurately, and their belief about the current price level.[94] Households

then after ten periods they will overestimate the price level by 20 percentage points.

[93]Technically if the coefficient of risk aversion $\sigma = 2$, the expression $E[\pi]$ shows up in the the households' first order equations. I choose to make the certainty equivalence assumption more explicit.

[94]Households in this model do not know their real income accurately each period, although they do know their nominal income. This process could represent a nominal wage that adjusts with a one-period lag: households believe that a higher inflation rate erodes the purchasing power of their income, but this process does not compound over time.

choose their subjective bonds deflated by the price index $\tilde{A}_t$ and their subjective consumption $\tilde{C}$. However, if households over- (under-)estimate inflation, then they have less (more) real wealth than they believe. The true household budget constraint is

$$\frac{C_t^N}{P_t} + \frac{A_t^N}{P_t} = \frac{(1+i)A_t^N}{P_t} + \frac{Y_t^N}{P_t}$$
$$C_t + A_t = \frac{(1+i)}{\pi_t} A_t + Y_t.$$

In my setup, I assume that households allocate spending towards consumption goods C_t^N and nominal bonds A_t^N first, and then realize the true price level when they obtain C_t consumption goods. The relationship between the households' planned consumption $\tilde{C}_t$ and their realized consumption C_t is

$$C_t = \frac{E_t^*[\pi_t]}{\pi_t} \tilde{C}_t.$$

Likewise the relationship between $\tilde{A}_t$ and A_t, and the relationship between $\tilde{C}_t$ and C_t, depends on the ratio of perceived inflation to actual inflation. In practice, this means that if households plan to purchase $\tilde{C}_t$ real consumption goods and save $\tilde{A}_t$ price-deflated bonds, they will end up with C_t and A_t bonds without violating their budget constraint. Equation 11 is a combination of 35 and $Y_t = \frac{E^*[\pi_t]}{\pi_t} \tilde{Y}_t$.