BLS WORKING PAPERS

U.S. Department of Labor
U.S. Bureau of Labor Statistics
Office of Prices and Living Conditions

Changing Tastes Versus Specification Error in Cost-of-Living Measurement

Robert Martin
U.S. Bureau of Labor Statistics

Working Paper 531
August 2020

Changing tastes versus specification error in cost-of-living measurement*

Robert S. Martin†

August 2020

Abstract

PRELIMINARY

Several recent papers aim to account for changing preferences in cost-of-living indexes (COLI). Workhorse models like Constant Elasticity of Substitution (CES) attribute the errors in demand regressions entirely to preferences, leaving no room for other sources of error. Using a Monte Carlo experiment and retail scanner data, I find evidence that model misspecification can lead to misleading conclusions about the degree of taste change reflected in CES-based price indexes. Nevertheless, under misspecification, a Sato-Vartia index still approximates a conditional COLI that fixes tastes to an intermediate level.

1 Introduction

Many recently proposed methods for cost-of-living index (COLI) measurement rely on specification and estimation of a model of consumer preferences. For example, a workhorse model

*The views expressed herein are those of the author and not necessarily those of the Bureau of Labor Statistics or the U.S. Department of Labor.

†Division of Price and Index Number Research, Bureau of Labor Statistics, 2 Massachusetts Ave, NE, Washington, DC 20212, USA. Email: Martin.Robert@bls.gov

is constant elasticity of substitution (CES).[1] The impact of changing preferences has been an object of recent attention in the price index literature. Redding and Weinstein (2020) (henceforth RW) estimate that the pure contribution of changing tastes to welfare is economically important—lowering an unconditional COLI by 0.4 percentage points per year when based on the CES model.[2] Using retail scanner data and a simple CES model, Martin (2020) found that the choice of taste vector could raise or lower a conditional COLI for a food category between half a percentage point and three percentage points per year.

As discussed by Fisher and Shell (1972), it is difficult to assess the role of tastes in full generality—analysts must focus attention on one or two items, or assume a full model for preferences. In the CES model, time-varying tastes make up the idiosyncratic error of the regression of log expenditure shares on log prices. The obvious issue in drawing inference on taste change from regression residuals is that this leaves no room for other sources of error.[3] One potential source of error is from model misspecification. CES preferences are quite tractable, but they are restrictive. Substitution elasticities are held constant across different pairs of goods, income elasticities are all equal to one, and complementary goods are ruled out in some applications. COLI can be derived, of course, for more flexible models, but some parsimony is usually required.[4] A hypothetical index user might therefore reasonably view the underlying model as an approximation for a more complicated, unknown preference structure.

[1]See, for example, Feenstra (1994) and Broda and Weinstein (2010) use this model to account for welfare gains from new and disappearing goods. The Bureau of Labor Statistics (BLS) implicitly assumes CES-like preferences by using a geometric mean formula for its elementary item indexes, and uses a CES formula for initial estimates of the Chained CPI. See US Department of Labor (2018).

[2]An earlier version, Redding and Weinstein (2018) found much higher differences, on the order of two to three percentage points per year. Martin (2020) discusses how changes in the set of common goods may have contributed to the change in RW's results. Incorporating pure taste effects also requires the assumption of cardinal utility.

[3]Feenstra and Reinsdorf (2007) also point out this possibility in describing the challenges of merging the stochastic and economic approaches to index numbers. See also Nevo (2003) regarding difficulty in distinguishing taste change from quality change.

[4]Even flexible cost functions like the translog may not actually be estimated in a fully flexible manner due to the curse of dimensionality. Applications may restrict own and cross price elasticities to depend on a relatively small set of parameters. For example, see Feenstra and Weinstein (2017), where the $N(N+1)/2$ elasticities are restricted to depend on only one free parameter.

It is prudent, therefore to ask if and to what extent specification errors can impact model-based price indexes. As a starting point, I use a Monte Carlo experiment to explore the performance of several CES-related price indexes, including the Sato-Vartia price index, Lloyd-Moulton index, the Backwards Lloyd-Moulton index, and RW's Common-Goods Price Index (CCV). The first three of these are exact conditional COLI in the CES case (Feenstra and Reinsdorf, 2007; Martin, 2020), while RW's CCV index estimates an unconditional COLI under the assumption of cardinal utility and the normalization that the taste parameters have a time-constant geometric mean. The experiment's true model is in the CES family, but has a neglected nesting structure reflecting heterogeneity in substitution elasticities across groups of goods. The goal is to see how misspecification contributes to the performance of price indexes and indicators of taste change. Of course, a model can be made more flexible; indeed, nested CES preferences are quite common in empirical studies. Still, one might be concerned that imposing some degree of parsimony will impact the measurement of the COLI and lead to improper conclusions regarding taste change.

To preview the results, I find that this form of misspecification leads to economically significant bias and root mean squared error (RMSE) in the Lloyd-Moulton variants and the CCV index. Moreover, the Lloyd-Moulton variants, as well as the CCV and Sato-Vartia indexes can differ significantly from each other regardless of whether there is taste change or not. This raises the empirical possibility that index comparisons may severely misdiagnose the extent of changing tastes. An empirical application to retail scanner data on fresh produce also suggests that model specification can have a significant impact on the Lloyd-Moulton indexes and the CCV index. As a consequence, researchers using model-based indexes to draw conclusions about tastes should estimate a variety of models and report a range of results.[5] At the same time, the Monte Carlo experiment and empirical results suggest that the Sato-Vartia index, as well as the geometric mean of the two Lloyd-Moulton indexes (which is also a quadratic mean of order-r index), are relatively robust to

[5] In supporting this recommendation, this paper complements the findings of Nevo (2003).

misspecification.

2 Cost-of-Living Index Theory with CES Preferences

A COLI is based on the expenditure function from classical consumer theory.[6] We assume a representative agent has preferences over N narrowly-defined commodities, indexed by i, that can be represented by a CES utility function. The function is governed by an elasticity of substitution parameger σ, as well as quantity augmenting or diminishing parameters, $\varphi_1, \ldots, \varphi_N$, which I collect in the vector $\boldsymbol{\varphi}$. Denote the set of goods available as $\mathcal{I}$. As in RW, I assume that σ is constant between different price situations being compared.[7] Under the assumption of optimizing behavior, the expenditure function gives the minimum expenditure required to achieve utility level $\bar{u}$ when facing prices $\boldsymbol{p} = (p_1, \ldots, p_N)$. These assumptions are collected below:

Assumption 2.1 *The representative agent's expenditure function has the form:*

$$C(\boldsymbol{p}, \bar{u}; \boldsymbol{\varphi}) = \bar{u} \left[\sum_{i \in \mathcal{I}} \left(\frac{p_i}{\varphi_i} \right)^{1-\sigma} \right]^{\frac{1}{1-\sigma}} \tag{1}$$

Due to homotheticity of CES preferences, we focus attention on the unit expenditure function $c(\boldsymbol{p}; \boldsymbol{\varphi}) \equiv C(\boldsymbol{p}, 1; \boldsymbol{\varphi})$ without further loss of generality. Using Shephard's Lemma, the optimal expenditure shares have the form.

$$s_i(\boldsymbol{p}; \boldsymbol{\varphi}) = \frac{(p_i/\varphi_i)^{1-\sigma}}{\sum_{j \in \mathcal{I}} (p_j/\varphi_j)^{1-\sigma}} = \frac{(p_i/\varphi_i)^{1-\sigma}}{[c(\boldsymbol{p}; \boldsymbol{\varphi})]^{1-\sigma}} \tag{2}$$

A bilateral COLI uses the expenditure function to compare two price situations.[8] Sup-

[6] The concept dates back at least to Konüs (1924). For more information, see Bureau of Labor Statistics (2018), National Research Council (2002), or ILO (2004).

[7] Despite the loss of generality, constant σ simplifies the derivation of price indexes and allows for identification using panel variation in prices and expenditure shares.

[8] Most commonly the comparison is intertemporal, but the general theory accommodates other possibilities (e.g., regional comparisons).

posing the comparison is intertemporal, constant tastes is easily seen to be an inelegant assumption empirically. Adding time subscripts t, the log-share equation for item i with respect to the actual price and vectors $\boldsymbol{p}_t$ and $\boldsymbol{\varphi}_t$ is:

$$\ln s_{it} = (1-\sigma)\ln p_{it} + (\sigma-1)\ln\left[c(\boldsymbol{p}_t;\boldsymbol{\varphi}_t)\right] + (\sigma-1)\ln\varphi_{it} \tag{3}$$

The equation has two unobservables—the log of the unit expenditure, which is constant across items, and the log of φ_{it}. We define an item i narrowly enough so that potential time series variation in $\ln\varphi_{it}$ is attributable to taste change and not changes in product attributes or quality. As RW point out, time-varying tastes are more realistic as they provide a source of idiosyncratic error, yet exact price indexes for CES preferences are typically derived for the case where φ_{it} is time-constant. However, the basic theory of the COLI, outlined in the next subsection, makes no such assumption as it relates to a data generating process.

2.1 Conditional COLI

A conditional COLI is defined as the minimum expenditure required for an agent to be indifferent between two price situations. I label the reference situation 0 and the comparison situation 1.

Definition 2.1 *A (conditional or ordinal) Cost-of-living Index (Fisher and Shell, 1972; Heien and Dunn, 1985; Pollak, 1989)*

$$\Phi(\boldsymbol{p}_0, \boldsymbol{p}_1; \boldsymbol{\varphi}) = \frac{c(\boldsymbol{p}_1;\boldsymbol{\varphi})}{c(\boldsymbol{p}_0;\boldsymbol{\varphi})}, \tag{4}$$

for a given $\boldsymbol{\varphi}$.

The $\boldsymbol{\varphi}$ parameters determine the specific indifference surface on which Φ is based. Two immediate candidates for preferences to plug in are $\boldsymbol{\varphi}_0$ and $\boldsymbol{\varphi}_1$, corresponding to the reference and comparison periods, respectively. In principle, however, other choices are possible.

A number of price indexes are associated with CES preferences. Of particular interest is the Sato-Vartia (SV) index, proposed independently by Sato (1976) and Vartia (1976). In addition to prices, the following formulas depend on observed quantities q_{it} and expenditure shares $s_{it} = p_{it}q_{it}/\sum_{j=1}^{N} p_{jt}q_{jt}$, $t = 0, 1$.

Definition 2.2 *The Sato-Vartia price index*

$$P_{SV}(\boldsymbol{p}_0, \boldsymbol{p}_1, \boldsymbol{q}_0, \boldsymbol{q}_1) = \prod_{i \in \mathcal{I}} \left(\frac{p_{i1}}{p_{i0}} \right)^{w_i}, \tag{5}$$

where $w_i = \left[\frac{s_{i1} - s_{i0}}{\ln s_{i1} - \ln s_{i0}} \right] / \left[\sum_{k \in \mathcal{I}} \frac{s_{k1} - s_{k0}}{\ln s_{k1} - \ln s_{k0}} \right]$.

When tastes are constant between periods 0 and 1, the SV index is exact for the CES model, meaning $P_{SV}(\boldsymbol{p}_0, \boldsymbol{p}_1, \boldsymbol{q}_0, \boldsymbol{q}_1) = \Phi(\boldsymbol{p}_0, \boldsymbol{p}_1; \boldsymbol{\varphi})$. This is notable because it does not require knowledge or estimation of σ or $\boldsymbol{\varphi}$. In the case of changing tastes, however, the SV index still estimates a COLI that may be of interest. From Feenstra and Reinsdorf (2007), there exists a vector $\bar{\boldsymbol{\varphi}}$, where each element $\bar{\varphi}_i$ lies between normalized φ_{i0} and φ_{i1}, such that $P_{SV}(\boldsymbol{p}_0, \boldsymbol{p}_1, \boldsymbol{q}_0, \boldsymbol{q}_1) = \Phi(\boldsymbol{p}_0, \boldsymbol{p}_1; \bar{\boldsymbol{\varphi}})$. While I am unaware of any prior efforts to explicitly study the SV index under misspecification, previous studies[9] have found it performs similarly to superlative indexes like the Tornqvist, which suggests some flexibility with respect to functional form.[10]

Two other indexes of note are the Lloyd-Moulton index, proposed by Lloyd (1975) and Moulton (1996), and what Martin (2020) calls the Backwards Lloyd-Moulton index (also due to Lloyd, 1975), both of which depend on the parameter σ.

Definition 2.3 *Lloyd-Moulton Index*

$$P_{LM}(\boldsymbol{p}_0, \boldsymbol{p}_1, \boldsymbol{q}_0, \boldsymbol{q}_1, \sigma) = \left\{ \sum_{i \in \mathcal{I}} s_{i0} \left(\frac{p_{i1}}{p_{i0}} \right)^{1-\sigma} \right\}^{\frac{1}{1-\sigma}} \tag{6}$$

[9] E.g., Redding and Weinstein (2018) and Martin (2020)

[10] In fact, a similar index, the Vartia I of Vartia (1976), has been shown to approximate superlative indexes to the second order (Diewert, 1978).

Definition 2.4 *Backwards Lloyd-Moulton Index*

$$P_{BLM}(\boldsymbol{p}_0, \boldsymbol{p}_1, \boldsymbol{q}_0, \boldsymbol{q}_1, \sigma) = \left\{ \sum_{i \in \mathcal{I}} s_{i1} \left(\frac{p_{i0}}{p_{i1}} \right)^{1-\sigma} \right\}^{\frac{-1}{1-\sigma}} \tag{7}$$

The Lloyd-Moulton variants are also exact for CES preferences. If preferences are constant, then $P_{LM}(\boldsymbol{p}_0, \boldsymbol{p}_1, \boldsymbol{q}_0, \boldsymbol{q}_1, \sigma) = P_{BLM}(\boldsymbol{p}_0, \boldsymbol{p}_1, \boldsymbol{q}_0, \boldsymbol{q}_1, \sigma) = \Phi(\boldsymbol{p}_0, \boldsymbol{p}_1; \boldsymbol{\varphi})$. In the case of changing preferences, Martin (2020) showed that $P_{LM}(\boldsymbol{p}_0, \boldsymbol{p}_1, \boldsymbol{q}_0, \boldsymbol{q}_1, \sigma) = \Phi(\boldsymbol{p}_0, \boldsymbol{p}_1; \boldsymbol{\varphi}_0)$, and $P_{BLM}(\boldsymbol{p}_0, \boldsymbol{p}_1, \boldsymbol{q}_0, \boldsymbol{q}_1, \sigma) = \Phi(\boldsymbol{p}_0, \boldsymbol{p}_1; \boldsymbol{\varphi}_1)$. As a result, one may view the difference between the two Lloyd-Moulton variants as an indicator of taste change. Another quantity of interest may be the geometric mean of the LM and BLM indexes, denoted $P_{LMM}(\boldsymbol{p}_0, \boldsymbol{p}_1, \boldsymbol{q}_0, \boldsymbol{q}_1, \sigma)$ which is exact for the geometric mean of $\Phi(\boldsymbol{p}_0, \boldsymbol{p}_1; \boldsymbol{\varphi}_0)$ and $\Phi(\boldsymbol{p}_0, \boldsymbol{p}_1; \boldsymbol{\varphi}_1)$. While not associated with a COLI, per se, $P_{LMM}(\boldsymbol{p}_0, \boldsymbol{p}_1, \boldsymbol{q}_0, \boldsymbol{q}_1, \sigma)$ is equivalent to quadratic mean of order $2(1-\sigma)$ price index (Martin, 2020). Since this index is superlative, it is likely to be somewhat robust to misspecification.

However, exactness of P_{LM} and P_{BLM} depends on the CES assumption through Eq. 2. As each only relies on one period's expenditures, they rely on the model to predict the demand response to relative price change. Consequently, their relative performance under misspecification may not accurately reflect differential tastes.

2.2 Cardinal COLI

RW focus attention on a different concept which aligns more closely to what National Research Council (2002) or ILO (2004) call an unconditional COLI, or what Muellbauer (1975) calls a cardinal COLI.

Definition 2.5 *The Cardinal COLI*

$$\Phi_C(\boldsymbol{p}_0, \boldsymbol{p}_1; \boldsymbol{\varphi}_0, \boldsymbol{\varphi}_1) = \frac{c(\boldsymbol{p}_1; \boldsymbol{\varphi}_1)}{c(\boldsymbol{p}_0; \boldsymbol{\varphi}_0)}, \tag{8}$$

This ratio represents the change in expenditure required to maintain a constant utility level, even though the associated consumption bundles do not lie on the same indifference surface. It relies, therefore, on a stronger interpretation of utility than what is implied by the usual axioms of preferences.

RW propose the following, which they call the CES Common Varieties Index (CCV).[11]

Definition 2.6 *RW's CES Common Varieties Index (CCV)*

$$P_{CCV}(\boldsymbol{p}_0, \boldsymbol{p}_1, \boldsymbol{q}_0, \boldsymbol{q}_1, \sigma) = \exp\left[\frac{1}{N}\sum_{i=1}^{N}\ln\left(\frac{p_{i1}}{p_{i0}}\right) + \frac{1}{\sigma-1}\frac{1}{N}\sum_{i=1}^{N}\ln\left(\frac{s_{i1}}{s_{i0}}\right)\right], \tag{9}$$

Under the normalization that tastes have a time-constant geometric mean, i.e. $\prod_{i\in\mathcal{I}}\varphi_{i0} = \prod_{i\in\mathcal{I}}\varphi_{i1}$, $P_{CCV}(\boldsymbol{p}_0, \boldsymbol{p}_1, \boldsymbol{q}_0, \boldsymbol{q}_1, \sigma) = \Phi_C(\boldsymbol{p}_0, \boldsymbol{p}_1; \boldsymbol{\varphi}_0, \boldsymbol{\varphi}_1)$[12] The need for normalization arises from the fact that $s(\boldsymbol{p}, \boldsymbol{\varphi})$ is homogeneous of degree zero in $\boldsymbol{\varphi}$, while $c(\boldsymbol{p}, \boldsymbol{\varphi})$ is homogeneous of degree -1. In other words, a simple re-scaling of the taste vector would affect $\Phi_C(\boldsymbol{p}_0, \boldsymbol{p}_1; \boldsymbol{\varphi}_0, \boldsymbol{\varphi}_1)$, but have no effect on expenditure shares.

The CCV index can be derived by plugging in the following expression for $\ln\varphi_{it}$, which follows from the expression for $s(\boldsymbol{p}_t, \boldsymbol{\varphi}_t)$ given by Eq. 2 and the normalization that the geometric mean of the φ_{it} equals one.

$$\ln\varphi_{it} = \left[\ln\left(\frac{s_{it}}{\widetilde{s}_t}\right) - (1-\sigma)\ln\left(\frac{p_{it}}{\widetilde{p}_t}\right)\right]/(\sigma-1) \tag{10}$$

The right-hand-side of Eq. 10 is equivalent to the scaled error term of a regression of $\ln(s_{it}/\tilde{s}_t)$ on $\ln(p_{it}/\tilde{p}_t)$. Thus, any misspecification picked up by this error will affect the price index.

The cardinal COLI differs from a given ordinal COLI by a term that values the pure

[11] RW's proposed CES Unified Price Index consists of the CCV plus a new goods adjustment in the style of Feenstra (1994). I abstract from the issue of new and disappearing goods because the taste change issue exists regardless. I calculate indexes with new goods adjustments as part of my empirical application.

[12] Redding and Weinstein (2018) motivates the normalization with the assumption that the natural logs of the φ_{it} are drawn from a distribution with constant mean and variance. Under this assumption, P_{CCV} and Φ_C have the same probability limit as N grows large.

effect of tastes on expenditure. For instance, let $\bar{\boldsymbol{\varphi}}$ be the intermediate taste vector such that $P_{SV}(\boldsymbol{p}_0, \boldsymbol{p}_1, \boldsymbol{q}_0, \boldsymbol{q}_1) = \Phi(\boldsymbol{p}_0, \boldsymbol{p}_1; \bar{\boldsymbol{\varphi}})$. Then we can write

$$\ln \Phi_C(\boldsymbol{p}_0, \boldsymbol{p}_1; \boldsymbol{\varphi}_0, \boldsymbol{\varphi}_1) - \ln \Phi(\boldsymbol{p}_0, \boldsymbol{p}_1; \bar{\boldsymbol{\varphi}}) = \ln \left[\frac{c(\boldsymbol{p}_0; \bar{\boldsymbol{\varphi}})}{c(\boldsymbol{p}_0; \boldsymbol{\varphi}_0)} \frac{c(\boldsymbol{p}_1; \boldsymbol{\varphi}_1)}{c(\boldsymbol{p}_1; \bar{\boldsymbol{\varphi}})} \right] \tag{11}$$

The term on the right hand side of Eq. 11 is the product of two expenditure ratios. One uses reference period prices to value a taste change from $\boldsymbol{\varphi}_0$ to $\bar{\boldsymbol{\varphi}}$, while the other uses comparison period prices to value a taste change from $\bar{\boldsymbol{\varphi}}$ to $\boldsymbol{\varphi}_1$.[13] If tastes are constant, then $\Phi_C(\boldsymbol{p}_0, \boldsymbol{p}_1; \boldsymbol{\varphi}_0, \boldsymbol{\varphi}_1) = \Phi(\boldsymbol{p}_0, \boldsymbol{p}_1; \bar{\boldsymbol{\varphi}}) = P_{CCV}(\boldsymbol{p}_0, \boldsymbol{p}_1, \boldsymbol{q}_0, \boldsymbol{q}_1, \sigma) = P_{SV}(\boldsymbol{p}_0, \boldsymbol{p}_1, \boldsymbol{q}_0, \boldsymbol{q}_1)$. Therefore, the price index analog to the left hand side of Eq. 11, $\ln P_{CCV}(\boldsymbol{p}_0, \boldsymbol{p}_1, \boldsymbol{q}_0, \boldsymbol{q}_1, \sigma) - \ln P_{SV}(\boldsymbol{p}_0, \boldsymbol{p}_1, \boldsymbol{q}_0, \boldsymbol{q}_1)$, is another indicator of taste change.[14]

Tables 1 and 2 summarize the CES-based COLI and price indexes, as well as the indicators of taste change that will be the basis of the analysis in later sections. For readability of tables and figures, I abbreviate Φ_C for $\Phi_C(\boldsymbol{p}_0, \boldsymbol{p}_1; \boldsymbol{\varphi}_0, \boldsymbol{\varphi}_1)$, $\bar{\Phi}$ for $\Phi(\boldsymbol{p}_0, \boldsymbol{p}_1; \bar{\boldsymbol{\varphi}})$, Φ_0 for $\Phi(\boldsymbol{p}_0, \boldsymbol{p}_1; \boldsymbol{\varphi}_0)$, Φ_1 for $\Phi(\boldsymbol{p}_0, \boldsymbol{p}_1; \boldsymbol{\varphi}_1)$, and drop arguments from P_{CCV}, P_{SV}, P_{LM}, P_{BLM}, and P_{LMM}.

3 Specification error from a missing nest

This section describes the potential missspecification—a neglected nesting structure—that this paper investigates. The nested CES model is a common extension of the model introduced in the last section. For the purposes of this exercise, the nested preferences represent the true model that is either unknown or infeasible to estimate, forcing the analyst to specify Eq. 1 as an approximation. Otherwise, it would be straightforward to estimate versions of P_{SV}, P_{LM}, P_{BLM}, and P_{CCV} for the nested model. In addition to rendering Eq.'s 1 and 2 incorrect, this introduces an additional source of time-varying error into the differenced

[13] The negative of the term on the right hand side of Eq. 11 is the true analog to what RW term "consumer valuation bias," which they estimate as $\ln P_{SV}(\boldsymbol{p}_0, \boldsymbol{p}_1, \boldsymbol{q}_0, \boldsymbol{q}_1) - \ln P_{CCV}(\boldsymbol{p}_0, \boldsymbol{p}_1, \boldsymbol{q}_0, \boldsymbol{q}_1, \sigma)$.

[14] The difference between Φ_C and any ordinal index is an indicator of the pure effect of tastes. I choose to study Eq. 11 because the difference between the CCV and SV indexes is a major focus of RW.

log-share equation.

Suppose the set of goods $\mathcal{I}$ is partitioned in to G exhaustive and mutually exclusive groups $\mathcal{I}_g$, $g = 1, \ldots, G$. The elasticity of substitution between goods now depends on their group membership, which has the effect of allowing some heterogeneity in the closeness of substitutes and relaxing the independence of irrelevant alternatives (IIA) property.

The unit-utility expenditure function is given by:

$$c(\boldsymbol{p}_t, \boldsymbol{\zeta}_t, \boldsymbol{\delta}_t) = \left[\sum_{g=1}^{G} \left(\frac{c_g(\boldsymbol{p}_{gt}, \boldsymbol{\delta}_{gt})}{\zeta_{gt}} \right)^{1-\sigma_a} \right]^{\frac{1}{1-\sigma_a}}, \tag{12}$$

where g indexes group, ζ_{gt}, $g = 1, \ldots, G$, is a group-level taste shifter, and σ_a is the elasticity of substitution across groups. The item-level prices enter the expenditure function through a group-level price index given by:

$$c_g(\boldsymbol{p}_{gt}, \boldsymbol{\delta}_{gt}) = \left[\sum_{i \in \mathcal{I}_g} \left(\frac{p_{it}}{\delta_{it}} \right)^{1-\sigma_g} \right]^{\frac{1}{1-\sigma_g}}, \tag{13}$$

where δ_{it}, $i = 1, \ldots, N_g$, is the item-level taste shifter, and σ_g is the group-specific elasticity of substitution between items in the same group. If $\sigma_1 = \sigma_2 = \cdots = \sigma_G = \sigma_a$, then Eq. 12 collapses to Eq. 1 with $\varphi_{it} = \zeta_{gt}\delta_{it}$.

Consider what happens when the group structure is not taken into account. In this case, the period t share of total (not group) expenditure on good i in group g is, in natural logs:

$$\begin{aligned} \ln s_{it} = {} & (1 - \sigma_g) \ln p_{it} + (\sigma_g - \sigma_a) \ln c_g(\boldsymbol{p}_{gt}, \boldsymbol{\delta}_{gt}) \\ & + (\sigma_a - 1) \ln \left[c(\boldsymbol{p}_t, \boldsymbol{\zeta}_t, \boldsymbol{\delta}_t) \right] + (\sigma_g - 1) \ln \delta_{it} + (\sigma_a - 1) \ln \zeta_{gt} \end{aligned} \tag{14}$$

Eq. 14 implies that in addition to error originating from the taste parameters ζ_{gt} and δ_{it}, a simple regression of $\ln(s_{it}/\tilde{s}_t)$ on $\ln(p_{it}/\tilde{p}_t)$ will include additional sources of time-varying error related to neglected heterogeneity in the substitution elasticities. For this example, the error is straightforward to derive, depending in part on interactions between $\ln(p_{it}/\tilde{p}_t)$ and group indicators. With other models, the form would likely be more complicated. As a consequence, differences such as $CCV - SV$, or $BLM - LM$ may not be indicative of taste change.

4 Monte Carlo Experiment

I evaluate the CES-based price indexes through Monte Carlo experiment, whereby I approximate their behavior by calculating them repeatedly over different, independent draws of the data. The advantages of this method are that the true COLI are observed (since the underlying preference structure is under my control) and that I can repeat the experiment many times. The disadvantage is that the results ultimately depend on the experiment design, and so they are difficult to generalize. Nevertheless, I examine a wide range of conditions and arrive at some interesting conclusions.

4.1 Design

Each replication, I draw time series of prices and taste parameters. For simplicity, I abstract from simultaneity issues and generate the prices exogenously.[15] These feed into the expenditure shares and unit cost equations implied by the nested CES model described in the previous section. Then, I calculate the price indexes as if the group structure were unknown. For each replication, I compare these index values against the different true cost of living change they target.

The type of data I have in mind are disaggregated prices and quantities of the sort found household or retail scanner datasets.[16] For simplicity, I consider a panel of products $i = 1, \ldots, N$ that exist in two time periods, $t = 0, 1$. Log-prices and taste shifters are

[15] The theory in Section 2 does not take a stand on simultaneity per se, but simultaneity would complicate estimation of σ.

[16] I treat the set of available goods as fully observed. This abstracts from issues of item sampling in the CPI, but might reasonable for large scanner datasets.

generated to be independent in the cross-section. Log prices follow a random walk with drift, with the mean log price change set to 0.02 and the standard deviation set to 0.1.[17]

$$\ln p_{i0} = 0.02 + r_{i0} \tag{15}$$

$$\ln p_{i1} = 0.02 + \ln p_{i0} + r_{i1} \tag{16}$$

$$r_{it} \sim \text{i.i.d. Normal}(0, 0.01) \tag{17}$$

The N goods are divided in equal proportion into two groups, $g = 1, 2$. I set the group-level taste shifters both equal to one, and generate the individual taste shifters as:[18]

$$\ln \delta_{it} = \ln \delta_i + \ln \theta_{it}, \tag{18}$$

$$\ln \delta_i \sim \text{Normal}(0, 0.25), \tag{19}$$

$$\ln \theta_{it} \sim \text{i.i.d. Normal}(0, \chi^2). \tag{20}$$

The utility-maximizing quantities are then generated as:

$$q_{it} = \frac{\delta_{it}^{\sigma_g - 1} p_{it}^{-\sigma_g} M_{gt}}{c_g(\boldsymbol{p}_{gt}, \boldsymbol{\delta}_{gt})^{1-\sigma_g}}, \tag{21}$$

where

$$M_{gt} = M_t c_g(\boldsymbol{p}_{gt}, \boldsymbol{\delta}_{gt})^{1-\sigma_a} / c(\boldsymbol{p}_t, \boldsymbol{\zeta}_t, \boldsymbol{\delta}_t)^{1-\sigma_a}, \tag{22}$$

and M_t is income, which is constant and set to 100 without loss of generality (since preferences are homothetic).

The target measures for the experiment are the true conditional and unconditional COLI defined in Section 2, but using the nested CES model (i.e., using Eq. 12 with the appropriate

[17] The standard deviation of price change is reasonable for a product group from a dataset like Nielsen's Scantrack and generates enough variance for substantial substitution effects.

[18] The equation for $\ln \delta_{it}$ satisfies the assumption for $\ln \phi_{it}$ from RW 2018 (a population version of the constant geometric mean normalization), but there will still be finite sample variation between the geometric means of the φ in each period.

taste parameters plugged in).[19] I calculate the price indexes acting as if the group structure is unknown, however, by using shares of total expenditure. Estimation of a substitution elasticity is required for P_{CCV}, P_{LM}, and P_{BLM}. I accomplish this by calculating one minus the OLS slope coefficient of a simple regression of $\Delta \ln s_{it}$ on $\Delta \ln p_{it}$ and a constant.[20] This would be appropriate if the homogeneous model were correct and prices were exogenous, though a real application would surely account for simultaneity. Of course, the regression with homogeneous σ is misspecified when the nested model is true.

Because of the underlying randomness in prices and tastes, the true COLIs, in addition to the two candidate indexes, will change with different draws of the data, as will the differences between the indexes. By design, no index is exact for their target COLI under misspecification, and will generally differ from each other. I hope to approximate the distribution of these random variables with repeated, independent draws of $\{\boldsymbol{p}_0, \boldsymbol{p}_1, \boldsymbol{\delta}_0, \boldsymbol{\delta}_1\}$ (Wooldridge, 2010). I conduct experiments for two main cases. First, as a benchmark, I set $\sigma_1 = \sigma_2 = \sigma_a = 4$. This provides a baseline for evaluating the price indexes when the implied model is correctly specified. Second, I consider varying degrees of misspecification by setting $\sigma_a = 3$, $\sigma_1 = 4$, and varying σ_2 over the interval $[4, 10]$. The empirical application in Section 5 suggests these are reasonable parameter values. For each case, I vary the degree of taste change, with $\chi \in \{0, 0.25, 0.5\}$. The $\chi = 0$ case represents no taste change, $\chi = 0.25$ represents low to moderate taste change (the idiosyncratic variance is less than the cross-section variation), and the $\chi = 0.5$ case represents a high degree of taste change (the idiosyncratic variance is equal to the cross-section variance).[21]

My results are for a cross-section of size $2,000$, which is on the order of a moderately-sized

[19] The intermediate level of tastes, $\bar{\varphi}$ for which the SV index is exact were calculated using the appendix to Feenstra and Reinsdorf (2007).

[20] Note: in this two period model, a time dummy is collinear with a constant in the differenced equation.

[21] RW report percentiles of the distributions of estimates of $var\left[(\sigma-1)\log\varphi_{it}\right]$ and $var\left[(\sigma-1)\Delta\log\varphi_{it}\right]$ for 104 product groups from Nielsen's household scanner data. My taste parameter choices produce estimates of these demand variances that fall in RW's reported range when the model is correctly specified model. When $\chi = 0.25$, this continues to hold for all but the $\sigma_2 = 10$ case. When $\chi = 0.50$, higher values of σ_2 produce estimates of $var\left[(\sigma-1)\Delta\log\varphi_{it}\right]$ that exceed those reported by RW. Note: None of the price indexes considered place restrictions on either of these variances.

product group in scanner datasets. All statistics are based on 5,000 replications using Stata.

4.2 Results

As discussed in Section 2, when tastes vary, different true COLI can be defined for the CES model (summarized in Table 1), and certain differences between them reflect the impact of changing tastes (Table 2). All simulation tables are natural logs or differences in natural logs, so units can be interpreted as approximate percentage change or percentage point differences when multiplied by 100. Appendix Tables A1, A2, and A3 give the means and standard deviations of the true COLI themselves. Table 3 presents means and standard distributions for these true taste change indicators for the simulations described in the previous subsection. Note that when taste parameters are constant over time, both taste change indicators are identically zero in each replication, as all four true COLI are identical. The ordinal taste change effect, $\ln \Phi_1 - \ln \Phi_0$ has a mean close to zero regardless of the model or the degree of time series variation in tastes, but the standard deviation indicates significant dispersion that increases both as tastes vary more and as substitution is more heterogeneous between groups. The cardinal taste change effect, $\ln \Phi_C - \ln \bar{\Phi}$ has a mean slightly below zero, between -0.07 and -0.24 percentage points, and this decreases as χ or σ_2 increase. This indicator is highly dispersed, with standard deviations between 2.8 and 11.2 percentage points, roughly three to four times as large as their ordinal counterpart's.[22]

I define an estimation error as the natural log of the price index minus the natural log of the appropriate true COLI (see Table 1). I then calculate the bias and root mean squared error (RMSE) for each price index for each set of model parameters, which are presented in Appendix Tables A4-A6. Figures 1 and 2 summarize these results. The bars show the bias or RMSE of an index for a particular level of χ as the substitution heterogeneity increases, with the first bar from the left representing the simple CES model and subsequent bars representing the nested model with varying σ_2.

[22] Appendix Tables A1-A3 suggest this variability might be related dispersion in $\ln \Phi_C$, though I did not calculate covariances between true indexes for a complete decomposition.

As would be expected, bias for all of the price indexes is zero or virtually zero in the simple CES case (regardless of the degree of taste change), for which the underlying models are correctly specified. Bias is also very small for the least heterogeneous nested model, where $\sigma_2 = 4$. As σ_2 in the true nested model increases, the simple CES model is a poorer approximation. Bias in the BLM, LM, and CCV indexes increases substantially, and in many cases exceeds the mean and standard deviation of the associated true COLI. Bias appears to be flat or slightly decreasing as χ increases, with the exception of the SV index, for which bias is low, but slightly increasing. BLM and LM are clearly biased in opposite directions, with BLM tending to miss high and LM tending to miss low. CCV errors were more evenly split between positive and negative, but as the figure indicates, tended to be negative. Finally, both the LMM indexes and SV indexes performed relatively well under misspecification. For the LMM index, it suggests that the biases in BLM and LM offset when taking the geometric mean, and the result conforms with what is known about Quadratic Mean of Order r price indexes. For the SV index, the bias increases with σ_2, but remains relatively low. This suggests, perhaps, that the SV index is approximately consistent in aggregation in the sense described by Diewert (1978).

The patterns for the RMSE (Figure 2) are similar to the bias, though in this case, greater degrees of taste change lead to larger errors. It is also notable that errors in the CCV index are substantially larger than those of the other indexes, though the comparison is less dramatic if we normalize by the standard deviations of the associated true COLI. The CCV index also has significant errors when the model is correctly specified, which is possibly due to the normalization of the true taste parameters holding in expectation (as per Eq. 2 of RW) rather than in finite samples.

As a consequence, the estimated index differences, which under correct specification are indicators of taste change, lead to misleading inferences when the model is incorrectly specified. Table 4 presents the biases and RMSEs of the estimated indicators, whiles Figure 3 and 4 illustrate their performance graphically. Specification errors push apart the the price index pairs even if the underlying COLI are close, meaning BLM will tend to exceed LM, and SV will tend to exceed CCV, to an excess degree reflecting the neglected substitution heterogeneity. This can occur even if tastes are constant. On the other hand, the results for the SV and LMM indexes are encouraging for the estimation of conditional COLI in models with taste change and potential misspecification. Even though it appears difficult to separately identify the contributions of changing tastes, COLI reflecting different notions of "average" tastes are still estimable.

5 Application: Fresh Produce

The previous sections suggest model choice may have a significant impact on price index results related to taste change. In this section, I illustrate the issue by estimating a simple and nested CES model for fresh produce using retail scanner data (sometimes called Scantrack) from Nielsen.[23] The data cover the fourth quarter of 2005 through the second quarter of 2010, and include total sales and quantities for about 4500 unique universal product codes (UPC) or equivalent. Because products are defined by UPC, characteristics or quality are (in theory), constant (Broda and Weinstein, 2010). The dataset includes mainly large retail grocery and drug store chains, but still covers roughly 85% of total consumer expenditures on fresh fruits and vegetables as estimated by the BLS (Bureau of Labor Statistics, 2013). Within this product group, Nielsen organizes items into 22 product modules. For example, salad mix, potatoes, and apples are all separate modules. The number of items in each module ranges from 3 to 958, with a median of 128. Though RW use Nielsen's household scanner data, they are classified similarly. Their methodology estimates a single elasticity of substitution for the entire fresh produce product group. A priori, we might think substitution behavior within and across more finely defined strata is heterogeneous. For instance, a consumer might be more willing to substitute within varieties of onions than within varieties of apples,

[23] CES preferences have the property that optimization over a subset of goods does not depend on goods outside the subset (Deaton and Muellbauer, 1980).

and might be even less willing to substitute apples for onions. If we view the true model as CES nested by module, how much might our conclusions change if we ignore the nest? To get at this, I estimate both versions of the model and calculate the CES-based price indexes for each.

Prior papers (including prior work by RW, though not calculating a price index with taste change) has recognized potential heterogeneity and have nested by module, producing firm or even brand.[24] Often finer product classifications are modeled at the expense of having to impose numerous parameter restrictions due to small sample sizes.[25] In the Scantrack data, estimating elasticities within and across modules for every product group would be similarly challenging, but Fresh Produce is one of the better candidates. Moreover, since products are perishable, I am less concerned about threats to the model from neglected dynamics (i.e., as explored in Osborne (2018)).

Assembly and data cleaning follow Broda and Weinstein (2010), which is similar in this regard to RW. As such, I use national totals by UPC, aggregating across markets outlets.[26] As in Broda and Weinstein (2010), identification and estimation of the substitution elasticities follows the "double-differencing" method of Feenstra (1994), using panel variation in prices and expenditure shares as opposed to instrumental variables, which are often unavailable in scanner datasets.

Tables 5 through 8 present the results. To begin, the estimate of σ based on the simple model is 2.94, which is reasonable given the aforementioned papers using similar data. The CCV index based on this estimate of σ is given in the second column of Table 7. The base period is the same quarter in the previous year, and the values are presented as percent changes. The simple average of CCV index over 2006Q4-2010Q2 is -2.4%. In contrast,

[24] See Jaravel (2016), Hottman, Redding, and Weinstein (2016), and Broda and Weinstein (2010).

[25] Broda and Weinstein (2010) restrict elasticities within and across brand module to be the same within the entire product group, adding only one more parameter. Hottman, Redding, and Weinstein (2016) similarly restrict the elasticities within and across firm. Jaravel (2016) estimates a separate elasticity within each module, but does not estimate elasticities of substitution across modules.

[26] Handbury (2013) and Lecznar and Smith (2018) are two examples where heterogeneity by consumer group, outlet, or geography play a significant role in COLI estimation.

the SV index estimates an average 1.1% yearly rise in the cost-of-living, a difference of 3.2 percentage points. The differences in any particular quarter ranges from 0.1 to 8.9 percentage points. This suggests an interpretation that the pure effect of changing tastes is to lower the estimated cost-of-living significantly, so much in fact that costs for these products have actually fallen. The differences between the CCV and SV values are of a similar magnitude for what RW report for a more broad range of food, beverage, and general merchandise products. Meanwhile, columns 2-4 of Table 8 compare the two Lloyd-Moulton variants based on $\sigma = 2.94$, which correspond to conditional COLI where tastes are fixed at different points–for BLM, the taste vector corresponds to the current quarter, whereas for LM, it corresponds to four quarters prior. On average, using current tastes to evaluate price change leads to 2.1% higher inflation than using reference tastes.

The results change substantially when I estimate the nested model. First, Panel B of Table 5, as well as Table 6 indicate considerable heterogeneity in the substitution elasticity estimates across modules.[27] The within-module point estimates ranged from 3.37 to 55.64, with a share-weighted median of 5.16. The elasticity of substitution of products across modules was estimated as 4.37. Because they are based on smaller samples, some of the module-specific estimates are imprecisely estimated, but many are still significantly different from the simple model's estimate. In terms of within-module versus across-module substitution, the estimates tend to fit the expected pattern, with the within-elasticity exceeding the across-elasticity for 17 product groups.

The resulting indexes are compared in columns 5-7 of Tables 7 and 8. As expected the SV index is little changed by adding the nest, still averaging 1.1% year-over-year inflation. The nested CCV index, on the other hand, gives an average inflation figure of 0.2%, about 2.6 percentage points higher than the simple version. The quarterly differences between the nested SV and nested CCV range from -3.1 to 4.6 percentage points, with the average difference at 0.9 percentage points. Not only does this imply a much lower "consumer

[27] There are only 21 estimates listed, as the "Cranberries" module had insufficient observations.

valuation bias" on average, but unlike the simple CCV, where accounting for taste change always pushes the COLI lower, the nested CCV implies taste change actually pushes the COLI higher in four of the quarters. Figure 5 illustrates this graphically. Both models could be interpreted as implying a significant role for tastes, but the magnitudes, as well as the bottom line direction of the cardinal COLI are quite different.[28]

While the nested model implies greater similarity between SV and CCV, these data imply a greater difference between BLM and LM than suggested by the simple model, as illustrated in Figure 6. The nested LM index is lower and the nested BLM index is higher than their respective non-nested counterparts. The average gap between BLM and LM jumps to 8.0 percentage points with the nested model, with individual quarterly differences ranging from 5.7 to 11.9 percentage points. At face value, these results suggest the choice of taste vector has a larger impact on the conditional COLI when the model is nested. If we view the nested CES model as closer to the truth, then these results run contrary to what the Monte Carlo experiment's finding that misspecification was associated with a wider BLM-LM gap.

Of course, the nested CES model could also be misspecified, sampling error in the estimation of the elasticities could play a role, so those results should be interpreted with caution. There are clearly practical trade-offs an analysts faces if implementing a model-based COLI estimate like CCV, LM or BLM. The imprecision of the module-specific elasticity estimates is just one example. Model selection would ideally be justified by substantive robustness checks. This exercise raises an encouraging point, however. Figures 5 and 6 suggest that both the SV and LMM indexes are much less sensitive to the nesting structure chosen. Again, the SV is exact for a COLI for an average level of tastes, and the LMM index exact for the average of two COLI evaluated at different tastes. This suggests, therefore, that while model specification plays a large role in quantifying the effects of taste change, it may be less consequential for estimating a conditional COLI which reflects average tastes.

[28]These results hold qualitatively if I add a Feenstra-style adjustment for entering and exiting goods. The results are available from the author by request.

6 Conclusion

This paper shows that the assumed preference structure needs to be taken seriously if many CES-based COLI estimates are of interest. Even in a case where true preferences are in the CES family, neglected heterogeneity in substitution elasticities across groups of products can lead to errors in the measurement of inflation. As a result, it is possible for empirical studies to misdiagnose or overestimate the impact of changing tastes. One can always employ a richer nested CES model, or even a more flexible cost function like the translog, but model estimation will always require the analyst to draw the line somewhere between generality and tractability. On the other hand, while this paper suggests that specification errors confound estimates of taste change effects, indexes like the Sato-Vartia, which estimate COLI that reflect average tastes, appear relatively robust to model misspecification.

This paper explores a missing nest as a potential misspecification because it is not only a simple extension of RW's framework, but also is convenient to simulate. Of course, there are many other such forms of heterogeneity that have been explored in the price index literature in other contexts—varying tastes across consumers, nonhomotheticity, outlet substitution, and geographic aggregation, to name a few—and these should be examined carefully as well. Nailing down these issues, in addition to critically examining the conceptual target, will be a crucial in the development of model-based COLI measures for use in national statistical programs.

References

Broda, Christian and David E. Weinstein (2010). "Product Creation and Destruction: Evidence and Price Implications". In: *American Economic Review* 100, pp. 691–723.

Bureau of Labor Statistics (Aug. 2013). *Average annual expenditures and characteristics of all consumer units, Consumer Expenditure Survey, 2006-2012*. Tech. rep. URL: `https://www.bls.gov/cex/2012/standard/multiyr.pdf`.

— (2018). "Chapter 17. The Consumer Price Index". In: *Handbook of Methods*. Bureau of Labor Statistics.

Deaton, Angus and John Muellbauer (1980). *Economics and Consumer Behavior*. Cambridge University Press.

Diewert, W. Erwin (1978). "Superlative index numbers and consistency in aggregation". In: *Econometrica* 46.4, pp. 883–900.

Feenstra, Robert C. (1994). "New Product Varieties and the Measurement of International Prices". In: *American Economic Review*, pp. 157–177.

Feenstra, Robert C. and Marshall B. Reinsdorf (2007). "Should Exact Index Numbers Have Standard Errors? Theory and Application to Asian Growth". In: *Hard-to-Measure Goods and Services: Essays in Honor of Zvi Griliches*. University of Chicago Press, pp. 483–513.

Feenstra, Robert C. and David E. Weinstein (2017). "Globalization, Markups, and U.S. Welfare". In: *Journal of Political Economy* 125.4, pp. 1040–1074.

Fisher, Franklin M. and Karl Shell (1972). "Taste and Quality Change in the Pure Theory of the True Cost-of-Living Index". In: *The Economic Theory of Price Indices*. Academic Press, pp. 1–48.

Handbury, Jessie (2013). "Are Poor Cities Cheap for Everyone? Non-Homotheticity and the Cost of Living Across U.S. Cities". Working paper.

Heien, Dale and James Dunn (1985). “The True Cost-of-Living Index with Changing Preferences”. In: *Journal of Business & Economic Statistics* 3.4, pp. 332–335.

Hottman, Colin J., Stephen J. Redding, and David E. Weinstein (2016). “Quantifying the Sources of Firm Heterogeneity”. In: *The Quarterly Journal of Economics* 131.3, pp. 1291–1364.

ILO (2004). *Consumer Price Index Manual: Theory and Practice*. Ed. by Peter Hill. Jointly published by ILO, IMF, OECD, UNECE, Eurostat, and the World Bank.

Jaravel, Xavier (2016). “The unequal gains from product innovations: Evidence from the US retail sector”. Unpublished. Version dated Apr. 7, 2017.

Lecznar, Jonathon and Arthur Smith (2018). “Geographic Aggregation and the Measurement of Real Consumption Growth and Volatility”. In: *Available at SSRN 3048600*.

Lloyd, Peter J. (1975). “Substitution effects and biases in nontrue price indices”. In: *The American Economic Review* 65.3, pp. 301–313.

Martin, Robert S. (2020). “Revisiting taste change in cost-of-living measurement”. BLS Working Paper 515.

Moulton, Brent R. (1996). “Constant Elasticity Cost-of-Living Index in Share Relative Form”. BLS Working Paper.

Muellbauer, John (1975). “The Cost of Living and Taste and Quality Change”. In: *Journal of Economic Theory* 10.3, pp. 269–283.

National Research Council (2002). *At What Price?: Conceptualizing and Measuring Cost-of-Living and Price Indexes*. Ed. by Charles Schultze and Christopher Mackie. National Academies Press.

Nevo, Aviv (2003). "New Products, Quality Changes, and Welfare Measures Computed from Estimated Demand Systems". In: *Review of Economics and Statistics* 85.2, pp. 266–275.

Osborne, Matthew (2018). "Approximating the cost-of-living index for a storable good". In: *American Economic Journal: Microeconomics* 10.2, pp. 286–314.

Pollak, Robert A. (1989). *The Theory of the Cost-of-Living Index*. Oxford University Press on Demand.

Redding, Stephen J. and David E. Weinstein (2018). "Measuring Aggregate Price Indexes with Demand Shocks: Theory and Evidence for CES Preferences". NBER Working Paper.

— (2020). "Measuring Aggregate Price Indices with Taste Shocks: Theory and Evidence for CES Preferences". In: *The Quarterly Journal of Economics* 135.1, pp. 503–560.

Sato, Kazuo (1976). "The ideal log-change index number". In: *The Review of Economics and Statistics*, pp. 223–228.

US Department of Labor, Bureau of Labor Statistics (2018). *BLS Handbook of Methods*.

Vartia, Yrjö O (1976). "Ideal Log-Change Index Numbers". In: *Scandinavian Journal of Statistics*, pp. 121–126.

Wooldridge, Jeffrey M. (2010). *Econometric analysis of cross section and panel data*. MIT Press.

Tables

Table 1: CES Price Index Summary

True COLI	Price Index
Φ_0	P_{LM}
Φ_1	P_{BLM}
$\bar{\Phi}$	P_{SV}
Φ_C	P_{CCV}

Table 2: CES Indicators of Taste Change

Truth	Estimator
$\ln \Phi_1 - \ln \Phi_0$	$\ln P_{BLM} - \ln P_{LM}$
$\ln \Phi_C - \ln \bar{\Phi}$	$\ln P_{CCV} - \ln P_{SV}$

Table 3: Empirical Distributions of True Taste Change Indicators

		CES	Nested CES: $\sigma_a = 3, \sigma_1 = 4$			
		$\sigma = 4$	$\sigma_2 = 4$	$\sigma_2 = 6$	$\sigma_2 = 8$	$\sigma_2 = 10$
		$\chi = 0.25$				
$\ln \Phi_C - \ln \bar{\Phi}$	Mean	-0.0007	-0.0007	-0.0013	-0.0015	-0.0015
	SD	0.0282	0.0278	0.0398	0.0445	0.0468
$\ln \Phi_1 - \ln \Phi_0$	Mean	0.0000	0.0000	0.0001	0.0001	0.0002
	SD	0.0080	0.0078	0.0125	0.0147	0.0159
		$\chi = 0.50$				
$\ln \Phi_C - \ln \bar{\Phi}$	Mean	-0.0016	-0.0016	-0.0022	-0.0024	-0.0024
	SD	0.0762	0.0742	0.1012	0.1090	0.1122
$\ln \Phi_1 - \ln \Phi_0$	Mean	0.0000	0.0000	0.0001	0.0002	0.0003
	SD	0.0203	0.0195	0.0282	0.0311	0.0324

Table 4: Empirical Performance of Taste Change Indicators

		CES	Nested CES: $\sigma_a = 3, \sigma_1 = 4$			
		$\sigma = 4$	$\sigma_2 = 4$	$\sigma_2 = 6$	$\sigma_2 = 8$	$\sigma_2 = 10$
		$\chi = 0$				
$\ln P_{CCV} - \ln P_{SV}$	Bias	0.0000	0.0000	-0.0021	-0.0049	-0.0069
	RMSE	0.0000	0.0001	0.0078	0.0188	0.0285
$\ln P_{BLM} - \ln P_{LM}$	Bias	0.0000	0.0000	0.0063	0.0155	0.0251
	RMSE	0.0000	0.0001	0.0064	0.0158	0.0255
		$\chi = 0.25$				
$\ln P_{CCV} - \ln P_{SV}$	Bias	0.0001	0.0001	-0.0025	-0.0058	-0.0081
	RMSE	0.0083	0.0083	0.0299	0.0629	0.0896
$\ln P_{BLM} - \ln P_{LM}$	Bias	0.0000	0.0000	0.0062	0.0154	0.0249
	RMSE	0.0024	0.0024	0.0072	0.0163	0.0260
		$\chi = 0.50$				
$\ln P_{CCV} - \ln P_{SV}$	Bias	0.0000	0.0001	-0.0017	-0.0037	-0.0050
	RMSE	0.0204	0.0202	0.0618	0.1111	0.1476
$\ln P_{BLM} - \ln P_{LM}$	Bias	0.0000	0.0001	0.0060	0.0147	0.0238
	RMSE	0.0047	0.0047	0.0091	0.0175	0.0269

Table 5: Substitution Elasticity Estimates for Fresh Produce

Panel A: No Nesting

Estimate	Value	95% CI
$\hat{\sigma}$	2.94	$[2.81, 3.06]$

Panel B: Nested by Module

Estimate	Value	95% CI
$\hat{\sigma}_a$	4.37	$[3.14, 5.60]$

Distribution of $\hat{\sigma}_g$

Mean	7.78
P25	3.73
Med.	5.16
P75	11.08

Note: Based on data provided by the Nielsen Company.

Table 6: Elasticity Estimates by Module

Module	Exp. share	$\hat{\sigma}_g$	95% CI
Salad Mix	0.18	3.73	$[2.62, 4.84]$
Other Fruit	0.13	6.87	$[4.96, 8.79]$
Potatoes	0.12	4.04	$[3.32, 4.76]$
Strawberries	0.10	18.59	$[8.85, 28.34]$
Lettuce	0.07	17.42	$[10.94, 23.90]$
Carrots	0.07	11.08	$[6.50, 15.66]$
Other Vegetables	0.06	3.51	NA
Tomatoes	0.05	4.17	$[3.62, 4.71]$
Mushrooms	0.05	3.37	$[2.30, 4.45]$
Apples	0.04	8.31	$[6.70, 9.94]$
Onions	0.03	8.55	$[4.59, 12.51]$
Celery	0.03	5.16	$[2.86, 7.46]$
Oranges	0.02	6.19	$[5.90, 6.48]$
Spinach	0.02	3.71	$[1.99, 5.45]$
Herbs	0.01	8.86	$[-1.41, 19.13]$
Cauliflower	0.01	7.72	$[2.02, 13.42]$
Grapefruit	0.00	12.71	$[10.46, 14.96]$
Garlic	0.00	4.05	$[1.81, 6.30]$
Sprouts	0.00	18.01	$[2.38, 33.64]$
Radishes	0.00	5.94	$[1.25, 10.63]$
Kiwi	0.00	55.64	$[-0.44, 111.71]$

Note: Based on data provided by the Nielsen Company.

Table 7: SV and CCV Index Comparison (base period = same quarter a year ago)

	No Nest			Nested by Module		
Quarter	SV	CCV	Difference	SV	CCV	Difference
2006Q4	3.6%	3.6%	0.1%	3.6%	3.4%	0.2%
2007Q1	4.9%	3.0%	2.0%	5.0%	4.1%	0.9%
2007Q2	4.4%	1.3%	3.1%	4.4%	3.0%	1.4%
2007Q3	1.3%	-0.9%	2.2%	1.3%	0.5%	0.8%
2007Q4	1.3%	-4.0%	5.2%	1.2%	-1.2%	2.4%
2008Q1	0.0%	-2.1%	2.1%	0.0%	-0.7%	0.6%
2008Q2	0.6%	0.3%	0.3%	0.6%	1.0%	-0.4%
2008Q3	8.2%	1.9%	6.3%	8.1%	3.8%	4.3%
2008Q4	7.1%	-1.8%	8.9%	7.1%	2.5%	4.6%
2009Q1	1.6%	-4.9%	6.4%	1.5%	-1.6%	3.1%
2009Q2	-2.1%	-7.3%	5.3%	-2.0%	-3.7%	1.7%
2009Q3	-7.9%	-9.7%	1.8%	-7.9%	-4.8%	-3.1%
2009Q4	-6.2%	-7.3%	1.1%	-6.2%	-3.7%	-2.5%
2010Q1	-0.8%	-6.0%	5.2%	-0.7%	-0.7%	0.0%
2010Q2	1.0%	-2.0%	2.9%	1.0%	1.7%	-0.7%
Average	1.1%	-2.4%	3.5%	1.1%	0.2%	0.9%

Note: Based on data provided by the Nielsen Company.

Table 8: LM Index Comparison (base period = same quarter a year ago)

	No Nest			Nested by Module		
Quarter	BLM	LM	Difference	BLM	LM	Difference
2006Q4	4.4%	3.0%	1.4%	8.5%	2.2%	6.4%
2007Q1	6.5%	3.9%	2.6%	13.6%	1.8%	11.9%
2007Q2	6.0%	3.3%	2.6%	9.5%	-0.2%	9.7%
2007Q3	2.1%	0.7%	1.4%	5.7%	-1.6%	7.4%
2007Q4	1.9%	0.6%	1.4%	4.3%	-3.6%	7.9%
2008Q1	0.6%	-0.8%	1.5%	3.1%	-2.9%	6.0%
2008Q2	1.3%	-0.3%	1.6%	4.4%	-3.3%	7.8%
2008Q3	9.6%	6.8%	2.8%	13.2%	4.2%	9.0%
2008Q4	8.7%	5.5%	3.2%	11.0%	3.2%	7.8%
2009Q1	2.7%	0.7%	1.9%	5.3%	-3.0%	8.3%
2009Q2	-1.5%	-2.7%	1.2%	0.3%	-5.5%	5.8%
2009Q3	-7.1%	-8.7%	1.6%	-5.5%	-11.2%	5.7%
2009Q4	-4.9%	-7.8%	2.9%	-1.5%	-11.0%	9.5%
2010Q1	0.5%	-2.2%	2.7%	3.2%	-5.1%	8.3%
2010Q2	2.3%	-0.1%	2.4%	5.5%	-2.6%	8.2%
Average	2.2%	0.1%	2.1%	5.4%	-2.6%	8.0%

Note: Based on data provided by the Nielsen Company.

Figures

Figure 1: Empirical Bias

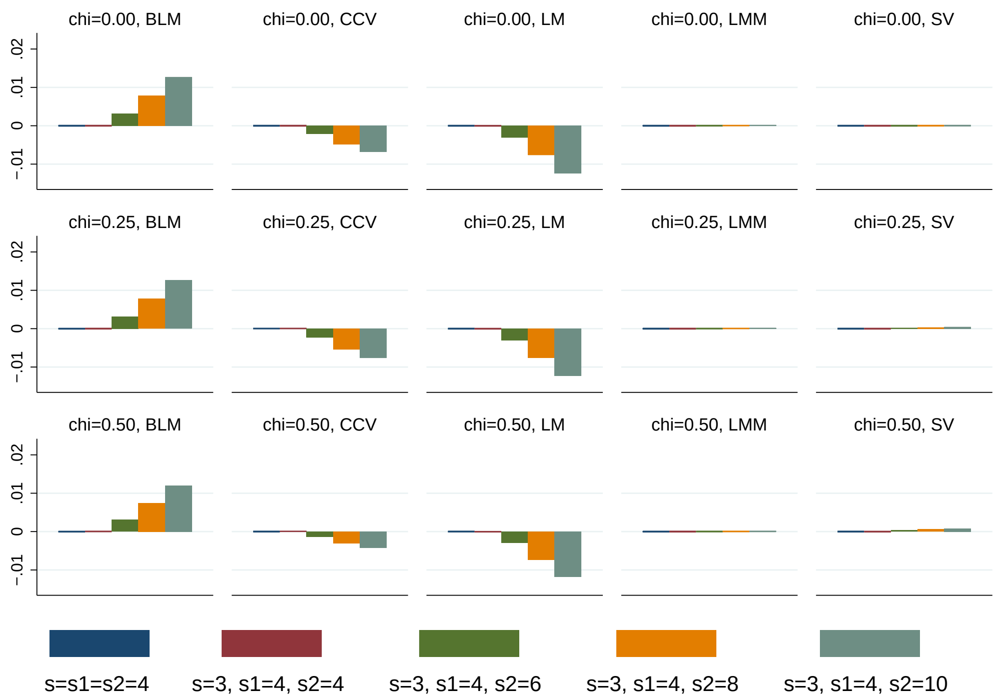


Figure 2: Empirical Root Mean Squared Errors

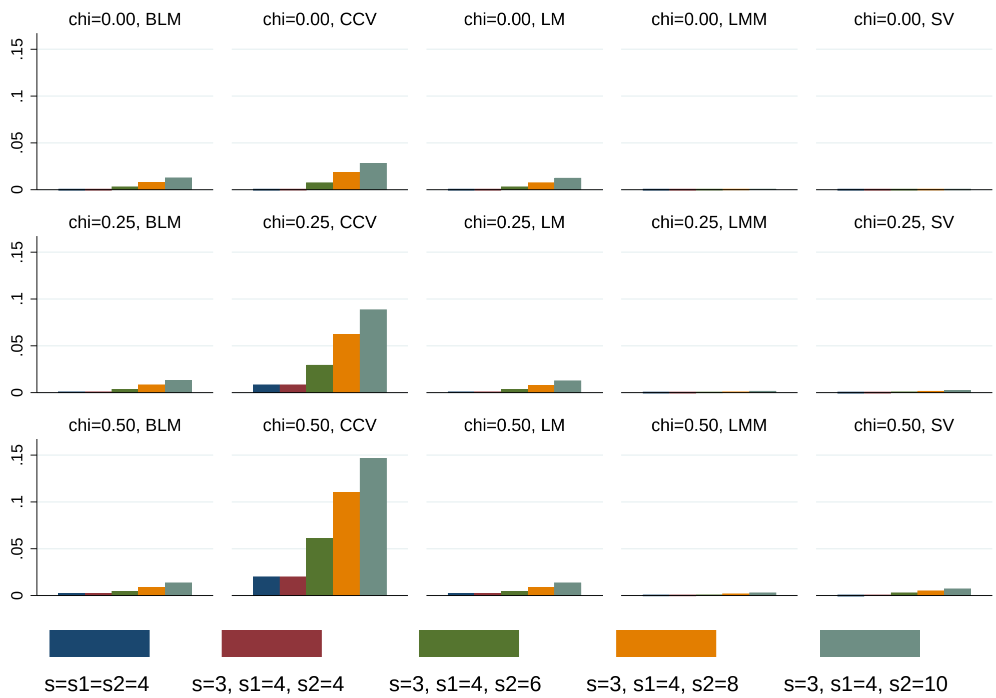


Figure 3: Empirical Bias for Taste Change Indicators

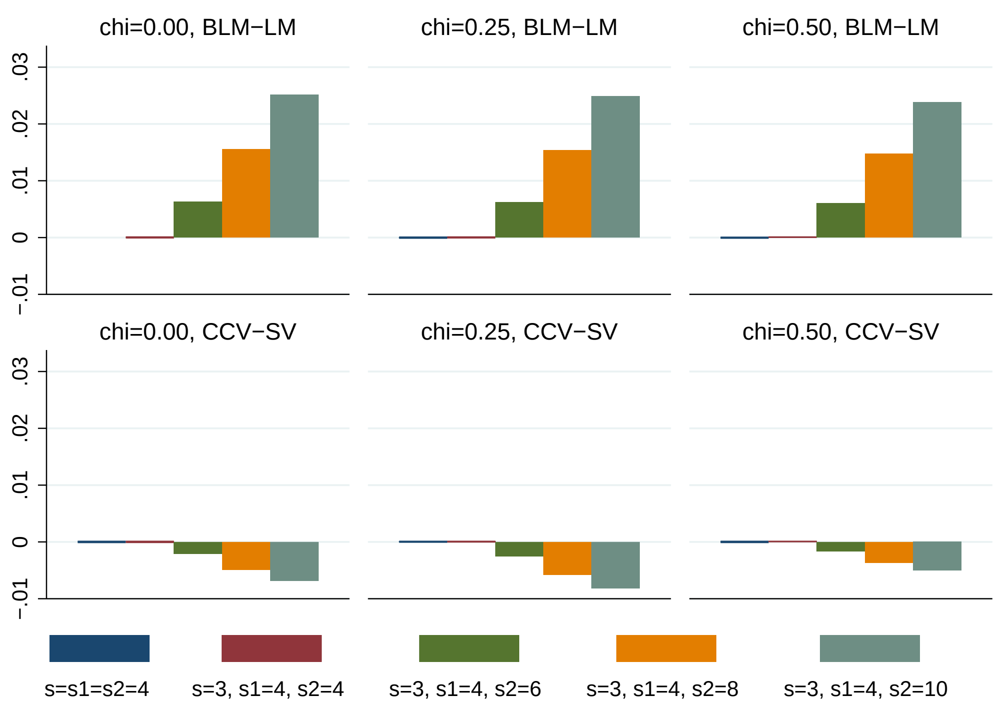


Figure 4: Empirical RMSE for Taste Change Indicators

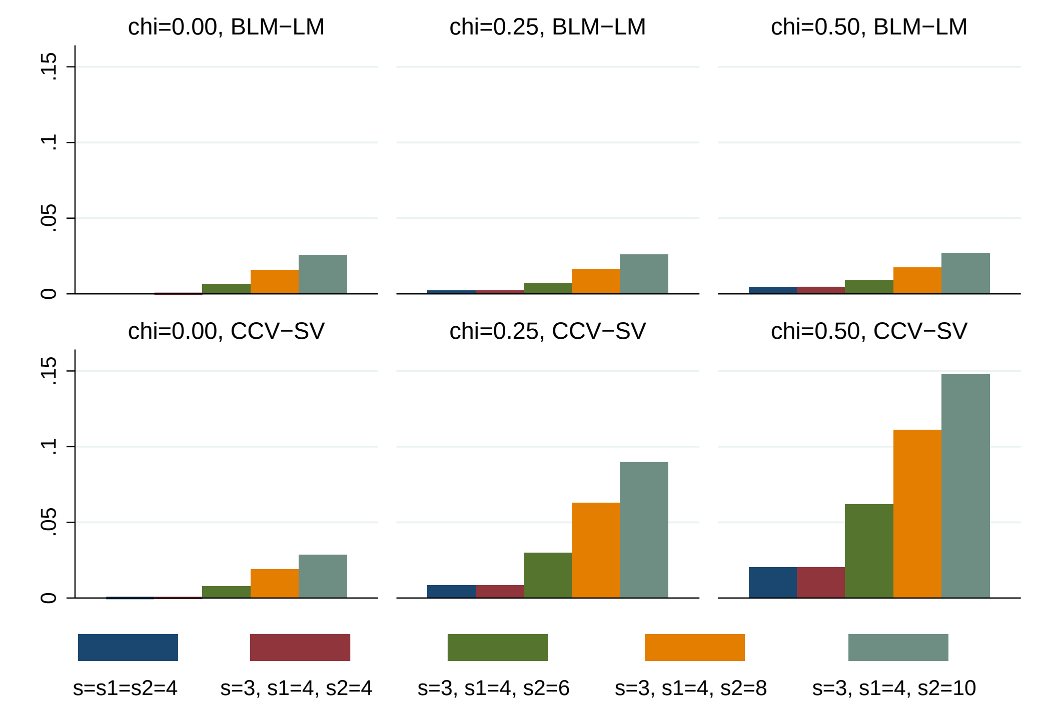


Figure 5: Index Comparison (base = same quarter a year ago)

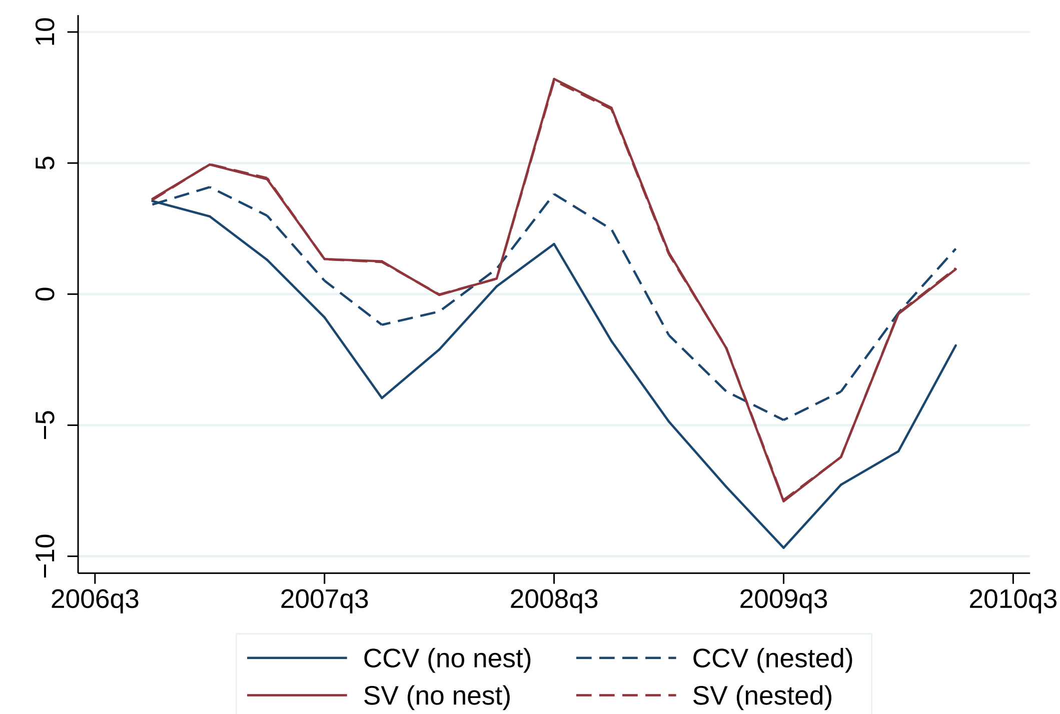
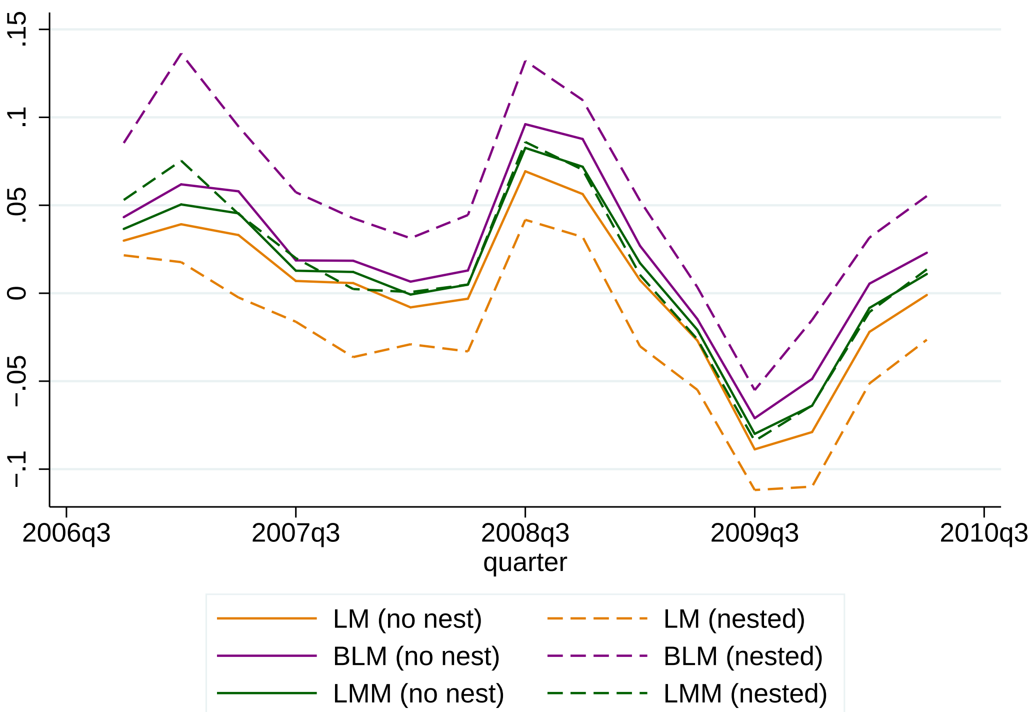


Figure 6: Lloyd-Moulton Index Comparison (base = same quarter a year ago)



A Additional Simulation Tables

Table A1: Empirical Distribution of true COLI, $\chi = 0$ (in natural logs)

		CES	Nested CES: $\sigma_a = 3, \sigma_1 = 4$			
		$\sigma = 4$	$\sigma_2 = 4$	$\sigma_2 = 6$	$\sigma_2 = 8$	$\sigma_2 = 10$
Φ_C	Mean	0.0051	0.0051	0.0033	0.0031	0.0031
	SD	0.0071	0.0071	0.0102	0.0118	0.0126
$\bar{\Phi}$	Mean	0.0051	0.0051	0.0033	0.0031	0.0031
	SD	0.0071	0.0071	0.0102	0.0118	0.0126
Φ_0	Mean	0.0051	0.0051	0.0033	0.0031	0.0031
	SD	0.0071	0.0071	0.0102	0.0118	0.0126
Φ_1	Mean	0.0051	0.0051	0.0033	0.0031	0.0031
	SD	0.0071	0.0071	0.0102	0.0118	0.0126
$\sqrt{\Phi_0 \Phi_1}$	Mean	0.0051	0.0051	0.0033	0.0031	0.0031
	SD	0.0071	0.0071	0.0102	0.0118	0.0126

Table A2: Empirical Distribution of true COLI, $\chi = 0.25$ (in natural logs)

		CES	Nested CES: $\sigma_a = 3, \sigma_1 = 4$			
		$\sigma = 4$	$\sigma_2 = 4$	$\sigma_2 = 6$	$\sigma_2 = 8$	$\sigma_2 = 10$
Φ_C	Mean	0.0045	0.0045	0.0021	0.0017	0.0015
	SD	0.0294	0.0290	0.0415	0.0463	0.0486
$\bar{\Phi}$	Mean	0.0052	0.0052	0.0034	0.0031	0.0031
	SD	0.0080	0.0079	0.0114	0.0129	0.0137
Φ_0	Mean	0.0052	0.0052	0.0034	0.0031	0.0031
	SD	0.0088	0.0087	0.0128	0.0146	0.0155
Φ_1	Mean	0.0052	0.0052	0.0035	0.0033	0.0032
	SD	0.0091	0.0089	0.0132	0.0149	0.0157
$\sqrt{\Phi_0 \Phi_1}$	Mean	0.0052	0.0052	0.0034	0.0032	0.0031
	SD	0.0080	0.0079	0.0114	0.0128	0.0135

Table A3: Empirical Distribution of true COLI, $\chi = 0.50$ (in natural logs)

		CES	Nested CES: $\sigma_a = 3, \sigma_1 = 4$			
		$\sigma = 4$	$\sigma_2 = 4$	$\sigma_2 = 6$	$\sigma_2 = 8$	$\sigma_2 = 10$
Φ_C	Mean	0.0036	0.0038	0.0015	0.0011	0.0010
	SD	0.0771	0.0751	0.1023	0.1101	0.1132
$\bar{\Phi}$	Mean	0.0053	0.0054	0.0037	0.0035	0.0035
	SD	0.0110	0.0107	0.0154	0.0171	0.0180
Φ_0	Mean	0.0054	0.0055	0.0039	0.0037	0.0037
	SD	0.0155	0.0150	0.0214	0.0235	0.0245
Φ_1	Mean	0.0054	0.0055	0.0040	0.0039	0.0040
	SD	0.0160	0.0154	0.0218	0.0238	0.0247
$\sqrt{\Phi_0 \Phi_1}$	Mean	0.0054	0.0055	0.0040	0.0038	0.0038
	SD	0.0121	0.0117	0.0164	0.0179	0.0185

Table A4: Empirical Performance of Price Indexes, $\chi = 0$

		CES	Nested CES: $\sigma_a = 3, \sigma_1 = 4$			
		$\sigma = 4$	$\sigma_2 = 4$	$\sigma_2 = 6$	$\sigma_2 = 8$	$\sigma_2 = 10$
CCV	Bias	0.0000	0.0000	-0.0021	-0.0049	-0.0068
	RMSE	0.0000	0.0001	0.0078	0.0187	0.0282
SV	Bias	0.0000	0.0000	0.0000	0.0000	0.0001
	RMSE	0.0000	0.0000	0.0001	0.0002	0.0003
LM	Bias	0.0000	0.0000	-0.0031	-0.0077	-0.0124
	RMSE	0.0000	0.0000	0.0032	0.0078	0.0126
BLM	Bias	0.0000	0.0000	0.0032	0.0079	0.0127
	RMSE	0.0000	0.0000	0.0032	0.0080	0.0130
LMM	Bias	0.0000	0.0000	0.0000	0.0001	0.0002
	RMSE	0.0000	0.0000	0.0002	0.0004	0.0008

Note: Statistics are for the natural logs of the price indexes.

Table A5: Empirical Performance of Price Indexes, $\chi = 0.25$

		CES	Nested CES: $\sigma_a = 3, \sigma_1 = 4$			
		$\sigma = 4$	$\sigma_2 = 4$	$\sigma_2 = 6$	$\sigma_2 = 8$	$\sigma_2 = 10$
CCV	Bias	0.0001	0.0001	-0.0023	-0.0055	-0.0077
	RMSE	0.0083	0.0083	0.0296	0.0624	0.0889
SV	Bias	0.0000	0.0000	0.0002	0.0004	0.0005
	RMSE	0.0000	0.0001	0.0008	0.0018	0.0026
LM	Bias	0.0000	0.0000	-0.0031	-0.0076	-0.0123
	RMSE	0.0012	0.0012	0.0036	0.0082	0.0130
BLM	Bias	0.0000	0.0000	0.0031	0.0078	0.0126
	RMSE	0.0012	0.0012	0.0037	0.0084	0.0133
LMM	Bias	0.0000	0.0000	0.0000	0.0001	0.0002
	RMSE	0.0001	0.0001	0.0005	0.0011	0.0018

Note: Statistics are for the natural logs of the price indexes.

Table A6: Empirical Performance of Price Indexes, $\chi = 0.50$

		CES	Nested CES: $\sigma_a = 3, \sigma_1 = 4$			
		$\sigma = 4$	$\sigma_2 = 4$	$\sigma_2 = 6$	$\sigma_2 = 8$	$\sigma_2 = 10$
CCV	Bias	0.0000	0.0001	-0.0013	-0.0030	-0.0042
	RMSE	0.0204	0.0201	0.0613	0.1102	0.1466
SV	Bias	0.0000	0.0000	0.0003	0.0006	0.0008
	RMSE	0.0000	0.0005	0.0030	0.0054	0.0071
LM	Bias	0.0000	-0.0001	-0.0030	-0.0073	-0.0118
	RMSE	0.0023	0.0024	0.0046	0.0089	0.0137
BLM	Bias	0.0000	0.0001	0.0030	0.0074	0.0120
	RMSE	0.0024	0.0024	0.0047	0.0091	0.0139
LMM	Bias	0.0000	0.0000	0.0000	0.0001	0.0001
	RMSE	0.0003	0.0003	0.0011	0.0021	0.0031

Note: Statistics are for the natural logs of the price indexes.