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Revisiting Taste Change in Cost-of-Living Measurement

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Abstract

This paper compares conditional and unconditional cost-of-living indexes (COLI) when tastes change, focusing on the Constant Elasticity of Substitution model. A consumer price index typically targets a conditional COLI, which evaluates price change given set of preferences. An unconditional COLI aims to also capture the welfare effects of changing tastes, but it requires stronger assumptions. Using retail scanner data for food and beverage products, I find COLIs conditioning on current period tastes exceed those conditioning on prior period tastes. Consistent with previous studies, I find an unconditional COLI tends to reflect negative direct contributions from taste change.

Keywords: Cost of living index; price index; taste change

JEL Codes: C43, E31

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1 Introduction

A cost-of-living index (COLI) is based on the expenditure function from classical consumer theory. The evolution of expenditures over time may reflect changes in many factors in addition to prices. Preferences (which may also be referred to as tastes) may themselves be influenced by climate, crime, or fashion, to name a few. Index users may wish to account for preferences in different ways. Users of a consumer price index (CPI), for instance, may not want to adjust payments (for example) in response to non-price factors. For reasons like this, the target of a CPI is commonly (though not universally) agreed to be a *conditional* COLI, which holds preferences fixed (National Research Council, 2002; ILO, 2004). For example, the Tornqvist formula used by the Bureau of Labor Statistics (BLS) for the U.S. Chained Consumer Price Index for All Urban Consumers (C-CPI-U) approximates the conditional COLI pertaining to the set of average tastes (Caves, Christensen, and Diewert, 1982). On the other hand, researchers may be interested in how changing prices and changing preferences combine to impact an *unconditional* COLI. Notably, Redding and Weinstein (2020) (henceforth RW), find using household scanner data that changing preferences lower costs of living significantly.

This paper reviews the literature on COLI measurement in the presence of taste change and compares the conditional and unconditional COLI approaches theoretically and empirically. I focus on the Constant Elasticity of Substitution (CES) model for its tractability. The CES model is relevant because it is widely used in cost-of-living analysis and for initial estimates of the C-CPI-U (Klick, 2018). The indexes I consider are for the most part well-known in the literature, though some, like the Lloyd (1975) and Moulton (1996) variants, have not been widely discussed in the context of conditional COLIs or estimated alongside unconditional COLIs. While there has been a recent literature on taste change exemplified by RW, it focuses primarily on unconditional COLIs.[1] Many earlier studies of taste change

[1] Among others, Hottman and Monarch (2018), Zadrozny (2019), and Ueda, K. Watanabe, and T. Watanabe (2019) also explore indexes of the unconditional type.

(e.g. Phlips and Sans-Ferrer, 1975; Heien and Dunn, 1985) either use a model other than CES, a specific parameterization of the taste change, or allow taste changes for a small subset of products.

A few key themes emerge from the analysis which may be of use to practitioners. First, conditional COLIs generally depend on the indifference surface at which they are measured, and average tastes might not be the only interesting reference point for price-level comparisons, particularly if tastes are changing rapidly. Using retail scanner data for food and beverage products, I estimate that COLIs conditioning on current period tastes exceed those conditioning on past tastes by an average of 0.6 to 3.0 percentage points per year, depending on the product category, with COLIs conditioning on intermediate tastes falling roughly in the middle. Faced with different conditional COLIs, a researcher may desire a single estimate that does not give primacy to one period's tastes. Notably, I find it matters little whether they condition on intermediate tastes (an interpretation of the index of Sato (1976) and Vartia (1976)) or simply average the past and current taste conditional COLIs. Accounting for tastes at the individual product level is infeasible within current data constraints at the BLS. However, in Appendix B, I find that Tornqvist aggregates of U.S. CPI data that similarly account for broader item-stratum level tastes are affected to a lesser extent by the choice of taste vector than are the detailed product group indexes formed using the retail data.

Unconditional COLIs deliver a potentially more comprehensive measure, though at a cost of imposing stronger assumptions on preferences. The latter point has not been widely discussed in the recent literature. Incorporating the direct effects of preference change necessarily treats utility as cardinal (Balk, 1989) and requires a restriction on the taste change process. Under these assumptions and using RW's method, I find the unconditional COLI is, on average, within 0.05 percentage points per year of a COLI that conditions on past preferences. I also find the unconditional COLI averages 1.1 percentage points lower than a COLI that conditions on average tastes. This is qualitatively similar to concurrent findings in Ehrlich, Haltiwanger, Jarmin, Johnson, Olivares, et al. (2021) and is of the same sign, though of a larger magnitude than RW. RW interpret this wedge as a "taste-shock bias" which afflicts indexes like the Tornqvist or Sato-Vartia which do include direct taste effects. However, it might validly be interpreted as reflecting the distinct theoretical targets of the indexes in question. With available items tending to change over time, the unconditional COLI estimate is more sensitive (relative to the conditional COLIs) to which items are considered to be "common" to the periods being compared.

2 Existing literature

The economic approach to consumer price indexes, dating to Konüs (1924), is based on the expenditure function of an optimizing agent. The final version of the C-CPI-U, for example, uses the Tornqvist formula for upper-level aggregation (Cage, Greenlees, and Jackman, 2003). The Tornqvist is an example of a "superlative" index (Diewert, 1976), meaning it approximates an arbitrary expenditure function. The seminal work of F. M. Fisher and Shell (1972) analyzes conditional and unconditional COLIs in an environment with changing preferences. Follow-up studies by Muellbauer (1975), Phlips and Sanz-Ferrer (1975), Heien and Dunn (1985), among others, explore conditional and unconditional COLI for different models under various assumptions about tastes. This paper highlights methods for estimating conditional COLI for the CES model.[2] The appropriate target for a consumer price index is generally considered to be a conditional COLI (National Research Council, 2002; ILO, 2004), which isolates the effect of changing prices by holding preferences (or anything else affecting well-being) fixed. By referencing a specific indifference surface, a conditional COLI requires only the ordinal properties of utility functions.

An unconditional COLI, on the other hand aims to track changes in expenditure whether driven by price change or preference change. As a consequence, the index may increase or decrease even if prices are constant. An unconditional COLI must reference a cardinal utility

[2]The main focus of the paper is CES, but Appendix E derives conditional COLIs for the translog model (which underlies the Tornqvist index). I do not implement them empirically because they have many more parameters to estimate.

level in order to compare expenditures across varying indifference maps.[3] Furthermore, because utility is only identified up to a positive monotone transformation, a normalization or restriction on the taste change process is required for index calculation (Muellbauer, 1975). In RW, the unweighted geometric mean of the taste parameters is assumed to be time constant. Kurtzon (2020) explores the robustness of RW's method to this normalization. Ueda, K. Watanabe, and T. Watanabe (2019) also uses the CES model. Taste changes (which are called fashion effects) are only allowed in the first τ periods after a product's introduction, and follow stationary functions of the product's age.[4] Section 4 shows conditional COLIs do not require either restriction. Interested in an unconditional-like COLI concept, but wishing to avoid these drawbacks, Balk (1989) proposes an index that attempts to hold constant some notion of well-being without fixing the cardinal utility level. The method tracks the change in expenditure required to reach the period-specific indifference surface that passes through a fixed bundle. Gábor-Tóth and Vermeulen (2018) apply this method to European scanner data and find the average annual contribution of taste change to be −1.1 percentage points. Unconditional COLI seek to answer interesting questions, and may be more comprehensive as *cost-of-living* concepts.[5] However, ILO (2004) emphasizes they do not contain any additional information on *prices* than what is already conveyed by a conditional COLI.

As noted by F. M. Fisher and Shell (1972), deriving the relationship between preference change and either type of COLI is difficult without either assuming a specific parameterization (allowing changes on a small subset of items only), or restricting attention to a particular utility function (as this paper does). Tastes do not pose much of a measurement challenge when the objective is a conditional COLI, however, and average tastes are an acceptable

[3]For this reason, unconditional COLI are sometimes called "cardinal," whereas conditional COLI are sometimes called "ordinal" (Muellbauer, 1975).

[4]While not a normalization per se, this restriction allows the fashion effects to be accounted for by Feenstra (1994)-style product turnover adjustments while the remaining fixed-taste component of the COLI is estimated using a Sato-Vartia index over the remaining set of common varieties.

[5]National Research Council (2002) gives several situations where a conditional COLI may be inadequate, including medical products whose quality is difficult to separate from general health status, and regional comparisons where fixing weather conditions may make little sense.

reference point. Caves, Christensen, and Diewert (1982), Diewert (2001), and Feenstra and Reinsdorf (2007) provide conditions under which the Tornqvist, Fisher, and Sato-Vartia price indexes, respectively, are exact for or approximate COLI that condition on some notion of average tastes. Section 3 discusses these results further, while Section 4 complements them by showing that variants of the Lloyd-Moulton index are also exact for conditional COLIs in the CES case. Considering a constant tastes model, Balk (1999) and Melser (2006) use the theoretical equivalence among these CES-based indexes to back out estimates of the elasticity of substitution. With changing preferences, however, we should expect these indexes to vary as they each correspond to a different conditional COLI.

In order to isolate the issue of preference change, I focus my empirical analysis on matched-model indexes, i.e., those defined over a fixed set of specific product varieties with constant tangible attributes. RW's taste-shock bias is defined in association with a matched-model index. This is not to suggest that improvements to a matched model index should not be pursued for reasons of representativity. For instance, product turnover can cause matched model indexes to miss initial price declines for new items (Feenstra, 1994), as well as selection bias in the set of matched items (Pakes, 2003). Applying Feenstra (1994), Appendix C shows how with product turnover, the matched model component of the CES conditional COLI is unchanged, and it is possible to bound the variety adjustment term. Preference change is also fundamentally different from quality change, though the two may have similar effects on relative demand. Price change for the matched model is measurable without quality adjustment, since the set of items and their associated bundles of attributes are constant by definition. Though the two issues are similar mechanically (F. M. Fisher and Shell, 1972), taste-shock bias should not be confused with quality bias. If item definitions are not constant, then whether or not demand shifts are attributed to quality changes or taste changes can have large effects on index estimation (Nevo, 2003).

3 Cost-of-living theory

It is helpful to first review the cost-of-living index theory and precisely state what conceptual target a price index is intended to measure when preferences are changing. A cost-of-living index is a ratio of two expenditure functions. It is helpful in this case to first specify a set of preferences rather than jump straight to a utility function. Consider an ordinal preference relation, denoted $\succeq$, on a commodity space $\mathcal{Q} \subseteq \mathbb{R}_+^N$, which is made up of bundles $\boldsymbol{q}$. We assume:

Assumption 3.1 (Ordinal Preferences) *A preference relation $\succeq$ is i) rational (complete and transitive), ii) continuous, iii) convex, and iv) monotone.*

Assumption 3.1 is sufficient for the existence of a function, $u : \mathcal{Q} \to \mathbb{R}$ which represents $\succeq$, in the sense that we have $\boldsymbol{q} \succeq \boldsymbol{q}' \Leftrightarrow u(\boldsymbol{q}; \succeq) \geq u(\boldsymbol{q}'; \succeq)$ (Mas-Colell, Whinston, Green, et al., 1995). Due to the ordinal nature of preferences, the function u, is not unique. Even when assuming a specific form for u (as in Section 4) the most that Assumption 3.1 would imply about an actual "utility-generating process" is that it follows a function $v = f \circ u$ for some unknown monotonic transformation f.

Let $\boldsymbol{p}$ denote a vector of prices. We then assume:

Assumption 3.2 (Optimization) *Facing prices $\boldsymbol{p}$ and having preferences $\succeq$ that satisfy Assumption 3.1, a representative consumer chooses $\boldsymbol{q}$ to maximize utility $u(\boldsymbol{q}; \succeq)$ subject to a budget constraint, or equivalently, to minimize expenditure subject to a utility constraint.*

Let $\boldsymbol{h}(\boldsymbol{p}, \bar{u}; \succeq) = \underset{\boldsymbol{q}}{\operatorname{argmin}}\ \boldsymbol{p} \cdot \boldsymbol{q}$ s.t. $u(\boldsymbol{q}; \succeq) \geq \bar{u}$ denote the Hicksian demand function, which represents the quantities that minimize expenditure. The expenditure function is given as $C(\boldsymbol{p}, \bar{u}; \succeq) = \boldsymbol{p} \cdot \boldsymbol{h}(\boldsymbol{p}, \bar{u}; \succeq)$.

The task of the price statistician is to compare price vectors across situations. Following price index convention, I label the reference situation 0 and the comparison situation 1. This paper focuses on bilateral, intertemporal comparisons, but the general theory accommodates other possibilities (e.g., regional comparisons). I maintain Assumptions 3.1 and

3.2 separately for both situations (i.e., $C(\boldsymbol{p}_t, \bar{u}; \succeq_t) = \boldsymbol{p}_t \cdot \boldsymbol{h}(\boldsymbol{p}_t, \bar{u}; \succeq_t)$, $t = 0, 1$) but take no stand on whether $\succeq_0 = \succeq_1$ unless stated explicitly. In the general case of varying preferences, the cost-of-living framework yields different measurement concepts, which I review in the following subsections.

3.1 Conditional COLI

A conditional COLI is defined as the minimum expenditure required for an agent to be indifferent between two price situations.

Definition 3.1 *(F. M. Fisher and Shell, 1972; Pollak, 1989) The class of conditional cost-of-living indexes is given by:*

$$\Phi(\boldsymbol{p}_0, \boldsymbol{p}_1, \bar{u}; \succeq) = \frac{C(\boldsymbol{p}_1, \bar{u}; \succeq)}{C(\boldsymbol{p}_0, \bar{u}; \succeq)}, \tag{1}$$

for a given $\bar{u}$ and $\succeq$.

The combination of $\succeq$ (preference relation) and $\bar{u}$ (location) determine the specific indifference surface on which Φ is based. COLIs are not unique except in special cases like constant, homothetic preferences. As noted by F. M. Fisher and Shell (1972), unless preferences are constant, a conditional COLI does not equal the compensation required to maintain a constant "standard of living" or utility, as COLIs are often described. Such a measurement is not generally possible under Assumptions 3.1 and 3.2.

Two immediate candidates for preferences to plug in are $\succeq_0$ and $\succeq_1$. It is worth emphasizing that the oft-cited bounding results for the Laspeyres and Paasche indexes are one-way only, i.e., the Laspeyres is an upper bound for $\Phi(\boldsymbol{p}_0, \boldsymbol{p}_1, u_0; \succeq_0)$, and the Paasche is a lower bound for $\Phi(\boldsymbol{p}_0, \boldsymbol{p}_1, u_1; \succeq_1)$, where $u_t = u(\boldsymbol{q}_t, \succeq_t)$ for $t = 0, 1$. F. M. Fisher and Shell (1972) argue that from the standpoint of intertemporal compensation, the most interesting COLI is

$$\Phi(\boldsymbol{p}_0, \boldsymbol{p}_1, \bar{u}_1^*; \succeq_1) = \frac{C(\boldsymbol{p}_1, \bar{u}_1^*; \succeq_1)}{C(\boldsymbol{p}_0, \bar{u}_1^*; \succeq_1)}, \tag{2}$$

where $\bar{u}_1^*$ is the hypothetical utility that the consumer would receive facing the period 0 budget constraint with period 1 preferences. F. M. Fisher and Shell (1972) and others argue that a COLI based on $\succeq_1$ is more relevant for public policy than one based on the obsolete preferences $\succeq_0$, but Pollak (1989) notes that in principle, $\succeq$ need not be linked to either the reference or comparison situations. Indeed, two of the parameter-free price indexes discussed in the next subsection are exact for COLI based on average indifference surfaces.

3.2 Parameter-free COLI

Under Assumption 3.2, the observed market expenditures $\boldsymbol{p}_0 \cdot \boldsymbol{q}_0$ and $\boldsymbol{p}_1 \cdot \boldsymbol{q}_1$ equal the expenditure levels $C(\boldsymbol{p}_0, u_0, \succeq_0)$ and $C(\boldsymbol{p}_1, u_1, \succeq_1)$, respectively. Since, Eq. 1 holds the indifference surface fixed, however, estimation generally requires knowledge of the expenditure function for the given set of preferences.

Nevertheless, some well-known price index formulas either approximate or are exact for conditional COLI, precluding any need for structural estimation.[6] These indexes and their components are defined below. Let i index items or varieties, and denote the set of items as $\mathcal{I}$, which has dimension N.

Definition 3.2 *The Fisher price index*

$$P_F(\boldsymbol{p}_0, \boldsymbol{p}_1, \boldsymbol{q}_0, \boldsymbol{q}_1) = \sqrt{P_L P_P}, \tag{3}$$

where $P_L(\boldsymbol{p}_0, \boldsymbol{p}_1, \boldsymbol{q}_0, \boldsymbol{q}_1) = \frac{\sum_{i\in\mathcal{I}} p_{i1} q_{i0}}{\sum_{i\in\mathcal{I}} p_{i0} q_{i0}}$ *is the Laspeyres index, and* $P_P(\boldsymbol{p}_0, \boldsymbol{p}_1, \boldsymbol{q}_0, \boldsymbol{q}_1) = \frac{\sum_{i\in\mathcal{I}} p_{i1} q_{i1}}{\sum_{i\in\mathcal{I}} p_{i0} q_{i1}}$ *is the Paasche index.*

Definition 3.3 *The Tornqvist price index*

$$P_T(\boldsymbol{p}_0, \boldsymbol{p}_1, \boldsymbol{q}_0, \boldsymbol{q}_1) = \prod_{i\in\mathcal{I}} \left(\frac{p_{i1}}{p_{i0}}\right)^{0.5(s_{i0}+s_{i1})}, \tag{4}$$

[6] Per Diewert (1976), a price index formula is exact if it equals a ratio of expenditure functions for a given model.

where $s_{it} = \frac{p_{it}q_{it}}{\sum_{j\in\mathcal{I}} p_{jt}q_{jt}}, t = 0, 1.$

Definition 3.4 *The Sato-Vartia price index*

$$P_{SV}(\boldsymbol{p}_0, \boldsymbol{p}_1, \boldsymbol{q}_0, \boldsymbol{q}_1) = \prod_{i\in\mathcal{I}} \left(\frac{p_{i1}}{p_{i0}}\right)^{w_i}, \tag{5}$$

where $w_i = \left[\frac{s_{i1}-s_{i0}}{\ln s_{i1}-\ln s_{i0}}\right] / \left[\sum_{k\in\mathcal{I}} \frac{s_{k1}-s_{k0}}{\ln s_{k1}-\ln s_{k0}}\right].$

Suppose tastes (or environmental variables, as referred to in Diewert, 2001) are represented by the vector of parameters $\boldsymbol{\varphi}$, and that $u()$ is continuous and increasing in $\boldsymbol{\varphi}$, as will be the case in the following section on CES preferences. Then Diewert (2001) showed that there exists a u^* and $\boldsymbol{\varphi}^*$ such that $\Phi(\boldsymbol{p}_0, \boldsymbol{p}_1, u^*; \boldsymbol{\varphi}^*)$ is bounded by the Laspeyres and Paasche indexes, where $u_0 \leq u^* \leq u_1$ and $\varphi_{i0} \leq \varphi_i^* \leq \varphi_{i1}$, $i \in \mathcal{I}$. If the Laspeyres and Paasche are close numerically, a symmetric average like the Fisher index approximates this COLI. In addition, under the assumption that the expenditure function is translog, Caves, Christensen, and Diewert (1982) showed that the Tornqvist price index is exact for the geometric average of the COLI based on period 0 preferences and the COLI based on period 1 preferences. Due to translog functional form, this is equivalent to the COLI evaluated at the geometric averages of the taste parameters and utilities, respectively. The Tornqvist index is also attractive because the translog expenditure function approximates arbitrary expenditure functions to the second order. Finally, the Sato-Vartia index is exact for the CES COLI that conditions on intermediate levels of the tastes (Feenstra and Reinsdorf, 2007). Each of these results relates to an indifference surface that is, loosely speaking, an average of the base and current period indifference surfaces. Of course, the measurement of substitution effects (responses to relative price change), may change depending on which indifference surface the COLI is based, and so interpretations should be made carefully.

3.3 Unconditional COLI

An unconditional COLI measures the change in expenditure required for the consumer to achieve the same standard-of-living, or utility level, in the comparison period as they experienced in the reference period.

Definition 3.5 *(Muellbauer, 1975) The class of unconditional or cardinal COLI is given by:*

$$\Phi_U(\boldsymbol{p}_0, \boldsymbol{p}_1, \bar{u}; \succeq_0, \succeq_1) = \frac{C(\boldsymbol{p}_1, \bar{u}; \succeq_1)}{C(\boldsymbol{p}_0, \bar{u}; \succeq_0)}, \tag{6}$$

for some $\bar{u}$.

As discussed by F. M. Fisher and Shell (1972), shifting preferences complicate the "constant standard-of-living" interpretation of Definition 3.5. In addition to Assumptions 3.1 and 3.2, this interpretation implicitly assumes location $\bar{u}$ under $\succeq_0$ is comparable to location $\bar{u}$ under $\succeq_1$ (utility is cardinal). In general, the associated quantities $\boldsymbol{h}(\boldsymbol{p}_0, \bar{u}; \succeq_0)$ and $\boldsymbol{h}(\boldsymbol{p}_1, \bar{u}; \succeq_1)$ lie on different indifference surfaces, even though both are labeled $\bar{u}$. As utility levels are not identified, normalizations are needed to pin down the relative expenditures associated with maintaining $\bar{u}$ utils. Section 4.2 and Appendix D discuss these further for the CES model.

As discussed in Sections 1 and 2, the unconditional COLI aims to account for the effect of non-price factors on standards of living. This can be illustrated through the following algebraic relationship, which is similar to decompositions in Balk (1989) and Gábor-Tóth and Vermeulen (2018).

$$\ln \Phi_U(\boldsymbol{p}_0, \boldsymbol{p}_1, \bar{u}; \succeq_0, \succeq_1) = \ln \Phi(\boldsymbol{p}_0, \boldsymbol{p}_1, \bar{u}; \succeq_1) + \ln \left[\frac{C(\boldsymbol{p}_0, \bar{u}; \succeq_1)}{C(\boldsymbol{p}_0, \bar{u}; \succeq_0)} \right] \tag{7}$$

Eq. 7 decomposes the unconditional COLI of Definition 3.5 into two parts; a price effect equal to a conditional COLI, and a pure taste change effect $C(\boldsymbol{p}_0, \bar{u}; \succeq_1)/C(\boldsymbol{p}_0, \bar{u}; \succeq_0)$. It is straightforward to compare the unconditional COLI with other conditional COLI in a similar fashion. Equation 7 describes the sense in which Φ_U is "unconditional" in that the last term aims to capture the impact of factors other than prices (National Research Council, 2002). It is also apparent that the contribution of price change is completely captured by the conditional index, and that the pure taste change component is what requires cardinal utility.[7]

4 CES Preferences

Section 3 described a few conditional COLI that can be estimated with prices and quantities only. In general, however, estimating a COLI requires specifying and estimating a model of preferences. For comparability to other studies, I focus on the CES model for the rest of this paper. The CES model is a workhorse for its tractability, though it implies significant restrictions on price and income elasticities. Appendix E derives similar results for the homothetic translog expenditure function, which is more flexible, but requires estimating many more parameters. Specification error is a potential issue for an unconditional COLI, as well as COLI that condition on a specific period's tastes, as these depend on the model's ability to separate price responses from preference shifts (Martin, 2020). While the price indexes considered in this section are not new, some of their relationships with specific conditional COLI when tastes are changing have not been widely discussed in the literature.

We now assume:

Assumption 4.1 *The representative agent's expenditure function has the form:*

$$C(\boldsymbol{p}, \bar{u}; \boldsymbol{\varphi}) = \bar{u} \left[\sum_{i \in \mathcal{I}} \left(\frac{p_i}{\varphi_i} \right)^{1-\sigma} \right]^{\frac{1}{1-\sigma}} \tag{8}$$

For the purposes of a COLI, we take $\bar{u} = 1$ without further loss of generality (preferences are homothetic) and suppress the argument from further notation. The parameter $\sigma \neq 1$ is the elasticity of substitution, which we assume is constant over time, and so the notation now

[7] If one defines an unconditional COLI using an alternative notion of "constant well-being", as in Balk (1989) then a pure taste change component can be estimated without cardinal utility.

refers to preferences through the vector of demand shifters $\boldsymbol{\varphi}$. I use $\boldsymbol{\varphi}$ to refer to a generic vector, while $\boldsymbol{\varphi}_t$ refers to the actual preferences of time t. The agent's optimal expenditure shares are given by

$$s_i(\boldsymbol{p};\boldsymbol{\varphi}) = \frac{p_i h_i(\boldsymbol{p};\boldsymbol{\varphi})}{\sum_{j\in\mathcal{I}} p_j h_j(\boldsymbol{p};\boldsymbol{\varphi})} = \frac{(p_i/\varphi_i)^{1-\sigma}}{\sum_{j\in\mathcal{I}} (p_j/\varphi_j)^{1-\sigma}} = \frac{(p_i/\varphi_i)^{1-\sigma}}{[C(\boldsymbol{p};\boldsymbol{\varphi})]^{1-\sigma}},\ i = 1,\ldots,N. \tag{9}$$

Under Assumptions 3.2 and 4.1, the observed expenditure shares $s_{it} = \frac{p_{it}q_{it}}{\sum_{j\in\mathcal{I}} p_{jt}q_{jt}}$ equal the optimal expenditure shares $s_i(\boldsymbol{p}_t;\boldsymbol{\varphi}_t)$. The indexes in the following subsections make use of this equation to estimate conditional and unconditional COLI.

Eq. 10 shows that under Assumption 4.1, the log expenditure share of item i in period t can be decomposed into its log price, the log expenditure function, and the log of the taste parameters.

$$\ln s_{it} = (1-\sigma)\ln p_{it} + (\sigma-1)\ln[c(\boldsymbol{p}_t;\boldsymbol{\varphi}_t)] + (\sigma-1)\ln\varphi_{it} \tag{10}$$

As RW note, the taste parameters provide a source of idiosyncratic error which is necessary for empirical analysis.

4.1 Exact Price Indexes for CES Preferences

As previously mentioned, the index proposed by Sato (1976) and Vartia (1976) (see Eq. 5) is exact for the CES COLI that conditions on an intermediate taste vector $\bar{\boldsymbol{\varphi}}$ (Feenstra and Reinsdorf, 2007).[8] The salient question then is how do price comparisons using $\bar{\boldsymbol{\varphi}}$ compare to price comparisons using $\boldsymbol{\varphi}_0$, $\boldsymbol{\varphi}_1$ or some other tastes? Exact price indexes for reference period or current period tastes already exist for the CES model, though to my knowledge, their interpretation as such is novel. Lloyd (1975) and Moulton (1996) developed the following price index in the setting of constant tastes.

[8] Each element $\bar{\varphi}_i$ of $\bar{\boldsymbol{\varphi}}$ lies between $\varphi_{i0}/\prod_{i\in\mathcal{I}}\varphi_{i0}^{w_i}$ and $\varphi_{i1}/\prod_{i\in\mathcal{I}}\varphi_{i1}^{w_i}$.

Definition 4.1 *Lloyd-Moulton Index*

$$P_{LM}(\boldsymbol{p}_0, \boldsymbol{p}_1, \boldsymbol{q}_0, \boldsymbol{q}_1, \sigma) = \left\{ \sum_{i \in \mathcal{I}} s_{i0} \left(\frac{p_{i1}}{p_{i0}} \right)^{1-\sigma} \right\}^{\frac{1}{1-\sigma}} \tag{11}$$

Similarly, the time-antithesis (I. Fisher, 1922), or "backwards" version of the Lloyd-Moulton index can be formed (Lloyd, 1975).

Definition 4.2 *Backwards Lloyd-Moulton Index*

$$P_{BLM}(\boldsymbol{p}_0, \boldsymbol{p}_1, \boldsymbol{q}_0, \boldsymbol{q}_1, \sigma) = \left\{ \sum_{i \in \mathcal{I}} s_{i1} \left(\frac{p_{i0}}{p_{i1}} \right)^{1-\sigma} \right\}^{\frac{-1}{1-\sigma}} \tag{12}$$

The Lloyd-Moulton and Backwards Lloyd-Moulton are exact for the COLI that condition on reference period tastes and comparison period tastes, respectively. To see this, start with Eq. 4.1 for $P_{LM}(\boldsymbol{p}_0, \boldsymbol{p}_1, \boldsymbol{q}_0, \boldsymbol{q}_1, \sigma)$. Use the right hand side of Eq. 9 to substitute for s_{i0}, re-arrange, and use Eq. 8. We then have

$$\begin{aligned}
P_{LM}(\boldsymbol{p}_0, \boldsymbol{p}_1, \boldsymbol{q}_0, \boldsymbol{q}_1, \sigma) &= \left\{ \sum_{i \in \mathcal{I}} s_{i0} \left(\frac{p_{i1}}{p_{i0}} \right)^{1-\sigma} \right\}^{\frac{1}{1-\sigma}} \\
&= \left\{ \sum_{i \in \mathcal{I}} \frac{(p_{i0}/\varphi_{i0})^{1-\sigma}}{[C(\boldsymbol{p}_0, \boldsymbol{\varphi}_0)]^{1-\sigma}} \left(\frac{p_{i1}}{p_{i0}} \right)^{1-\sigma} \right\}^{\frac{1}{1-\sigma}} \\
&= \frac{\left\{ \sum_{i \in \mathcal{I}} (p_{i1}/\varphi_{i0})^{1-\sigma} \right\}^{\frac{1}{1-\sigma}}}{C(\boldsymbol{p}_0, \boldsymbol{\varphi}_0)} \\
&= \frac{C(\boldsymbol{p}_1, \boldsymbol{\varphi}_0)}{C(\boldsymbol{p}_0, \boldsymbol{\varphi}_0)} \\
&= \Phi(\boldsymbol{p}_0, \boldsymbol{p}_1; \boldsymbol{\varphi}_0)
\end{aligned}$$

The case of $P_{BLM}(\boldsymbol{p}_0, \boldsymbol{p}_1, \boldsymbol{q}_0, \boldsymbol{q}_1, \sigma)$ is very similar.

Faced with these two options for conditional COLIs, a price researcher may not wish to give primacy to either period's preferences. An additional possibility for the case of CES preferences is to calculate the geometric mean of the Lloyd-Moulton indexes, which is exact for the geometric mean of two CES conditional COLI. This index, which I label P_{LMM}, has the form:

$$P_{LMM}(\boldsymbol{p}_0, \boldsymbol{p}_1, \boldsymbol{q}_0, \boldsymbol{q}_1, \sigma) = [P_{LM}(\boldsymbol{p}_0, \boldsymbol{p}_1, \boldsymbol{q}_0, \boldsymbol{q}_1, \sigma) P_{BLM}(\boldsymbol{p}_0, \boldsymbol{p}_1, \boldsymbol{q}_0, \boldsymbol{q}_1, \sigma)]^{\frac{1}{2}}$$

$$= \left[\frac{\sum_{i \in \mathcal{I}} s_{i0} \left(\frac{p_{i1}}{p_{i0}} \right)^{1-\sigma}}{\sum_{i \in \mathcal{I}} s_{i1} \left(\frac{p_{i0}}{p_{i1}} \right)^{1-\sigma}} \right]^{\frac{1}{2(1-\sigma)}} \tag{13}$$

As pointed out in Balk (1999) and Haan, Balk, and C. B. Hansen (2010), Eq. 13 shows P_{LMM} is, in fact, the Quadratic Mean of Order r price index, where $r = 2(1 - \sigma)$ Diewert (1976). This is attractive because superlative indexes like this one have been shown to approximate each other to the second order (Diewert, 1978). This implies P_{LMM} should be somewhat robust to errors in estimation of σ or departures from CES functional form.[9]

Additionally, the availability of both P_{LMM} and P_{SV} for the CES case offers an interesting potential contrast. One averages COLI evaluated at different tastes, while the other is a COLI evaluated at an average of the tastes. A priori, we would not necessarily expect them to give identical answers, though their estimates in Section 5 are very similar.

4.2 RW's CES Common Varieties Index

RW propose a price index to target the CES unconditional COLI. A practical challenge concerns the scale of tastes. Given knowledge of σ, Eq. 9 implies that observed expenditure shares and prices only identify φ_{it} up to a time-varying scale factor. RW address this issue by assuming the φ_{it} have a constant geometric mean over time.

Assumption 4.2

$$\prod_{i \in \mathcal{I}} \varphi_{i0}^{1/N} = \prod_{i \in \mathcal{I}} \varphi_{i1}^{1/N}$$

Under Assumption 4.2, RW show the following index is exact for $\Phi_U(\boldsymbol{p}_0, \boldsymbol{p}_1; \boldsymbol{\varphi}_0, \boldsymbol{\varphi}_1)$..

[9] This is provided that $|\sigma|$ is not too large (Hill, 2006).

Definition 4.3 *RW's CES Common Varieties Index (CCV)* [10]

$$P_{CCV}(\boldsymbol{p}_0, \boldsymbol{p}_1, \boldsymbol{q}_0, \boldsymbol{q}_1, \sigma) = \prod_{i=1}^{N} \left(\frac{p_{i1}}{p_{i0}}\right)^{1/N} \left[\prod_{i} \left(\frac{s_{i1}}{s_{i0}}\right)^{1/N}\right]^{\frac{1}{\sigma-1}} \tag{14}$$

P_{CCV} consists of a unweighted geometric mean of price relatives (also known as a Jevons index), multiplied by an equally-weighted geometric mean of expenditure share relatives.

As discussed in Section 3, P_{CCV} differs from a conditional COLI in that it estimates the expenditure change associated with constant utility. Assumption 4.2 acts as a normalization which identifies the relative expenditure associated with a given level of utility. Mechanically, it prohibits systematic increases or decreases in the agent's "efficiency" as a producer of utility (Muellbauer, 1975). Such changes would affect the magnitude and direction of the unconditional COLI. While clearly necessary to construct an index using prices and quantities, such an index will reflect only *relative* taste shifts. The broad effects of factors like crime, environment, or public health on standards of living cannot be accounted for. Furthermore, the normalization which uses the unweighted geometric mean (Assumption 4.2) may produce significantly different estimate from other normalizations (Kurtzon, 2020). Appendix D discusses these issues further and shows how estimating P_{CCV} over subsets of $\mathcal{I}$ implicitly changes the normalization.

Table 1 summarizes the price index formulas discussed in this and the previous section, which are compared in Section 5. [Insert Table 1 near here.]

4.3 Common Goods Rules

In applications with detailed transactions data over narrowly defined items, one might be concerned from Eq. 14 that P_{CCV} will be sensitive to items with small expenditure shares or experiencing extreme expenditure changes. This motivates RW to carefully select which items to consider as common goods out of all items which can possibly be matched between

[10] RW's proposed CES Unified Price Index consists of the CCV plus a product turnover adjustment in the style of Feenstra (1994).

periods 0 and 1. They argue, for example, that large increases in revenues during a products first few periods of sales may incorrectly imply rapidly increasing relative tastes. For this reason, their empirical application defines the set of common goods to include only items which were available for a long time, and which were neither newly introduced nor close to exiting. Ehrlich, Haltiwanger, Jarmin, Johnson, Olivares, et al. (2021) term this requirement a Common Goods Rule (CGR) and provide further discussion.

Denote set $\mathcal{I}$ as all items which expenditures are observed in periods 0 and 1, and let $\mathcal{I}^* \subseteq \mathcal{I}$ be the set which satisfy the CGR. RW and Ehrlich, Haltiwanger, Jarmin, Johnson, Olivares, et al. (2021) apply P_{CCV} only to the items in $\mathcal{I}^*$. They shift the contributions of the matched, but ineligible items to terms which also account for product turnover, using a decomposition along the lines of Feenstra (1994, Prop. 1). Eq. 21 in Appendix C describes a similar decomposition. To my knowledge, the primary theoretical consideration given when implementing the CGR in these papers is to tailor Assumption 4.2 to refer to the set $\mathcal{I}^*$. Assumption 4.1 is still implicitly maintained for the larger set of goods. For any set $\mathcal{I}^*$, with the revision to Assumption 4.2, Φ_U over the entire set $\mathcal{I}$ can be estimated by multiplying P_{CCV} for the set $\mathcal{I}^*$ by adjustment terms comparing expenditure on $\mathcal{I}^*$ to expenditure on $\mathcal{I}$. This decomposition depends on the CES demand equations holding for the ineligible items in $\mathcal{I} \setminus \mathcal{I}^*$. If such goods have small expenditure shares in each period, however, they will contribute relatively little to the product turnover adjustments. In addition, Appendix D shows that the effect of the CGR on the unconditional COLI estimate can be replicated empirically while maintaining Assumption 4.2 over all of $\mathcal{I}$, but by changing the weights of the geometric means.

Alternatively, one may view the the CES expenditure shares (Eq. 9) as representing demand for items $i \in \mathcal{I}^*$, but not for items $i \in \mathcal{I} \setminus \mathcal{I}^*$. In this case, one should also similarly limit the goods covered by P_{SV}, P_{LM}, P_{BLM}, or P_{LMM}. Proceeding with analysis of price change over the set $\mathcal{I}^*$ implicitly assumes separability of preferences between items in $\mathcal{I}^*$ and $\mathcal{I} \setminus \mathcal{I}^*$ as discussed in Chapter 5 of Deaton and Muellbauer (1980). In Section 5, I examine the differences in these indexes between using the full $\mathcal{I}$ and a smaller set $\mathcal{I}^*$ which is motivated by RW's CGR.

5 Application to Retail Scanner Data

5.1 Data and model estimation

To illustrate the potential for measurement differences when basing CES conditional COLI on alternative taste levels, I estimate quarterly price indexes for food and beverage product categories using Scantrack, a point-of-sale scanner dataset from The Nielsen Company.[11] The data are similar in scope to Nielsen's household panel, which RW use. The Neilsen retail scanner data has been proposed for use in CPI's by Ehrlich, Haltiwanger, Jarmin, Johnson, and Shapiro (2019) and Ehrlich, Haltiwanger, Jarmin, Johnson, Olivares, et al. (2021), and similar data is used by the Australian Bureau of Statistics to estimate some food components of its CPI. The data cover the fourth quarter of 2005 through the second quarter of 2010, and include expenditures and quantities for roughly 600,000 universal product codes (UPC) sold by participating grocery, drug, and mass merchandise store chains.[12] Because items are defined by UPC, their characteristics and quality are arguably constant over time (Broda and Weinstein, 2010; Redding and Weinstein, 2020). UPCs are classified according to a structure defined by Nielsen. For instance, UPC 003800040500 is described as "Kellogg's Eggo Round Chocolate Chip 10 count." It belongs the brand module "Kellogg's Eggo," product module

[11] I use data for food and beverage products only, though Scantrack data also covers general merchandise, personal care, and other non-food grocery items sold in grocery and drug stores. Scantrack expenditures on nonfood goods equal only about 19% and 12% of comparable Consumer Expenditure Survey and Personal Consumption Expenditure estimates, respectively, suggesting the majority of consumption on these products originates from non-covered retailers (Bureau of Labor Statistics, 2019; Bureau of Economic Analysis, 2019). Furthermore, the degree to which the simple CES model is a suitable approximation for the data may differ between food and nonfood categories. The model assumes no dynamic behavior, i.e., stockpiling or durable goods, and expenditure on a nonfood product (e.g., "Kitchen gadgets") may be a relatively poor proxy for consumption of that product, even at a quarterly frequency.

[12] According to a Nielsen representative, the sample covers 90% of such retail chains and is weighted to be nationally representative. Potential selection bias is a limitation of this and other studies using convenience samples of transactions.

"Frozen Waffles/Pancakes/French Toast," product group "Breakfast Foods - Frozen," and department "Frozen Foods." Like RW, I calculate quarterly expenditure shares (within product group) and unit value prices by UPC, treating the continental United States as one market. I then winsorize by dropping items whose change in price or value were in the top or bottom one percentile for a given quarter.

Table 2 describes some basic attributes of the dataset. Just over 54% of food and beverage expenditures are from the Dry Grocery department, comprising about two-thirds of the total number of UPCs. Dairy (15%) and Frozen Foods (11%) are the next largest departments by expenditure. Use of these data for consumer price indexes treats retail sales as proxies for consumer expenditures, but they also include purchases by non-households. Total food and beverage expenditures in Scantrack exceed the BLS's Consumer Expenditure Survey (CE) estimates over the same time period by about 66%, while they exceed the Bureau of Economic Analysis's Personal Consumption Expenditure (PCE) estimates by about 9% (Bureau of Labor Statistics, 2019; Bureau of Economic Analysis, 2019).[13] [Insert Table 2 near here.]

For each product group, I calculate a series of indexes of the form $P(\boldsymbol{p}_{t-4}, \boldsymbol{p}_t, \boldsymbol{q}_{t-4}, \boldsymbol{q}_t)$, where $P()$ is one of the formulas given in Section 3 or 4. The index for quarter t uses the same quarter in the year prior as its base period, so index values reflect year-over-year price changes. Table 3 presents summary statistics for the four-quarter price relatives $p_{it}/p_{i,t-4}$ pooled over the entire sample period. Average price relatives exceed one for all departments, ranging from 1.021 for Alcoholic Beverages to 1.037 for Dairy. The distributions are quite dispersed however, with standard deviations within departments ranging from 0.106 for Packaged Meat to 0.165 for Fresh Produce. Relatives are positively skewed in all departments, as one might expect if prices follow an upward trend over time. Compared to a normal distribution (which has kurtosis equal to 3), the distributions of price relatives have higher kurtosis, which indicates thicker tails. [Insert Table 3 near here.]

[13] CE and PCE cover slightly different target populations and rely on different survey methods. See Passero, Garner, and McCully (2014) for a discussion.

As discussed in Section 4.3, RW and Ehrlich, Haltiwanger, Jarmin, Johnson, Olivares, et al. (2021) restrict the goods considered to be common between index periods. In RW, items must be sold for a total of six years (though not necessarily continuously), and may not be within three quarters of their entry period or exit period. As my full dataset only covers about five years, my best replication of this CGR requires the item to be sold in all periods in the sample, and then I truncate the first and last three quarters of the dataset, computing year-over-year indexes only for the period 2007Q3 to 2009Q3.[14] For this shorter period, Table 4 shows the impact of the CGR on total expenditure as well as selected statistics for price relatives, $p_{it}/p_{i,t-4}$, and share relatives $s_{it}/s_{i,t-4}$. Imposing the CGR reduces the number of UPC-quarter observations by 44%, though these only represent about 17% of total expenditure. The price relative distribution is less affected than the share relative distribution. Its mean increases slightly (1.046 to 1.053), and it becomes somewhat less dispersed. For the share relatives, however, implementing the CGR lowers the mean from 1.457 to 1.165, and the standard deviation from 4.675 to 1.293. Likewise, the 10th and 90th percentiles compress from $(0.266, 2.157)$ to $(0.563, 1.649)$. As mentioned, taste parameters in the CES model are essentially residuals in a regression log-shares, and so extreme changes in shares may be interpreted as taste changes or perhaps indicative of an inappropriate model for those items. The next subsection will compute indexes assuming CES preferences represent only the items satisfying the CGR. Then, I will expand this assumption to cover all matched UPCs and evaluate the impact on the various CES COLIs. [Insert Table 4 near here.]

Estimation of the substitution elasticities follows the “double-differencing” method of Feenstra (1994), using panel variation in prices and expenditure shares. This method assumes $\sigma > 1$, which is reasonable for indexes over similar product varieties. I follow the weighting and estimation procedure of Broda and Weinstein (2010), though as in RW, I do not distinguish between within-brand and across-brand substitutions. Start with Eq. 10

[14] Earlier versions of this paper used all matched UPCs, as in Table 9 and Figure 3.

and difference over time and with respect to a reference variety k, which is chosen to be the variety with the largest average market share. Denote the doubled-difference of variable x_{it} as $\Delta^k x_{it} = (x_{it} - x_{i,t-1}) - (x_{kt} - x_{k,t-1})$. For varieties $i = 1, \ldots, N$ and time periods $t = 1, \ldots, T$, the double-differenced demand equation is:

$$\Delta^k \ln s_{it} = -(\sigma - 1)\Delta^k \ln p_{it} + \Delta^k \ln u_{it}, \tag{15}$$

where $\Delta^k \ln u_{it}$ is the double-differenced error. The inverse supply equation is derived by assuming each variety is produced by a distinct firm in monopolistic competition, leading to a pricing equation that is linear in log-expenditure share, with slope depending on the inverse supply elasticity parameter ω. The double-differenced inverse supply equation is then:

$$\Delta^k \ln p_{it} = \frac{\omega}{1+\omega}\Delta^k \ln s_{it} + \Delta^k \ln v_{it}, \tag{16}$$

where $\Delta^k \ln v_{it}$ is the double-differenced supply error. We assume that the double-differenced demand and supply errors are drawn from stationary distributions, have variances that differ by product variety, and are uncorrelated with each other. The parameters can then be estimated using Generalized Method of Moments (L. P. Hansen, 1982) based on the moment conditions

$$E\left[\left(\Delta^k \ln p_{it}\right)^2 - \theta_1 \left(\Delta^k \ln s_{it}\right)^2 - \theta_2 \Delta^k \ln p_{it} \Delta^k \ln s_{it}\right] = 0, i = 1, \ldots, N, \tag{17}$$

where $\theta_1 = \frac{\omega}{(1+\omega)(\sigma-1)}$ and $\theta_2 = \frac{\omega(\sigma-2)-1}{(1+\omega)(\sigma-1)}$. As in Feenstra (1994), Eq. 17 can be written as a regression of time averaged variables and estimated using weighted nonlinear least squares.[15] When analytical estimates are outside of theoretical bounds (e.g., $\sigma < 1$), parameters are estimated by a grid search over the parameter space.[16]

[15]From Broda and Weinstein (2006), I include the time average of $\left(q_{it}^{-1} + q_{i,t-1}^{-1}\right)$ as additional regressor to control for measurement error introduced by aggregating transaction prices into quarterly unit values. This means a product group must have at least four varieties for estimation.

[16]Following the code used for Broda and Weinstein (2010), the grid search, e.g., searches for the value of

Two product groups in the Dairy department have too few varieties for estimation and are dropped from the analysis. Of the remaining, the procedure yields 55 analytical and 15 grid-searched estimates, the summary of which is presented in Table 5. The overall median elasticity is 4.32, which is lower than what RW found using the Feenstra method and Homescan data (6.48), but reasonable given data and time period differences.[17] There is heterogeneity in estimates by product group, as the interquartile range is nearly three. [Insert Table 5 near here.]

5.2 Results

As described in the previous subsection, I calculate series of four-quarter CES price indexes for 70 food and beverage product groups in the Scantrack dataset. For ease of presentation, figures and tables show statistics that are weighted by comparison-period expenditure shares. For example, for a set of product groups $\mathcal{G}$ indexed by g, Figures 1 and 2 plot averages of the form $\sum_{g\in\mathcal{G}} s_{gt} P(\boldsymbol{p}_{g,t-4}, \boldsymbol{p}_{gt}, \boldsymbol{q}_{g,t-4}, \boldsymbol{q}_{gt})$, where $P()$ is the price index and s_{gt} is the period t share of product group g out of all expenditures on $\mathcal{G}$. Figure 1 shows averages across all product groups, while Figure 2 and Table 6 break these out by department. Table 7 gives the percentiles of the distributions of differences between price indexes, while Table 8 presents average differences by department. For Tables 6-8, product group-level indexes or index differences are first averaged over time, and then between-product group averages are taken using shares of average quarterly expenditure as weights.

As discussed in Sections 3 and 4, these indexes are derived from the same CES model with time-varying taste parameters. Comparisons among P_{LM}, P_{BLM}, P_{LMM}, and P_{SV} shed light on the degree to which the estimate of pure price change is affected by the choice of conditioning taste vector. A difference such as $P_{CCV} - P_{SV}$ estimates the partial effect of

$\sigma \in [1.04, 50.5]$, at 4% increments, that minimizes the sample objective function.

[17] Previously, Kurtzon (2016) found estimated elasticities from Scantrack to be lower than those estimated using Homescan. The Feenstra method is based on large-T asymptotic arguments, so estimates may have finite-sample bias from the relatively sample period. See Soderbery (2010) and Soderbery (2015). As discussed in Section 4, P_{LMM} is robust to small changes in σ used, and comparisons among P_{CCV}, P_{LM}, and P_{BLM} are qualitatively similar when using alternative values of σ (e.g. setting all equal to 6.48).

tastes on the unconditional COLI (like in Eq. 7), provided one treats utility as cardinal. [Insert Table 6 near here.] [Insert Figure 1 near here.] [Insert Table 7 near here.]

The Scantrack data show that, as in RW, the contributions of tastes tend to lower P_{CCV} relative to all of the conditional COLI save P_{LM}, which implies similar inflation. From Figure 1, the unconditional and conditional COLI estimates tend to agree in sign, and accelerate and decelerate in similar patterns. P_{CCV} implies an average four-quarter percent change in the unconditional cost-of-living of 3.05 percentage points per year. Similar to RW, I find P_{CCV} tends to imply substantially lower inflation than P_{SV}. I find an average difference of 1.12 percentage points, while RW report an average of 0.4 percentage points reported in RW. The difference may be due to the time periods covered (RW cover 2005 to 2013), the precise CGR (RW require items be present for at least six years), or the dataset used (RW use the Homescan consumer panel). Also using Scantrack data, but over the period 2006-2014, Figure 11 of Ehrlich, Haltiwanger, Jarmin, Johnson, Olivares, et al. (2021) finds taste-shock biases (also called consumer valuation bias) of 1 percentage point per year or greater depending on the CGR. While Figure 1 suggests P_{CCV} tends to be much lower than most of the other conditional COLI estimates, Figure 2 and Table 8 suggest considerable heterogeneity by department. The average $P_{SV} - P_{CCV}$ spread ranges from -0.36 percentage points for Packaged Meat to 3.20 percentage points for Fresh Meat. For Dry Grocery, Fresh Meat, and Frozen Foods, P_{CCV} fairly consistently implies lower inflation over time than P_{SV}, but for Alcohol, Dairy, Deli, Fresh Produce, and Packaged Meat, the sign of the gap is not consistent. [Insert Table 8 near here.] [Insert Figure 2 near here.]

Turning to the conditional COLI, the Scantrack estimates indicate that the choice of taste vector also impacts the measurement of price change. From Table 7, P_{BLM} exceeds P_{LM} by 2.05 percentage points on average and by 1.33 percentage points at the median. Looking across departments (Table 8), the average difference ranges from 0.59 for Packaged Meat, to 3.04 for Dairy. As Figures 1 and 2 illustrate, the average $P_{BLM} - P_{LM}$ gap is persistent and fairly stable over time. Conditioning on an intermediate level of tastes, P_{SV} in most cases is very close to P_{LMM}, the geometric average of reference-taste and comparison-taste indexes. Overall, they differ by 0.05 percentage points on average, and 0.008 percentage points at median, with average differences by all department less than 0.1 percentage points in magnitude. Conditioning on current preferences (as F. M. Fisher and Shell (1972) prefer), P_{BLM} implies that consumers during this time period needed to increase expenditure by 5.15% on average to be indifferent between current and year-ago food and beverage prices. This is 0.99 percentage points greater than if measured using intermediate tastes (P_{SV}).[18]

The results discussed thus far are based on a CGR intended to match RW's as closely as possible. Now, I evaluate the impact of the CGR on the various index formulas by re-estimating the indexes using all UPCs that could be matched between quarters t and $t-4$. Figure 3 plots versions of each index—one using the CGR, and another (marked with an "A") using all matched UPCs. The impact of extreme share changes on P_{CCV} is large. Without the CGR, the average inflation implied by P_{CCV} is 4.86 percentage points lower, implying deflation in most periods. Ehrlich, Haltiwanger, Jarmin, Johnson, Olivares, et al. (2021) also find that P_{CCV} is sensitive to the CGR. Table 9 and Figure 3 also show the effect of the common goods definition on the CES conditional COLIs is smaller and would not change the sign of the index in most cases, amounting to only 0.11 percentage points on average for P_{SV} and 0.04 percentage points on average for P_{BLM}. The average impact on P_{LM} (and hence P_{LMM}) is a bit larger (0.59 percentage points in magnitude). The fact that P_{BLM} is virtually unchanged, while P_{LM} is higher under the CGR perhaps implies that items on clearance (experiencing large price and share declines) have a greater impact than new entrants experiencing large share increases. [Insert Figure 3 and Table 9 close to here.]

The descriptions of conditional and unconditional COLI presented in this section are based on a relatively simple model. Functional form may matter when estimating either the impact of tastes on a conditional COLI (e.g., $P_{BLM} - P_{LM}$) or the pure taste effect on the

[18] Across the non-food products, where the data and model are perhaps less representative, differences among the CES indexes tend to be quite large. They imply department average $P_{SV} - P_{CCV}$ spreads of up to 6.9 percentage points and $P_{BLM} - P_{LM}$ spreads up to 30 percentage points. Results are available from the author by request.

unconditional COLI, $P_{CCV} - P_{SV}$. The reason is P_{CCV}, P_{LM}, and P_{BLM} can be written as recovering the taste parameters as residuals in the CES expenditure share equation. Martin (2020) uses a simulation study that suggests a neglected nesting structure causes negative biases in P_{CCV} and P_{LM}, positive bias in P_{BLM}, and negligible bias in P_{SV}. The indexes tend to have the same rankings relative to each other, but index differences $P_{CCV} - P_{SV}$ and $P_{BLM} - P_{LM}$ can be overestimated. Quantification of taste changes, therefore, may be sensitive to model fit. However, as discussed in Section 3, COLI estimates based on intermediate tastes (or an average of COLI like LMM) may be robust to specification errors (provided σ is not too large) because they fall in the class of superlative and quasi-superlative indexes (Diewert, 1978). Appendix A compares traditional indexes using the Scantrack data. Similarity between the Sato-Vartia and the Fisher and Tornqvist (which approximate an arbitrary expenditure function) implies the CES assumption is not restrictive for estimating a COLI the conditions on average tastes.

6 Conclusion

In a model with changing preferences, there is no longer one true COLI, so it is important to carefully consider the intended theoretical targets when comparing alternative price indexes. When tastes change, the Tornqvist, Sato-Vartia, Lloyd-Moulton and CCV indexes all correspond to different true COLIs. These differ in whether they are intended to reflect only price change (and if so, what preference relation is referenced), or are intended to also capture the pure effect of changing tastes. Users should bear in mind that conditional COLIs are based on reaching a constant indifference curve, while an unconditional COLI is based on reaching a constant utility level. The latter is measurable only under a very strong assumption about utility. This paper's empirical analysis suggests the set of common goods has a significant impact on the unconditional COLI. If all matched items are incorporated, the relative contribution of prices can be swamped by taste change effects. If there is interest in a COLI that conditions on a specific period's taste vector, then this paper provides a novel empirical comparison for the CES case. Improvements to the simple CES model are likely possible, and so future research should include more general demand models to more precisely separate taste changes from price-related substitutions.

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Tables

Table 1: How Price Indexes Treat Taste Change

Index	Formula	Model	Tastes
Fisher	$\left(\frac{\sum_i p_{i1}q_{i0}}{\sum_i p_{i0}q_{i0}}\frac{\sum_i p_{i1}q_{i1}}{\sum_i p_{i0}q_{i1}}\right)^{\frac{1}{2}}$	Un-specif.	Conditional, intermed.
Tornqvist	$\prod_i \left(\frac{p_{i1}}{p_{i0}}\right)^{.5(s_{i0}+s_{i1})}$	Translog	Conditional, geomean
Sato-Vartia	$\prod_i \left(\frac{p_{i1}}{p_{i0}}\right)^{w_i}$	CES	Conditional, intermed.
CCV	$\prod_i \left(\frac{p_{i1}}{p_{i0}}\right)^{\frac{1}{N}} \times \prod_i \left(\frac{s_{i1}}{s_{i0}}\right)^{\frac{1}{N(\sigma-1)}}$	CES	Unconditional, normalized
Lloyd-Moulton	$\left\{\sum_i s_{i0}\left(\frac{p_{i1}}{p_{i0}}\right)^{1-\sigma}\right\}^{\frac{1}{1-\sigma}}$	CES	Conditional, reference
Backwards Lloyd-Moulton	$\left\{\sum_i s_{i1}\left(\frac{p_{i0}}{p_{i1}}\right)^{1-\sigma}\right\}^{\frac{-1}{1-\sigma}}$	CES	Conditional, comparison

Table 2: Scantrack Food and Beverage Departments

Department	# PG	# UPC	Exp. Share
Alcoholic Beverages	4	46,656	0.073
Dairy	12	46,686	0.153
Deli	1	22,061	0.022
Dry Grocery	40	412,319	0.541
Fresh Meat	1	1,934	0.006
Fresh Produce	1	20,244	0.052
Frozen Foods	12	64,635	0.115
Packaged Meat	1	18,401	0.039
All	*72*	*632,936*	*1.000*

Note: Based on data provided by The Nielsen Company (U.S.), LLC.

Table 3: Summary Statistics for $p_{it}/p_{i,t-4}$ by Department (2006Q4 - 2010Q2)

	Obs	Mean	StDev	Skew	Kurt	Min	Max
Alcoholic Beverages	343,007	1.021	0.118	0.415	7.163	0.270	2.098
Dairy	383,519	1.037	0.137	0.908	6.577	0.469	2.256
Deli	128,081	1.025	0.117	0.322	6.578	0.500	1.635
Dry Grocery	2,941,985	1.036	0.141	0.777	9.977	0.210	2.782
Fresh Meat	12,162	1.028	0.117	0.767	6.633	0.557	1.759
Fresh Produce	113,895	1.033	0.165	0.844	6.490	0.458	1.992
Frozen Foods	454,187	1.027	0.125	0.253	6.360	0.281	1.807
Packaged Meat	148,512	1.025	0.106	0.516	5.623	0.625	1.618
All	*4,525,348*	*1.033*	*0.136*	*0.74*	*9.254*	*0.21*	*2.782*

Note: Based on data provided by The Nielsen Company (U.S.), LLC.

Table 4: Statistics by Common Goods Definitions (2007Q3 - 2009Q3)

	Matched UPCs	With CGR
Observations	2,725,348	1,532,988
Expend. (b$)	1,756.2	1,458.0
	Price Relatives	
Mean	1.046	1.053
SD	0.143	0.113
P10	0.913	0.948
P50	1.028	1.034
P90	1.204	1.185
	Share Relatives	
Mean	1.457	1.165
SD	4.675	1.293
P10	0.266	0.563
P50	0.974	1.001
P90	2.157	1.649

Note: Based on data provided by The Nielsen Company (U.S.), LLC.

Table 5: Summary of Elasticity of Substitution Estimates by Department

	# Prod. Gr.	P25	Med	P75
Alcoholic Beverages	4	5.96	7.06	8.63
Dairy	10	3.31	3.65	4.05
Deli	1	3.96	3.96	3.96
Dry Grocery	40	3.85	4.68	6.50
Fresh Meat	1	3.37	3.37	3.37
Fresh Produce	1	2.94	2.94	2.94
Frozen Foods	12	3.31	3.94	6.33
Packaged Meat	1	3.12	3.12	3.12
All	*70*	*3.39*	*4.32*	*6.29*

Note: Based on data provided by The Nielsen Company (U.S.), LLC.

Table 6: Averages of CES Indexes, 2007Q3 - 2009Q3 (percent change)

	CCV	SV	LMM	LM	BLM
Alcoholic Beverages	2.2166	2.5866	2.5850	2.1971	2.9745
Dairy	4.1994	4.2703	4.2600	2.7608	5.8035
Deli	2.5535	2.2851	2.2440	1.5794	2.9134
Dry Grocery	3.3426	5.0194	4.9351	3.7795	6.1205
Fresh Meat	1.0696	4.2702	4.3044	3.7465	4.8658
Fresh Produce	0.5282	1.0948	1.0830	0.2152	1.9593
Frozen Foods	2.0840	3.4260	3.3966	2.9033	3.8943
Packaged Meat	2.7377	2.3790	2.3801	2.0854	2.6757
All	3.0468	4.1683	4.1167	3.1024	5.1551

Notes: Based on data provided by The Nielsen Company (U.S.), LLC. Product group-level indexes are first averaged over time. Department and overall averages are calculated using the product group's share of average quarterly expenditure as weights. CCV refers to RW's CES Common Varieties Index, SV refers to Sato-Vartia, LM refers to Lloyd-Moulton, BLM refers to Backwards Lloyd-Moulton, and LMM refers to the geometric mean of LM and BLM.

Table 7: Distribution of Average CES Index Differences Across Product Groups (percentage points)

	SV – CCV	SV – LMM	SV – BLM	BLM – LM
P5	−1.3823	−0.0892	−3.0150	0.2529
P10	−0.9184	−0.0587	−2.8766	0.3458
P25	0.0982	−0.0126	−1.0537	0.7227
P50	1.0619	0.0084	−0.7340	1.3341
P75	2.1459	0.0506	−0.3727	2.1260
P90	3.2087	0.1645	−0.1908	5.6108
P95	4.2060	0.1993	−0.1652	6.3208
Mean	1.1215	0.0516	−0.9869	2.0527

Notes: Based on data provided by The Nielsen Company (U.S.), LLC. Product group-level index differences are first averaged over time. Department and overall statistics are calculated using the product group's share of average quarterly expenditure as weights. CCV refers to RW's CES Common Varieties Index, SV refers to Sato-Vartia, LM refers to Lloyd-Moulton, BLM refers to Backwards Lloyd-Moulton, and LMM refers to the geometric mean of LM and BLM.

Table 8: Mean CES Index Differences, 2007Q3 - 2009Q3 (percentage points)

	SV – CCV	SV – LMM	SV – BLM	SV – LM	BLM – LM
Alcoholic Beverages	0.3701	0.0017	−0.3878	0.3895	0.7774
Dairy	0.0709	0.0103	−1.5332	1.5094	3.0427
Deli	−0.2684	0.0411	−0.6283	0.7058	1.3341
Dry Grocery	1.6768	0.0843	−1.1011	1.2399	2.3410
Fresh Meat	3.2006	−0.0343	−0.5956	0.5237	1.1193
Fresh Produce	0.5666	0.0118	−0.8645	0.8795	1.7441
Frozen Foods	1.3419	0.0294	−0.4684	0.5227	0.9911
Packaged Meat	−0.3586	−0.0011	−0.2966	0.2936	0.5902
All	1.1215	0.0516	−0.9869	1.0659	2.0527

Notes: Based on data provided by The Nielsen Company (U.S.), LLC. Product group-level index differences are first averaged over time. Department and overall averages are calculated using the product group's share of average quarterly expenditure as weights. CCV refers to RW's CES Common Varieties Index, SV refers to Sato-Vartia, LM refers to Lloyd-Moulton, BLM refers to Backwards Lloyd-Moulton, and LMM refers to the geometric mean of LM and BLM.

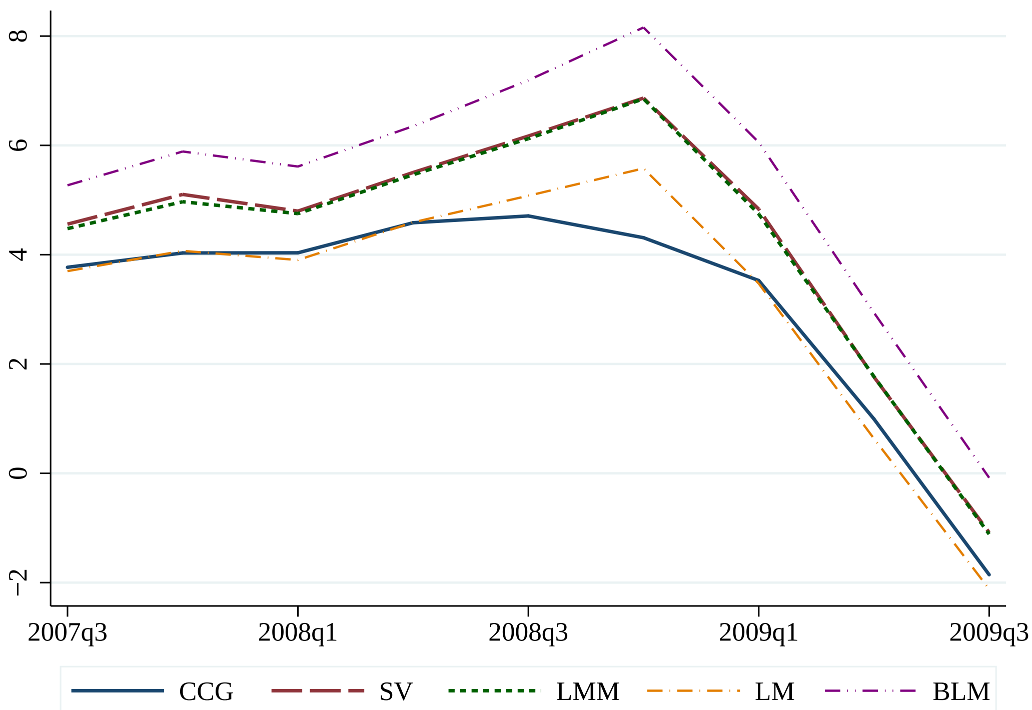
Table 9: Average CES Index by Common Goods Definitions, 2007Q3 - 2009Q3 (perc. points)

	With CGR	Matched UPCs	Diff.
CCV	3.0468	−1.8178	4.8646
SV	4.1683	4.0539	0.1143
LMM	4.1167	3.7908	0.3258
LM	3.1024	2.5168	0.5856
BLM	5.1551	5.1192	0.0359

Notes: Based on data provided by The Nielsen Company (U.S.), LLC. Product group-level indexes are first averaged over time. Overall averages are then calculated using the product group's share of average quarterly expenditure as weights. CCV refers to RW's CES Common Varieties Index, SV refers to Sato-Vartia, LM refers to Lloyd-Moulton, BLM refers to Backwards Lloyd-Moulton, and LMM refers to the geometric mean of LM and BLM.

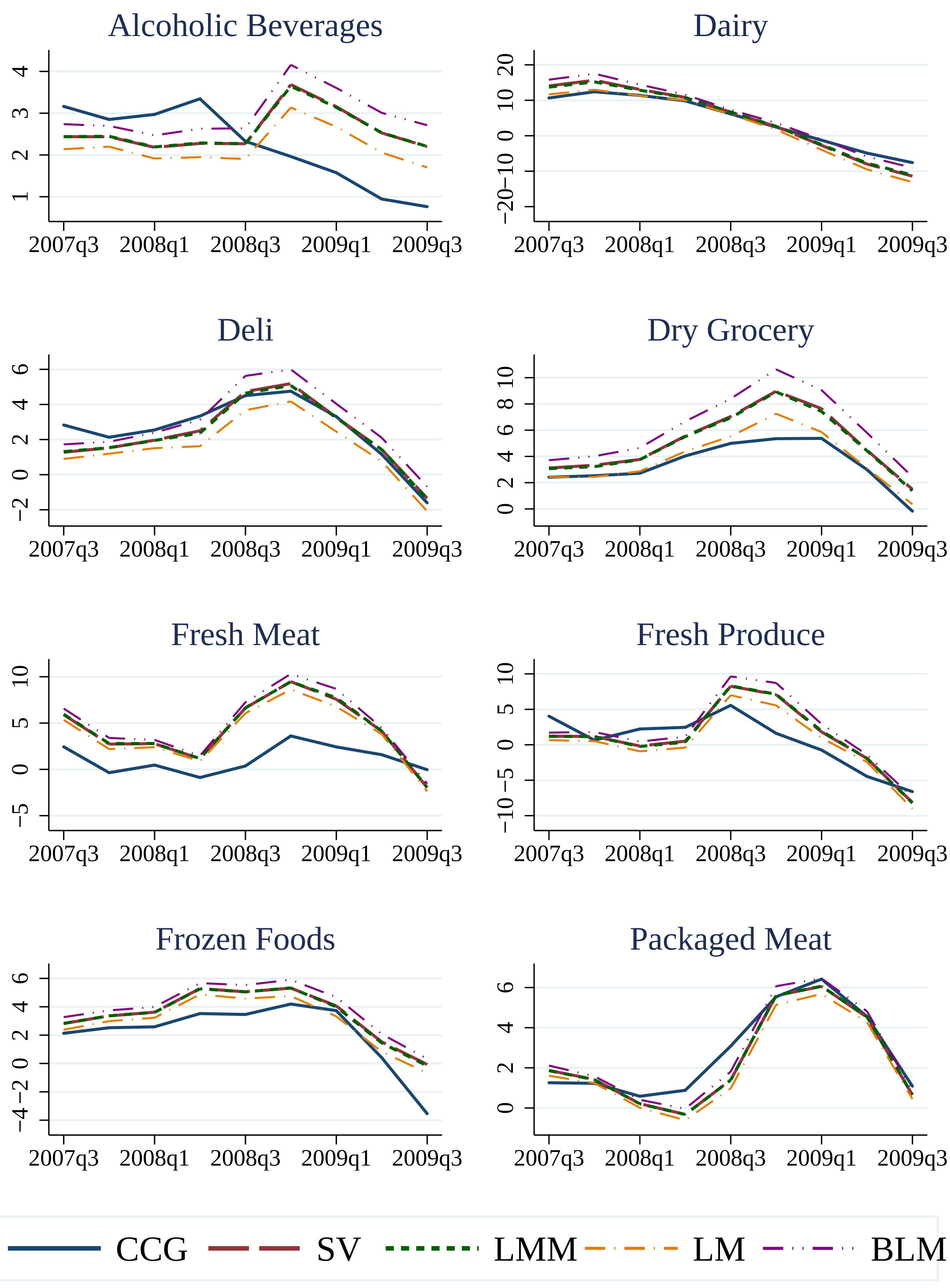
Figures

Figure 1: Scantrak CES Price Index Averages (% change versus year ago)



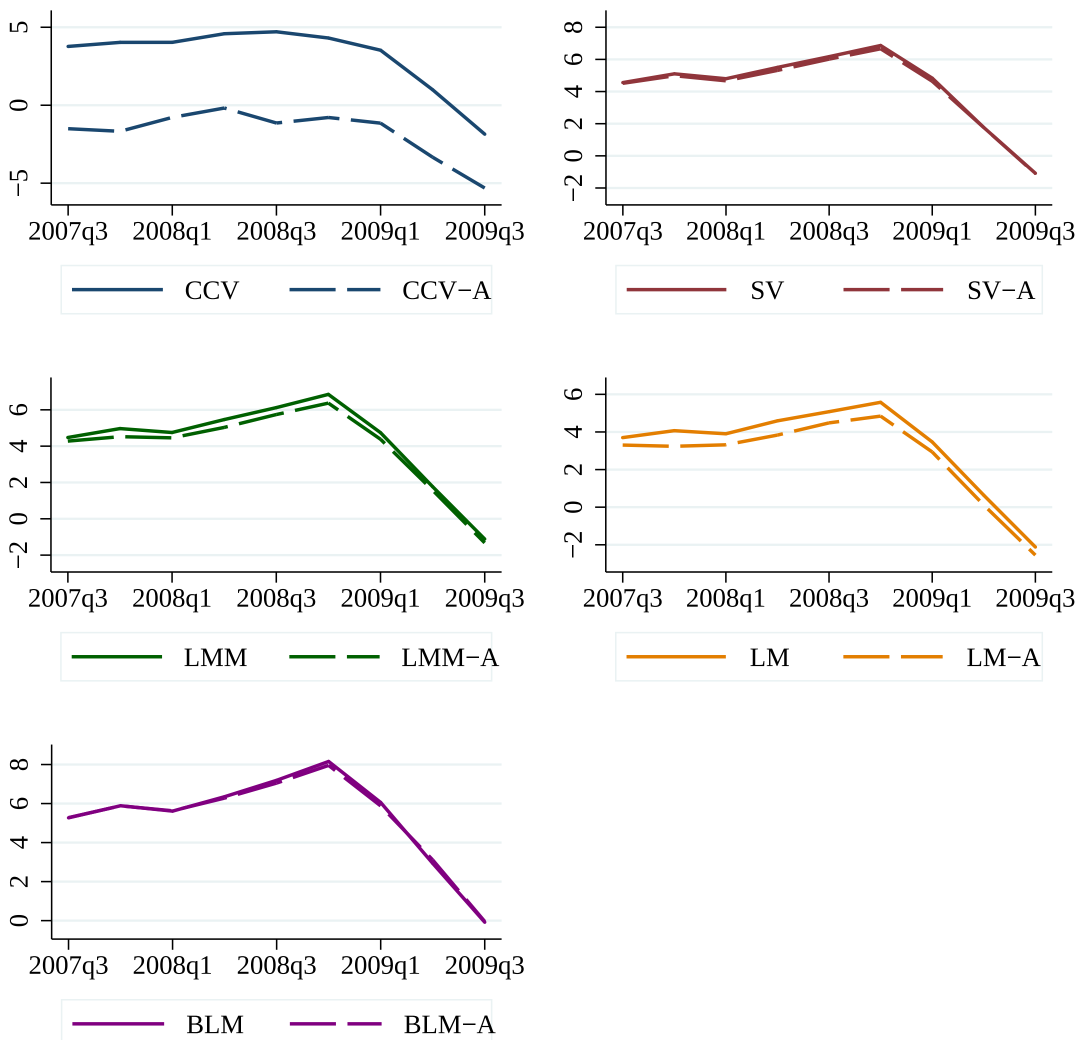
Note: Based on data provided by The Nielsen Company (U.S.), LLC. Plots are comparison period expenditure-weighted averages of the four-quarter proportional changes implied by product group-level indexes for food and beverage products. CCV refers to RW's CES Common Varieties Index, SV refers to Sato-Vartia, LM refers to Lloyd-Moulton, BLM refers to Backwards Lloyd-Moulton, and LMM refers to the geometric mean of LM and BLM. All but the SV indexes require estimated elasticities of substitution.

Figure 2: Scantrak CES Price Index Averages By Dept. (% change versus year ago)



Note: Based on data provided by The Nielsen Company (U.S.), LLC. See notes for Figure 1.

Figure 3: CES Price Indexes: CGR versus All Matched UPCs (% change versus year ago)



Note: Based on data provided by The Nielsen Company (U.S.), LLC. Plots are comparison expenditure-weighted averages of the four-quarter proportional changes implied by product group-level indexes for food and beverage products. CCV refers to RW's CES Common Varieties Index, SV refers to Sato-Vartia, LM refers to Lloyd-Moulton, BLM refers to Backwards Lloyd-Moulton, and LMM refers to the geometric mean of LM and BLM. All but the SV indexes require estimated elasticities of substitution. An "A" in the label indicates all matched UPCs were included. Otherwise, the indexes cover only UPCs satisfying the CGR.

A Traditional Price Indexes with Retail Scanner Data

As in Section 5, I calculate four-quarter price indexes for each food and beverage product group in the Scantrack data using the Fisher, Tornqvist, Sato-Vartia, and Laspeyres formulas. The CGR is not imposed, and so the indexes cover the period 2005Q4 to 2010Q2. All but the Laspeyres are variable-weight indexes, meaning they reflect consumer substitutions over time. While the Sato-Vartia reflects substitutions according to the CES expenditure function, the Fisher and Tornqvist are superlative, meaning they approximate an arbitrary homothetic expenditure function (Diewert, 1976). Figure A1 plots kernel density estimates of the differences between each index and the Fisher, while Table A1 lists mean differences and mean absolute differences within Scantrack department.

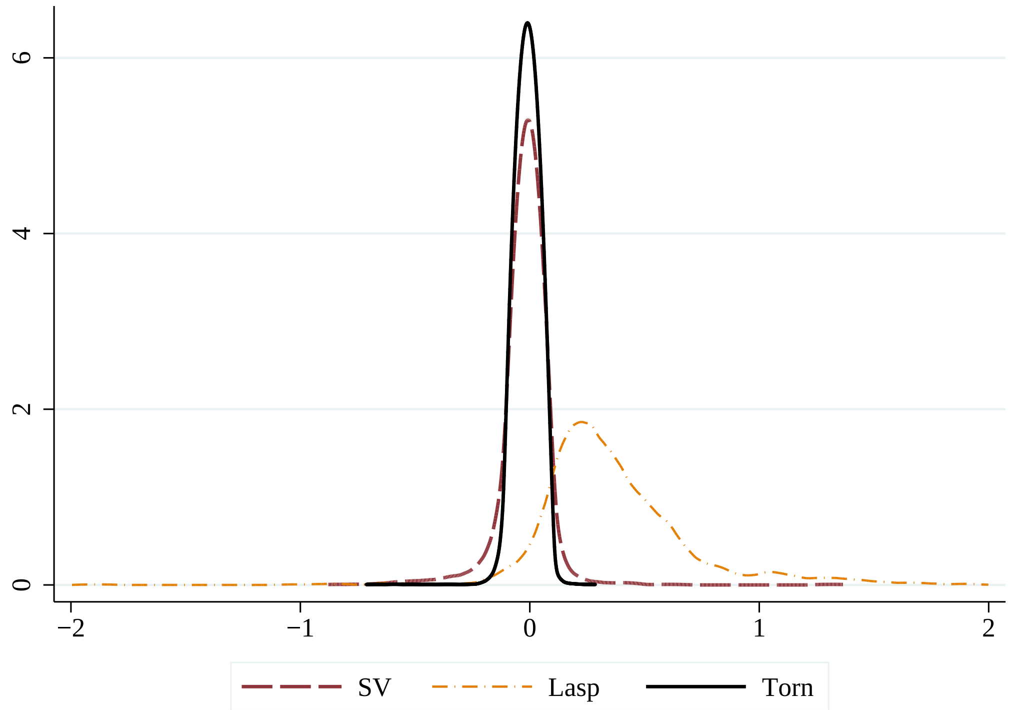
As many have found previously, there tends to be substantial agreement between the three traditional variable-weight indexes, with mean differences and mean absolute differences all less than one tenth of one percentage point in magnitude. In contrast, the Laspeyres indexes exceeds the Fisher index by about 0.4 percentage points, on average, which is on the order of estimates from Boskin, et. al. (1996) and others for lower-level substitution bias.

Table A1: Differences from a Fisher Index by Department (percentage points)

	Mean Difference			Mean Absolute Difference		
	Torn.	SV	Lasp.	Torn.	SV	Lasp.
Alcoholic Beverages	0.0005	0.0013	0.2093	0.0041	0.0101	0.2102
Dairy	−0.0019	−0.0021	0.2903	0.0071	0.0251	0.2916
Deli	−0.0029	0.0093	0.3257	0.0046	0.0252	0.3257
Dry Grocery	−0.0170	−0.0454	0.4043	0.0213	0.0683	0.4168
Fresh Meat	0.0031	0.0027	0.6404	0.0154	0.0276	0.6404
Fresh Produce	0.0097	0.0169	0.6203	0.0182	0.0428	0.6203
Frozen Foods	−0.0166	−0.0186	0.4255	0.0199	0.0416	0.4350
Packaged Meat	−0.0022	0.0012	0.4027	0.0046	0.0075	0.4027
All	*−0.0109*	*−.0257*	*0.3858*	*0.0165*	*0.0495*	*0.3940*

Note: Based on data provided by The Nielsen Company (U.S.), LLC. Statistics are average differences between index indicated and a Fisher index, weighted by the product group's share of expenditure in the comparison period.

Figure A1: Scanner Data: Differences from a Fisher Index



Note: Based on data provided by The Nielsen Company (U.S.), LLC. Data are differences between index indicated and a Fisher index at the product group level, expressed as percent changes over four quarters. Density estimates use Epanechnikov kernel with bandwidth of 0.05 percentage points.

B Application to CPI aggregation

The previous section, while making use of detailed transactions data, applies only to food and beverage products consumed at home, which constitute less than 10% of CPI-eligible expenditures (Bureau of Labor Statistics, 2020). To the extent possible, I now use CPI data to estimate what role changing tastes play in the calculation of price indexes over a broad consumption basket. Subject to the limitation described below, I find that year-over-year differences in CES indexes tend to be smaller and less persistent than those found in Section 5.

The basic unit of this analysis is the monthly elementary item-area index (e.g., Mens Suits in Pittsburgh), which is considerably more aggregated than the UPC-level data employed in Section 5. Such indexes are the inputs to both the headline Consumer Price Index for

Urban Consumers (CPI-U, which uses the Lowe formula for aggregation) and the C-CPI-U (which uses the Tornqvist). Currently, the BLS calculates elementary indexes for 243 item categories in 32 areas, for a total of 7,776 item-area indexes, though these dimensions have changed over time. Similar to Section 5, I consider an annual frequency of taste change by estimating a series of direct indexes where the base period for each is the same month during the prior year. For comparison with BLS methodology, I also include the Tornqvist index, P_T.[19]

A limitation of this analysis is that the elementary indexes are fixed. Weights for elementary indexes are available at a lag of up to four years, so indexes like P_T, P_{LM} and P_{BLM} are infeasible. The BLS uses either a weighted geometric mean or a modified Laspeyres formula (Bureau of Labor Statistics, 2018). Therefore, this paper's exercise is only informative about category-level tastes (e.g., for ground beef versus chicken) as opposed to variety-level tastes (e.g., for 85% ground beef versus 90% ground beef). Furthermore, the Lloyd-Moulton indexes require the elasticity of substitution σ. Estimation in the style of Feenstra (1994) requires $\sigma > 1$, which is not realistic for this application because it implies all varieties (or aggregates) are substitutes. Additionally, previous analysis of CPI elementary indexes, such as Klick (2018), have estimated elasticities less than one. RW also assume $\sigma > 1$, and for this reason, I do not present any estimates of P_{CCV} using CPI indexes. Given the importance of σ in the CES model's ability to separate price-related substitutions from taste-related substitutions, I present conditional COLI estimates for four different elasticities (0.6, 0.7, 0.8, and 0.9), and leave further inquiry into the correct choice of σ to future research.[20]

[19] In contrast, the published C-CPI-U is a series of one-month indexes multiplied together. When calculating monthly chained versions of the CES indexes, I find monthly percent change differences to be quite small, but levels can drift apart somewhat over time for some values of σ. Results are available from the author upon request.

[20] The initial and interim C-CPI-U use $\sigma = 0.6$, based on pooled, biennial regressions of logged, differenced shares on logged, differenced elementary indexes, in the style of Feenstra and Reinsdorf (2007). The goal in that case, however, is to predict the final value of the Tornqvist formula once updated expenditures are available, rather than estimating a true CES COLI.

B.1 Results

Table B2 presents average 12-month percent changes of P_T, P_{SV}, P_{LM}, P_{BLM}, and P_{LMM} over the period from December 2000 to December 2017. Table B3 gives the average differences between pairs of indexes.[21] Figures B2 to B4 plot these indexes separately for the different values of σ chosen. For readability, the graphs have been split into three time periods.

While the official C-CPI-U is a series of chained month-over-month indexes, these results suggest that an alternative accounting for preferences would have a relatively modest average effect on year-over-year measurements. The Tornqvist, SV and LMM indexes tend to be very close on average, differing by less 0.03 percentage points in magnitude, again reflecting how functional form is less important when conditioning on an intermediate taste level. In fact, average differences among all of the indexes tend to be less than one tenth of one percentage point. The only exception is when $\sigma = 0.9$, P_{BLM} exceeds P_{LM} by 0.16 percentage points on average. Taking current preferences (i.e., P_{BLM}) as the most relevant reference point, then P_T overstates this conditional COLI by 0.057 percentage points (3.1%) under the assumption that $\sigma = 0.6$, while it understates it by 0.079 percentage points (4.0%) under the assumption that $\sigma = 0.9$. Overall, compared to individual variety tastes, the effect of category-level tastes appears relatively small.

Figures B2 to B4 reveal that despite the indexes tending to give similar answers on average, short term divergences can occur. From November 2008 to September 2009, for example, with σ set to 0.9, P_{BLM} exceeds P_{LM} by an average of 0.8 percentage points each month, with individual month differences ranging from 0.34 to 1.22 percentage points. In other periods, however, differences are quite small. For instance, the average difference from January to November 2007 is only 0.01 percentage point, with individual month's differences ranging from -0.10 to 0.09 percentage points. This is different from Section 5,

[21] Item structure changes in 2008, 2010, and 2013 reduce the number of overlapping item-areas for those years by 0.47%, 2.84%, and 14.81%, respectively. The averages in Table B3 are qualitatively the same when excluding these years. Results available from the author upon request. Indexes ending in 2018 were not calculated due to implementation of a new CPI area sample.

where differences between P_{BLM} and P_{LM} indexes using Scantrack are found to be persistent over time.

As noted before, each formula uses the same elementary indexes, and so the most can be said is that tastes for broader item categories have relatively little impact on the the conditional COLI. Section 5 suggests a larger role of tastes at the individual item level, however.

Table B2: CPI: Mean of 12-mo. Indexes over 2000m12-2017m12 (perc. points)

σ	Torn.	SV	LMM	LM0	LM1
0.6	1.8834	1.8999	1.8676	1.9090	1.8263
0.7	1.8834	1.8999	1.8773	1.8733	1.8813
0.8	1.8834	1.8999	1.8815	1.8388	1.9243
0.9	1.8834	1.8999	1.8832	1.8045	1.9620

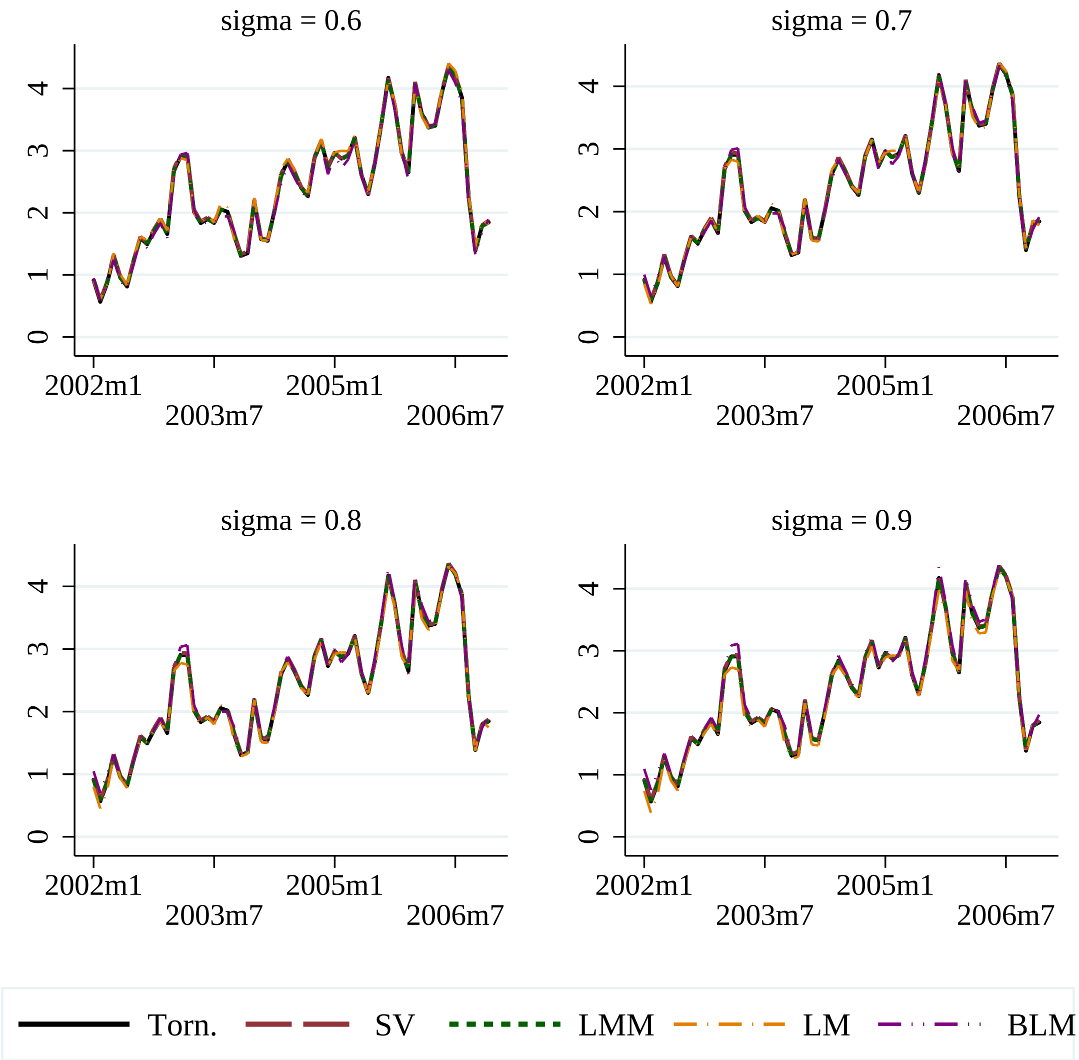
Note: Torn refers to the Tornqvist index, SV refers to Sato-Vartia, LM refers to Lloyd-Moulton, BLM refers to Backwards Lloyd-Moulton, and LMM refers to the geometric mean of LM and BLM.

Table B3: CPI: Mean Differences in 12-mo. Indexes over 2000m12-2017m12 (perc. points)

σ	Torn. − SV	SV − LMM	SV − LM	SV − BLM	BLM − LM
0.6	−0.0165	0.0323	−0.0090	0.0736	−0.0827
0.7	−0.0165	0.0227	0.0266	0.0186	0.0080
0.8	−0.0165	0.0184	0.0611	−0.0244	0.0856
0.9	−0.0165	0.0168	0.0954	−0.0621	0.1575

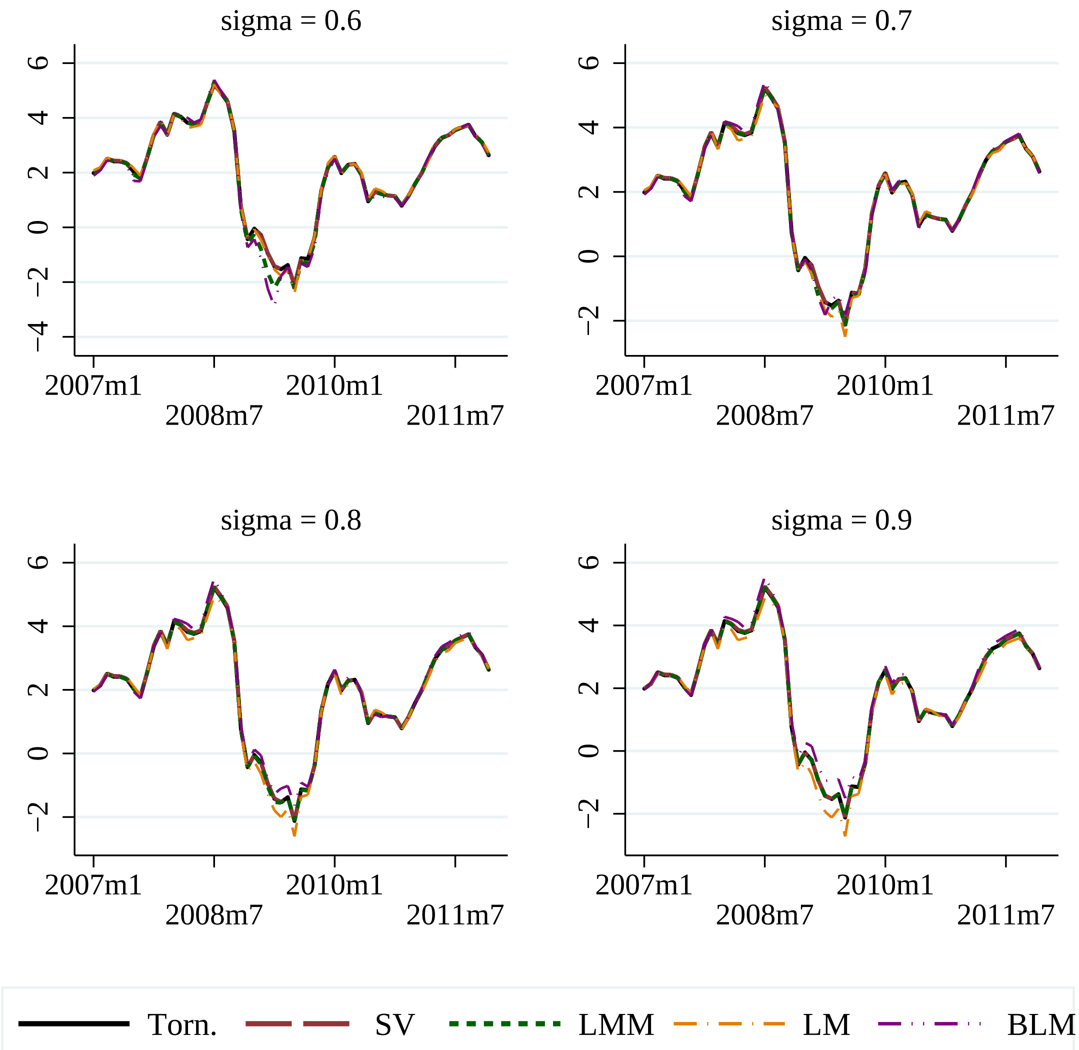
Note: Torn refers to the Tornqvist index, SV refers to Sato-Vartia, LM refers to Lloyd-Moulton, BLM refers to Backwards Lloyd-Moulton, and LMM refers to the geometric mean of LM and BLM.

Figure B2: Comparison of 12-mo. CPI Aggregates, 2002m1-2006m12



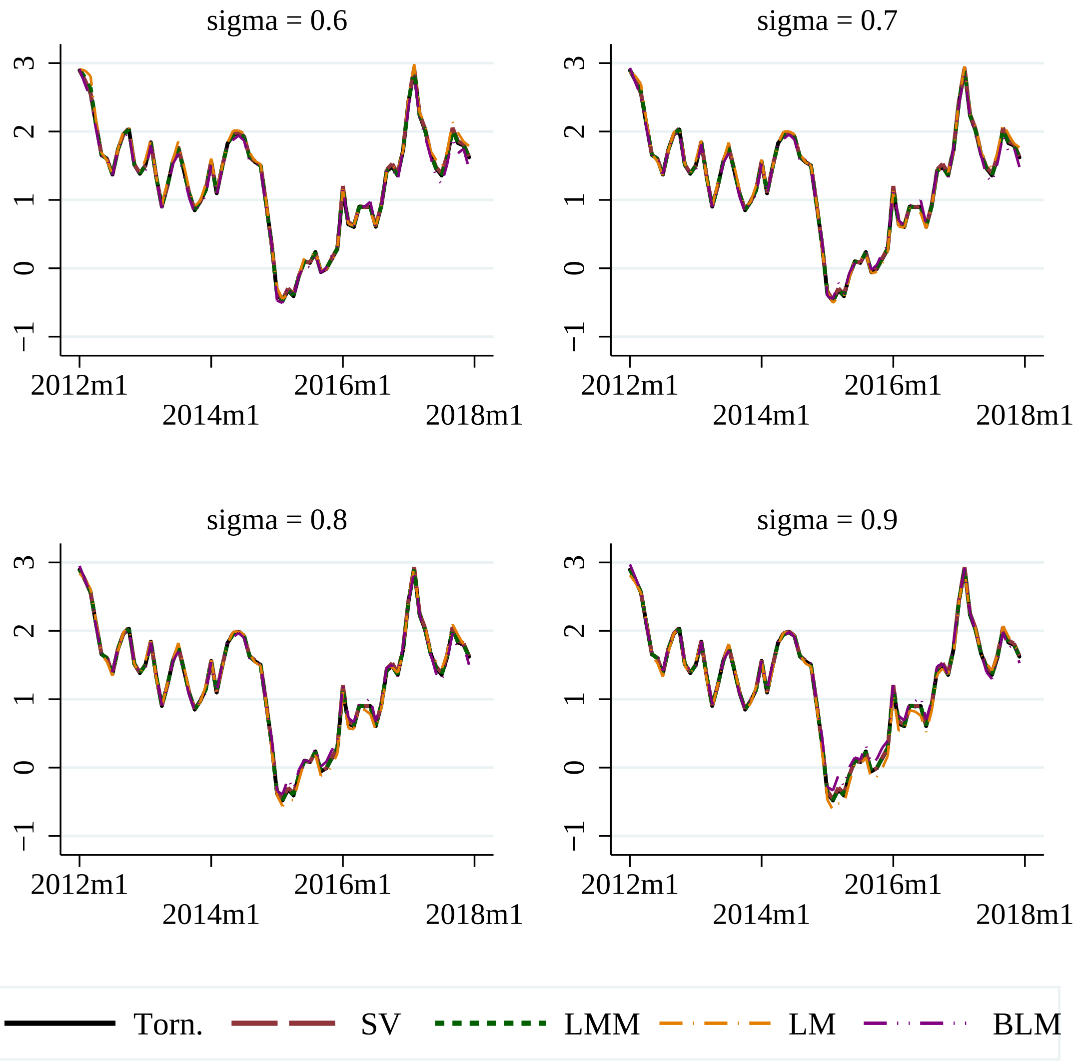
Note: Plots are twelve-month percent changes. Torn refers to the Tornqvist index, SV refers to Sato-Vartia, LM refers to Lloyd-Moulton, BLM refers to Backwards Lloyd-Moulton, and LMM refers to the geometric mean of LM and BLM. The LMM, LM, and BLM indexes are calculated using indicated elasticity of substitution (sigma).

Figure B3: Comparison of 12-mo. CPI Aggregates, 2007m1-2011m12



Note: Plots are twelve-month percent changes. Torn refers to the Tornqvist index, SV refers to Sato-Vartia, LM refers to Lloyd-Moulton, BLM refers to Backwards Lloyd-Moulton, and LMM refers to the geometric mean of LM and BLM. The LMM, LM, and BLM indexes are calculated using indicated elasticity of substitution (sigma).

Figure B4: Comparison of 12-mo. CPI Aggregates, 2012m1-2017m12



Note: Plots are twelve-month percent changes. Torn refers to the Tornqvist index, SV refers to Sato-Vartia, LM refers to Lloyd-Moulton, BLM refers to Backwards Lloyd-Moulton, and LMM refers to the geometric mean of LM and BLM. The LMM, LM, and BLM indexes are calculated using indicated elasticity of substitution (sigma).

C Product Turnover with CES Preferences

Dating back to Feenstra (1994), the CES function is convenient for modeling the cost-of-living effects of entering and exiting varieties. The challenge in constructing conditional COLIs with product turnover is that there really is no way to identify an item's taste parameter corresponding to a period prior to its entry or after its exit. Nevertheless, this section shows how under a reasonable assumption, one can form two-way bounds on the reference and comparison period conditional COLIs using Feenstra's (1994) adjustment term. Using the Scantrack data, I find these bounds are tighter for the comparison period COLI than for the reference period COLI.

We assume the consumer has CES preferences over the set of varieties $\bar{\mathcal{I}}$ (e.g., Eq.8 holds for $i \in \bar{\mathcal{I}}$), some items may be unavailable in one or more periods. Denote the set of varieties available in each period as $\mathcal{I}_0$ and $\mathcal{I}_1$, such that $\mathcal{I}_0 \cup \mathcal{I}_1 \subseteq \bar{\mathcal{I}}$. Denote the set of common varieties as $\mathcal{I} = \mathcal{I}_0 \cap \mathcal{I}_1$, as in the main text.

An important feature of CES preferences is that optimal expenditure on a subset of varieties depends only on prices and taste parameters for varieties in that subset. Adjusting notation to account for the varieties set, Eq. 8 becomes (taking $\bar{u}$ to be one):

$$C(\boldsymbol{p};\boldsymbol{\varphi},\mathcal{I}) = \left[\sum_{i\in\mathcal{I}} \left(\frac{p_i}{\varphi_i}\right)^{1-\sigma}\right]^{\frac{1}{1-\sigma}}. \tag{18}$$

Eq. 9 becomes

$$s_i(\boldsymbol{p};\boldsymbol{\varphi},\mathcal{I}) = \frac{(p_i/\varphi_i)^{1-\sigma}}{\sum_{j\in\mathcal{I}} (p_j/\varphi_j)^{1-\sigma}} = \frac{(p_i/\varphi_i)^{1-\sigma}}{[C(\boldsymbol{p};\boldsymbol{\varphi},\mathcal{I})]^{1-\sigma}} \tag{19}$$

If a variety is unavailable, the COLI framework uses the reservation price implied by the model of preferences. Assuming $\sigma > 1$, the CES model implies infinite reservation prices.[22] Together, these properties imply $C(\boldsymbol{p};\boldsymbol{\varphi},\bar{\mathcal{I}}) = C(\boldsymbol{p};\boldsymbol{\varphi}_t,\mathcal{I}_t)$.[23]

[22] When $\sigma < 1$, consumption of all commodities is necessary for positive utility.

[23] More precisely, as the prices of unavailable goods approach infinity, the limit of $C(\boldsymbol{p};\boldsymbol{\varphi},\bar{\mathcal{I}})$ equals $C(\boldsymbol{p};\boldsymbol{\varphi}_t,\mathcal{I}_t)$.

The class of conditional COLI is therefore given by

Definition C.1 *Conditional Cost-of-living Index with product turnover*

$$\Phi(\boldsymbol{p}_0, \boldsymbol{p}_1; \boldsymbol{\varphi}, \bar{\mathcal{I}}) = \frac{C(\boldsymbol{p}_1; \boldsymbol{\varphi}, \mathcal{I}_1)}{C(\boldsymbol{p}_0; \boldsymbol{\varphi}, \mathcal{I}_0)} \tag{20}$$

Let $\mathcal{I}^* \subseteq \mathcal{I}$. Similar to Feenstra (1994), we can rewrite Eq. 20 as

$$\begin{aligned}\Phi(\boldsymbol{p}_0, \boldsymbol{p}_1; \boldsymbol{\varphi}, \bar{\mathcal{I}}) &= \frac{C(\boldsymbol{p}_1; \boldsymbol{\varphi}, \mathcal{I}^*)}{C(\boldsymbol{p}_0; \boldsymbol{\varphi}, \mathcal{I}^*)} \frac{C(\boldsymbol{p}_0; \boldsymbol{\varphi}, \mathcal{I}^*)}{C(\boldsymbol{p}_0; \boldsymbol{\varphi}, \mathcal{I}_0)} \frac{C(\boldsymbol{p}_1; \boldsymbol{\varphi}, \mathcal{I}_1)}{C(\boldsymbol{p}_1; \boldsymbol{\varphi}, \mathcal{I}^*)} \\ &\equiv \Phi(\boldsymbol{p}_0, \boldsymbol{p}_1; \boldsymbol{\varphi}, \mathcal{I}^*) \lambda_0(\boldsymbol{\varphi})^{\frac{1}{1-\sigma}} \lambda_1(\boldsymbol{\varphi})^{\frac{1}{\sigma-1}}, \end{aligned} \tag{21}$$

where $\Phi(\boldsymbol{p}_0, \boldsymbol{p}_1; \boldsymbol{\varphi}, \mathcal{I}^*)$ is the conditional COLI over the common subset $\mathcal{I}^*$, and

$$\lambda_t(\boldsymbol{\varphi}) = \frac{\sum_{i \in \mathcal{I}^*} \left(\frac{p_{it}}{\varphi_i}\right)^{1-\sigma}}{\sum_{i \in \mathcal{I}_t} \left(\frac{p_{it}}{\varphi_i}\right)^{1-\sigma}},\ t = 0, 1. \tag{22}$$

The term $\lambda_0(\boldsymbol{\varphi})^{\frac{1}{1-\sigma}}$ adjusts the COLI for the welfare loss from exiting products, while $\lambda_1(\boldsymbol{\varphi})^{\frac{1}{\sigma-1}}$ adjusts it for the welfare gain from new products. A key implication of Equation 21 is that with the CES functional form, the matched model component of the conditional COLI is separable from the terms that account for product turnover.

As before, $\boldsymbol{\varphi}_0$ and $\boldsymbol{\varphi}_1$ are interesting choices. From Feenstra (1994), $\lambda_t(\boldsymbol{\varphi}_t) = \frac{\sum_{i \in \mathcal{I}^*} p_{it} q_{it}}{\sum_{i \in \mathcal{I}_t} p_{it} q_{it}} \equiv \lambda_t$, which is the share of common varieties expenditure out of total expenditure occurring in period t. A clear challenge arises, however, with the terms $\lambda_s(\boldsymbol{\varphi}_t), s \neq t$. Intuitively, Eq. 19 implies the taste parameters for absent varieties are not identified—given an infinite price, expenditure shares are zero for any finite value of φ_{it}.

The situation is helped somewhat by the fact that $\lambda_t(\boldsymbol{\varphi}) \in [0, 1]$, and so $\lambda_0(\boldsymbol{\varphi})^{\frac{1}{1-\sigma}} \geq 1$

and $\lambda_1(\boldsymbol{\varphi})^{\frac{1}{\sigma-1}} \leq 1$.[24] This implies the following bounds:

$$\bar{P}_{LM}(\boldsymbol{p}_0, \boldsymbol{p}_1, \boldsymbol{q}_0, \boldsymbol{q}_1, \sigma, \mathcal{I}^*) \equiv P_{LM}(\boldsymbol{p}_0, \boldsymbol{p}_1, \boldsymbol{q}_0, \boldsymbol{q}_1, \sigma, \mathcal{I}^*)\lambda_0^{\frac{1}{1-\sigma}} \geq \Phi(\boldsymbol{p}_0, \boldsymbol{p}_1; \boldsymbol{\varphi}_0, \bar{\mathcal{I}}) \tag{23}$$

$$\bar{P}_{BLM}(\boldsymbol{p}_0, \boldsymbol{p}_1, \boldsymbol{q}_0, \boldsymbol{q}_1, \sigma, \mathcal{I}^*) \equiv P_{BLM}(\boldsymbol{p}_0, \boldsymbol{p}_1, \boldsymbol{q}_0, \boldsymbol{q}_1, \sigma, \mathcal{I}^*)\lambda_1^{\frac{1}{\sigma-1}} \leq \Phi(\boldsymbol{p}_0, \boldsymbol{p}_1; \boldsymbol{\varphi}_1, \bar{\mathcal{I}}), \tag{24}$$

where $\bar{P}_{LM}(\boldsymbol{p}_0, \boldsymbol{p}_1, \boldsymbol{q}_0, \boldsymbol{q}_1, \sigma, \mathcal{I}^*)$ and $\bar{P}_{BLM}(\boldsymbol{p}_0, \boldsymbol{p}_1, \boldsymbol{q}_0, \boldsymbol{q}_1, \sigma, \mathcal{I}^*)$ are Lloyd-Moulton style indexes which include only adjustments for either exit or entry, not both.

Given these bounds, it is possible to apply the method of proof in Konüs (1924) and Diewert (2001) (Proposition 8) to show that there exists an intermediate taste vector $\check{\boldsymbol{\varphi}}$ such that either $\bar{P}_{LM}(\boldsymbol{p}_0, \boldsymbol{p}_1, \boldsymbol{q}_0, \boldsymbol{q}_1, \sigma, \mathcal{I}^*) \leq \Phi(\boldsymbol{p}_0, \boldsymbol{p}_1; \check{\boldsymbol{\varphi}}, \bar{\mathcal{I}}) \leq \bar{P}_{BLM}(\boldsymbol{p}_0, \boldsymbol{p}_1, \boldsymbol{q}_0, \boldsymbol{q}_1, \sigma, \mathcal{I}^*)$ or $\bar{P}_{BLM}(\boldsymbol{p}_0, \boldsymbol{p}_1, \boldsymbol{q}_0, \boldsymbol{q}_1, \sigma, \mathcal{I}^*) \leq \Phi(\boldsymbol{p}_0, \boldsymbol{p}_1; \check{\boldsymbol{\varphi}}, \bar{\mathcal{I}}) \leq \bar{P}_{LM}(\boldsymbol{p}_0, \boldsymbol{p}_1, \boldsymbol{q}_0, \boldsymbol{q}_1, \sigma, \mathcal{I}^*)$. Of course, these bounds may not be particularly tight, and a symmetric average (e.g., a geometric mean) might not be attractive because the missing factors $\lambda_1(\boldsymbol{\varphi}_0)^{\frac{1}{\sigma-1}}$ and $\lambda_0(\boldsymbol{\varphi}_1)^{\frac{1}{1-\sigma}}$ are not likely to be of comparable magnitudes. For instance, Feenstra (1994), Broda and Weinstein (2010), and others have found that $\lambda_1^{\frac{1}{\sigma-1}}$ dominates $\lambda_0^{\frac{1}{1-\sigma}}$, resulting in a net downward adjustment to the COLI. Consequently, $\bar{P}_{BLM}(\boldsymbol{p}_0, \boldsymbol{p}_1, \boldsymbol{q}_0, \boldsymbol{q}_1, \sigma, \mathcal{I}^*)$ might be a lot closer to its target than $\bar{P}_{LM}(\boldsymbol{p}_0, \boldsymbol{p}_1, \boldsymbol{q}_0, \boldsymbol{q}_1, \sigma, \mathcal{I}^*)$.

As market entry and exit decisions may themselves be tied to tastes, it seems reasonable that the welfare loss from exiting varieties is larger if conditioning on reference period preferences, and that the welfare gain from new varieties is larger if conditioning on comparison period preferences. This motivates the following assumption:

Assumption C.1 *Non-continuing varieties are valued more in the period in which they are available.*

$$\lambda_t(\boldsymbol{\varphi}_t) \leq \lambda_t(\boldsymbol{\varphi}_s), t = 0, 1;\ s \neq t$$

The implies $\lambda_0^{\frac{1}{1-\sigma}} \geq \lambda_0(\boldsymbol{\varphi}_1)^{\frac{1}{1-\sigma}}$ and $\lambda_1^{\frac{1}{\sigma-1}} \leq \lambda_1(\boldsymbol{\varphi}_0)^{\frac{1}{\sigma-1}}$. Assumption C.1 would be true,

[24] The term $\lambda_1(\varphi)^{\frac{1}{\sigma-1}} \leq 1$ is also greater than zero.

for example, if taste parameters for common varieties were constant while taste parameters were lower for varieties when they were absent from the market.

Suppose we use Feenstra's adjustment $\lambda_0^{\frac{1}{1-\sigma}}\lambda_1^{\frac{1}{\sigma-1}}$ on both P_{LM} and P_{BLM}. Define

$$P_{FLM}(\boldsymbol{p}_0,\boldsymbol{p}_1,\boldsymbol{q}_0,\boldsymbol{q}_1,\sigma,\bar{\mathcal{I}}) = P_{LM}(\boldsymbol{p}_0,\boldsymbol{p}_1,\boldsymbol{q}_0,\boldsymbol{q}_1,\sigma,\mathcal{I}^*)\lambda_0^{\frac{1}{1-\sigma}}\lambda_1^{\frac{1}{\sigma-1}} \tag{25}$$

and

$$P_{FBLM}(\boldsymbol{p}_0,\boldsymbol{p}_1,\boldsymbol{q}_0,\boldsymbol{q}_1,\sigma,\bar{\mathcal{I}}) = P_{BLM}(\boldsymbol{p}_0,\boldsymbol{p}_1,\boldsymbol{q}_0,\boldsymbol{q}_1,\sigma,\mathcal{I}^*)\lambda_0^{\frac{1}{1-\sigma}}\lambda_1^{\frac{1}{\sigma-1}}. \tag{26}$$

Under assumption C.1, we have:

$$P_{FLM}(\boldsymbol{p}_0,\boldsymbol{p}_1,\boldsymbol{q}_0,\boldsymbol{q}_1,\sigma,\bar{\mathcal{I}}) \leq \Phi(\boldsymbol{p}_0,\boldsymbol{p}_1;\boldsymbol{\varphi}_0,\bar{\mathcal{I}}) \leq \bar{P}_{LM}(\boldsymbol{p}_0,\boldsymbol{p}_1,\boldsymbol{q}_0,\boldsymbol{q}_1,\sigma,\mathcal{I}^*) \tag{27}$$

and

$$\bar{P}_{BLM}(\boldsymbol{p}_0,\boldsymbol{p}_1,\boldsymbol{q}_0,\boldsymbol{q}_1,\sigma,\mathcal{I}^*) \leq \Phi(\boldsymbol{p}_0,\boldsymbol{p}_1;\boldsymbol{\varphi}_1,\bar{\mathcal{I}}) \leq P_{FBLM}(\boldsymbol{p}_0,\boldsymbol{p}_1,\boldsymbol{q}_0,\boldsymbol{q}_1,\sigma,\bar{\mathcal{I}}). \tag{28}$$

A priori, neither of these bounds must be tight enough to be useful, but previous research has found that the effect of new varieties tends to dominate that of disappearing varieties in Feenstra-style CES indexes (Broda and Weinstein, 2010). Indeed, using the Nielsen Retail Scanner data (Table C1), I find the adjustment for exiting varieties is relatively small on average for many departments, ranging from 0.03 percentage points for Alcoholic Beverages to 1.43 percentage points for Deli. The adjustments for new varieties is between two and twelve times larger in magnitude, ranging from -0.38 percentage points for Alcoholic Beverages to -4.12 percentage points for Deli.[25] As a result, the bounds on $\Phi(\boldsymbol{p}_0,\boldsymbol{p}_1;\boldsymbol{\varphi}_1,\bar{\mathcal{I}})$ appear tighter than the bounds on $\Phi(\boldsymbol{p}_0,\boldsymbol{p}_1;\boldsymbol{\varphi}_0,\bar{\mathcal{I}})$. Figure C1 plots the average P_{BLM} index over common varieties versus the upper and lower bounds for $\Phi(\boldsymbol{p}_0,\boldsymbol{p}_1;\boldsymbol{\varphi}_1,\bar{\mathcal{I}})$. Overall, the average difference in bounds is 0.34 percentage points for food products and 0.26 percentage

[25] These adjustment magnitudes are significantly larger than what was reported in Broda and Weinstein (2010) using consumer scanner data over 1994-2003. As my estimates of σ using the retail data are lower, the larger adjustments are to be expected.

points for non-food products. As the graph indicates, this margin is small relative to the net adjustment for product turnover. Consequently, a geometric mean of the observable bounds seems reasonable to estimate the COLI conditional on comparison period tastes.[26] Note, such an index will imply a smaller welfare effect from exiting varieties, leading to a larger net adjustment or "new goods bias" for the common varieties index.

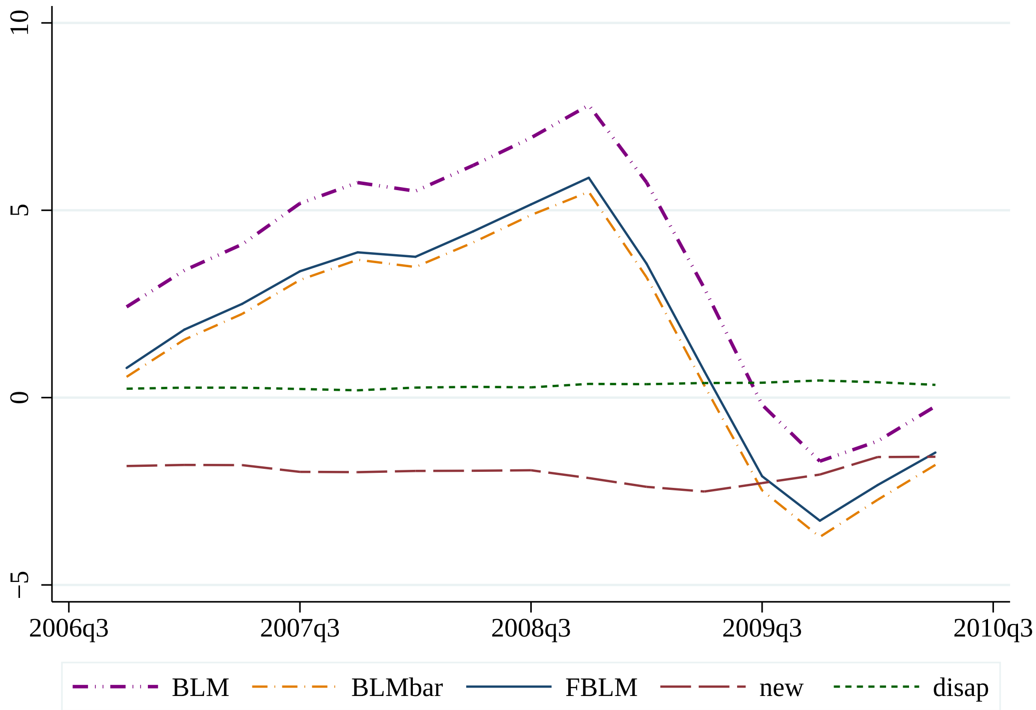
Table C1: Mean Product Turnover Adjustments and Bounds by Department

	Adj. New	Adj. Disapp.	LMbar – FLM	FBLM – BLMbar
Alcoholic Beverages	−0.3824	0.0332	0.3858	0.0336
Dairy	−1.4611	0.3869	1.4921	0.3797
Deli	−4.1236	1.4308	4.1879	1.3852
Dry Grocery	−1.9752	0.2197	2.0076	0.2235
Fresh Meat	−2.2178	1.0607	2.2682	1.0673
Fresh Produce	−2.0288	0.5090	2.0362	0.5073
Frozen Foods	−3.4847	0.4003	3.5433	0.3886
Packaged Meat	−1.8188	0.6936	1.8458	0.6886
All	*−1.9961*	*0.3169*	*2.0284*	*0.3153*

Notes: Based on data provided by The Nielsen Company (U.S.), LLC. Product group differences are weighted by comparison period expenditure share. BLM is the Backwards Lloyd-Moulton index over common varieties, BLMbar also includes the Feenstra adjustment for new varieties only, and FBLM uses the Feenstra adjustments for new and exiting varieties.

[26] In the style of Konüs (1924) and Diewert (2001), one can show P_{FBLM} and P_{FLM} bound a COLI evaluated at an intermediate taste level, though with retail scanner data, I found these bounds to also be wide for most departments.

Figure C1: Scanner Data BLM Indexes with Product Turnover (% change versus year ago)



Note: Based on data provided by The Nielsen Company (U.S.), LLC. Plots are averages of the four-quarter proportional changes implied by product group-level indexes, weighted by comparison period expenditure shares. BLM is the Backwards Lloyd-Moulton index over common varieties, BLMbar also includes the Feenstra adjustment for new varieties only, and FBLM uses the Feenstra adjustments for new and exiting varieties.

D Scale and Normalization of Tastes

This appendix discusses how the unconditional CES COLI depends on changes in the scale of tastes over time, as well as how alternative normalizations of tastes in the RW framework affect the index. The results I derive are similar to Kurtzon (2020).

For simplicity, suppose that σ is known. Re-arranging the CES expenditure share equation (Eq. 9) evaluated at $\boldsymbol{p}_t, \boldsymbol{\varphi}_t$, we see the φ_{it} are identified only up to a common scale factor.

$$\varphi_{it} = \left[C(\boldsymbol{p}_t, \boldsymbol{\varphi}_t)\right]^{-1} p_{it} s_{it}^{\frac{1}{\sigma-1}} \; i \in \mathcal{I}. \tag{29}$$

I consider geometric mean normalizations of the form[27]

$$\prod_{i\in\mathcal{I}} \varphi_{i0}^{w_i} = \prod_{i\in\mathcal{I}} \varphi_{i1}^{w_i}, \sum_{i\in\mathcal{I}} w_i = 1. \tag{30}$$

Taking geometric means of both sides of Eq. 29 and dividing Eq. 29 by the result yields Eq. 31 below.

$$\frac{\varphi_{it}}{\tilde{\varphi}_t} = \left(\frac{p_{it}}{\tilde{p}_t}\right)\left(\frac{s_{it}}{\tilde{s}_t}\right)^{\frac{1}{\sigma-1}} \equiv \ddot{\varphi}_{it},\, i \in \mathcal{I}, \tag{31}$$

where $\tilde{x}$ denotes a geometric mean across varieties i in set $\mathcal{I}$. Note we can plug in $\ddot{\varphi}_{it}$ (based on any normalization) into the conditional COLIs $\Phi(\boldsymbol{p}_0, \boldsymbol{p}_1; \boldsymbol{\varphi}_1)$ and $\Phi(\boldsymbol{p}_0, \boldsymbol{p}_1; \boldsymbol{\varphi}_0)$ and get exactly P_{BLM} and P_{LM}, respectively, since the scale factors cancel. In addition, it is straightforward to show that if we use the unweighted geometric mean as the normalization ($w_i = 1/N$) and plug $\ddot{\varphi}_{it}$ into the CES expenditure function Eq. 8, and then take the ratio $C(\boldsymbol{p}_1, \ddot{\boldsymbol{\varphi}}_1)/C(\boldsymbol{p}_0, \ddot{\boldsymbol{\varphi}}_0)$, we get Eq. 14 for P_{CCV} exactly.

While the conditional COLI estimates are invariant to the normalization, the unconditional estimate is not. Because the CES expenditure function, Eq. 8, is homogeneous of degree -1 in $\boldsymbol{\varphi}$, we have the following relationship:

$$\begin{aligned}\Phi_U(\boldsymbol{p}_0, \boldsymbol{p}_1; \boldsymbol{\varphi}_0, \boldsymbol{\varphi}_1) &= \frac{\tilde{\varphi}_0}{\tilde{\varphi}_1}\frac{\left[\sum_{i\in\mathcal{I}}\left(\frac{p_{i1}}{\ddot{\varphi}_{i1}}\right)^{1-\sigma}\right]^{\frac{1}{1-\sigma}}}{\left[\sum_{i\in\mathcal{I}}\left(\frac{p_{i0}}{\ddot{\varphi}_{i0}}\right)^{1-\sigma}\right]^{\frac{1}{1-\sigma}}} \\ &= \frac{\tilde{\varphi}_0}{\tilde{\varphi}_1}\Phi_U(\boldsymbol{p}_0, \boldsymbol{p}_1; \ddot{\boldsymbol{\varphi}}_0, \ddot{\boldsymbol{\varphi}}_1). \end{aligned} \tag{32}$$

The unconditional COLI based on normalized taste parameters therefore differs from the true unconditional COLI $\Phi_U(\boldsymbol{p}_0, \boldsymbol{p}_1; \boldsymbol{\varphi}_0, \boldsymbol{\varphi}_1)$ by the factor $\tilde{\varphi}_0/\tilde{\varphi}_1$, which is unidentified (RW assume it to be equal to one by setting $\tilde{\varphi}_0 = \tilde{\varphi}_1$). Therefore, the normalization imposed by

[27]Other types of means could be used, but as shown by Abe and Rao (2020), only geometric mean-type normalizations preserve invariance to units of measurement.

the P_{CCV} is not really "free", because it implicitly defines the estimand $\Phi_U(\boldsymbol{p}_0, \boldsymbol{p}_1; \ddot{\boldsymbol{\varphi}}_0, \ddot{\boldsymbol{\varphi}}_1)$. In reality, we do not even know if $\Phi_U(\boldsymbol{p}_0, \boldsymbol{p}_1; \boldsymbol{\varphi}_0, \boldsymbol{\varphi}_1)$ is greater than or less than one. In fact, Kurtzon (2020) shows that the taste-shock bias is measured to be identically zero if using the Sato-Vartia weights from Eq. 5 as w_i.

D.1 Alternative normalizations

It is clear from Eq. 32 that an unconditional COLI that uses normalized taste parameters is not invariant to the normalization chosen, of which there are infinitely many. Another aspect of the issue, relevant to Redding and Weinstein (2020), occurs when we assume the constant elasticity model (e.g., Eq. 8) for a commodity set $\mathcal{I}$, but estimate an unconditional sub-COLI over a smaller set. If normalizing using a geometric mean, then a natural question is whether this mean should be over all varieties or just the subset. RW's CES Universal Price Index (CUPI) consists of a common varieties index and product turnover adjustments from Feenstra (1994). RW (2018) calculates the common varieties index over the full set $\mathcal{I}$, while the published RW (2020) uses only products that had a lifespan of six years and were not within three quarters of birth or death in periods 0 or 1.

Restricting the set over which the P_{CCV} is calculated is made possible by the CES functional form. As in Feenstra (1994), for $\mathcal{I}^* \subseteq \mathcal{I}$, we can use Eq. 19 for $i \in \mathcal{I}$ and $i \in \mathcal{I}^*$ to write:

$$\Phi_U(\boldsymbol{p}_0, \boldsymbol{p}_1; \boldsymbol{\varphi}_0, \boldsymbol{\varphi}_1, \mathcal{I}) = \Phi_U(\boldsymbol{p}_0, \boldsymbol{p}_1; \boldsymbol{\varphi}_0, \boldsymbol{\varphi}_1, \mathcal{I}^*) \left(\frac{\sum_{i \in \mathcal{I}^*} p_{i0} q_{i0}}{\sum_{i \in \mathcal{I}} p_{i0} q_{i0}} \right)^{\frac{1}{1-\sigma}} \left(\frac{\sum_{i \in \mathcal{I}^*} p_{i1} q_{i1}}{\sum_{i \in \mathcal{I}} p_{i1} q_{i1}} \right)^{\frac{1}{\sigma-1}} \tag{33}$$

For the RW (2020) CUPI, $\Phi_U(\boldsymbol{p}_0, \boldsymbol{p}_1; \boldsymbol{\varphi}_0, \boldsymbol{\varphi}_1, \mathcal{I}^*)$ is estimated using Eq. 14 over $i \in \mathcal{I}^*$, and then the Feenstra-style product turnover adjustments absorb the second two factors on the right hand side of Eq. 33. Using the smaller common varieties set, the taste-shock bias estimate in RW (2020) is much lower in magnitude (around 0.4 percentage points per year), than that reported in RW (2018) (around 2-4 percentage points per year).

I attempt to replicate RW's index over a more restricted set of varieties in the results presented in Section 5. Table 9 and Figure 3 imply that using the smaller common goods set results in lower taste shock bias. One way in which restricting the common varieties set affects the index is that it implicitly changes the normalization, given by Assumption 4.2. To estimate $\Phi_U(\boldsymbol{p}_0, \boldsymbol{p}_1; \boldsymbol{\varphi}_0, \boldsymbol{\varphi}_1, \mathcal{I}^*)$, we could use this assumption written either for the full set $\mathcal{I}$ or the smaller set $\mathcal{I}^*$. We could also modify Assumption 4.2 to cover all of $\mathcal{I}$, but to assign weight $1/N^R$ to items $i \in \mathcal{I}^*$ and weight 0 to all other items, as in Eq. 30. This effectively assumes the constant scale for the restricted set, but is exact for $\Phi_U(\boldsymbol{p}_0, \boldsymbol{p}_1; \boldsymbol{\varphi}_0, \boldsymbol{\varphi}_1, \mathcal{I})$. Under this assumption, P_{CCV} follows Eq. 14 by taking means of price and share relatives over $\mathcal{I}^*$, but the expenditure shares are out of the entire $\mathcal{I}$.

To examine the issue of normalization and common goods definition, I estimate some additional versions of P_{CCV} and compare them against versions of P_{SV}. In Figure D1, SV-R and CCV-R (1) correspond to the indexes from Table 6 and Figure 1, which cover the restricted common goods only. The indexes SV-A and CCV-A (1) were presented in Table 3 and Figure 3 and use the entire set of matched UPCs. The normalizations CCV-R (1) and CCV-A (1) use is just Assumption 4.2 over taken over the appropriate set with weight equal to the inverse of the size of each set. Alternatively, CCV-R (2) $P_{CCV}(R, 2)$ uses the restricted set $\mathcal{I}^*$, but maintains $\prod_{i\in\mathcal{I}} \varphi_{i0}^{1/N} = \prod_{i\in\mathcal{I}} \varphi_{i1}^{1/N}$. This can be implemented in the following way. First, calculate $\ddot{\varphi}_{it}$ for $i \in \mathcal{I}^*$ using Eq. 31, but taking geometric means over all $i \in \mathcal{I}$. Then plug in the $\ddot{\varphi}_{it}$ into the equations for $C\left(\boldsymbol{p}_t, \ddot{\boldsymbol{\varphi}}_t; \mathcal{I}^*\right)$, $t = 0, 1$ (Eq. 18).[28]. Additionally, CCV-A (2) uses the entire set of matched UPCs, but changes the weights of the normalization in Assumption 4.2 so that the items not in $\mathcal{I}^*$ receive a weight of zero.

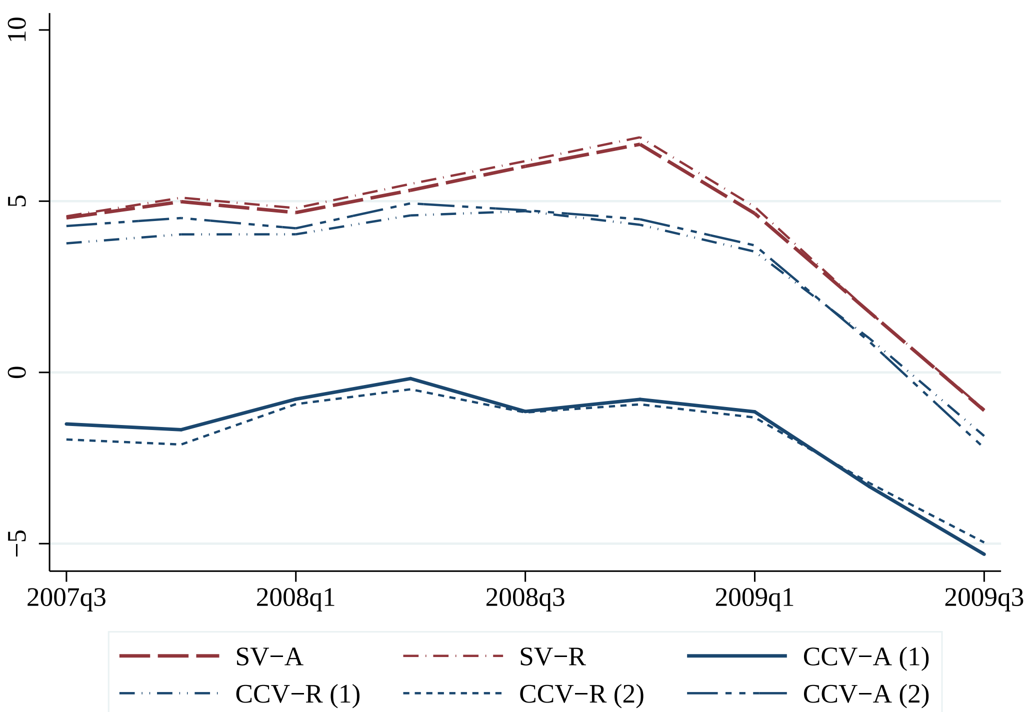
In Figure D1, the results indicate a large influence of the choice of normalization on the CCV index. Comparison of SV-A and SV-A, CCV-A and CCV-R (2), or CCV-R (1) and CCV-A (2) suggest that there is little difference in the average price changes across the two sets of varieties. However, the wide gap between, for example, CCV-R (2) and

[28] This is algebraically equivalent to estimating $\Phi_U(\boldsymbol{p}_0, \boldsymbol{p}_1; \boldsymbol{\varphi}_0, \boldsymbol{\varphi}_1, \mathcal{I})$ using Eq. 14 over $\mathcal{I}$, and then inverting Eq. 33 to get $\Phi_U(\boldsymbol{p}_0, \boldsymbol{p}_1; \boldsymbol{\varphi}_0, \boldsymbol{\varphi}_1, \mathcal{I}^*)$

CCV-(R) suggests that the choice of normalization is playing a large role in the SV to CCV comparison. Also interesting is the observation that the effect of the common goods rule is nearly replicated simply by changing the weights of the normalization

When proposing their restricted common goods set, RW argue that expenditure patterns near the beginning and end of a product's life may "make it appear as if consumer tastes for a common variety are changing rapidly when in fact they are not." This would seem to suggest that the CES model fits the data poorly for varieties in the set $\mathcal{I}^C \setminus \mathcal{I}^*$, and so CCV-A, CCV-A (2) and CCV-R (2) may be unreliable because they include (either directly or through the normalization) expenditure information for these varieties. Under the CES assumption for the set $\mathcal{I}$, however, these are all equally valid.

Figure D1: Scanner Data SV and CCV Index Comparison (% change versus year ago)



Note: Based on data provided by The Nielsen Company (U.S.), LLC. Plots are averages of the four-quarter proportional changes implied by product group-level indexes, weighted by comparison period expenditure shares. SV-A, CCV-A, and CCV-A (2) cover all common varieties. SV-R, CCV-R (1) and CCV-R (2) cover a restricted set of varieties with a lifespan of 2005Q3-2010Q2. CCV-A and CCV-R (2) assume that the taste parameter geomean across all common varieties is constant across periods. CCV-R (1) and CCV-A (2) assumes the taste parameter geomean across the restricted set of varieties is constant across periods.

E Homothetic Translog

This section shows how to derive conditional COLI for the homothetic translog model. To match RW's parameterization of tastes, the representative agent's minimized unit expenditure function is given in the following definition.

Assumption E.1 *Homothetic translog expenditure function*

$$\ln C(\boldsymbol{p};\boldsymbol{\varphi}) = \ln \alpha_0 + \sum_{i\in\mathcal{I}} \alpha_i \ln\left(\frac{p_i}{\varphi_i}\right) + \frac{1}{2}\sum_{i\in\mathcal{I}}\sum_{j\in\mathcal{I}} \gamma_{ij} \ln\left(\frac{p_i}{\varphi_i}\right)\ln\left(\frac{p_j}{\varphi_j}\right),\ t=0,1. \tag{34}$$

where the restriction $\gamma_{ij} = \gamma_{ji}$ *is made without loss of generality.*

After some algebra, we can rewrite Eq. 34 as

$$\ln C(\boldsymbol{p};\boldsymbol{\varphi}) = \ln\left[a_0(\boldsymbol{\varphi})\right] + \sum_{i\in\mathcal{I}} a_i(\boldsymbol{\varphi}) \ln p_i + \frac{1}{2}\sum_{i\in\mathcal{I}}\sum_{j\in\mathcal{I}} \gamma_{ij} \ln p_i \ln p_j, \tag{35}$$

where $\ln\left[a_0(\boldsymbol{\varphi})\right] = \ln \alpha_0 - \sum_{i\in\mathcal{I}} \alpha_i \ln \varphi_i + \frac{1}{2}\sum_{i\in\mathcal{I}}\sum_{j\in\mathcal{I}} \ln \varphi_i \ln \varphi_j$ and $a_i(\boldsymbol{\varphi}) = \alpha_i - \sum_{j\in\mathcal{I}} \gamma_{ij} \ln \varphi_j$. From Diewert (1976), homogeneity and symmetry then imply the restrictions $\sum_{i\in\mathcal{I}} a_i(\boldsymbol{\varphi}) = 1$ and $\sum_{j\in\mathcal{I}} \gamma_{ij} = 0$. These facts imply that conditional COLI using ratios of Eq. 34 are invariant to the scale of $\boldsymbol{\varphi}$, while unconditional COLI using Eq. 34 evaluated at $\boldsymbol{\varphi}_1$ and $\boldsymbol{\varphi}_1$ require a constant-scale assumption as in RW.

Eq. 35 reveals two additional salient points. First, the time variation in $\boldsymbol{\varphi}$ affects the parameter on the first order $\ln p$ terms only, and so the Caves, Christensen, and Diewert (1982) result on the Tornqvist index applies. Second, the $\ln\left[a_0(\boldsymbol{\varphi})\right]$ term captures the pure effect of tastes on unit expenditure, but cancels from the conditional index.

Under Assumption E.1, the $a_i(\boldsymbol{\varphi}_0)$ and $a_i(\boldsymbol{\varphi}_1)$ are recoverable up to estimates of the γ_{ij}.

To see this, the expenditure share equation for variety i is given by:

$$
\begin{aligned}
s_i(\boldsymbol{p};\boldsymbol{\varphi}) =& \alpha_i + \sum_{j\in\mathcal{I}} \gamma_{ij} \ln\left(\frac{p_j}{\varphi_j}\right) \\
=& a_i(\boldsymbol{\varphi}) + \sum_{j\in\mathcal{I}} \gamma_{ij} \ln p_j. \qquad (36)
\end{aligned}
$$

This implies the following counterfactual expenditure shares do not depend on the α_i.

$$
s_i(\boldsymbol{p_1};\boldsymbol{\varphi}_0) = s_{i0} + \sum_{j\in\mathcal{I}} \gamma_{ij} \ln\left(\frac{p_{j1}}{p_{j0}}\right), \qquad (37)
$$

$$
s_i(\boldsymbol{p_0};\boldsymbol{\varphi}_1) = s_{i1} - \sum_{j\in\mathcal{I}} \gamma_{ij} \ln\left(\frac{p_{j1}}{p_{j0}}\right). \qquad (38)
$$

Denote $s_{it} = s_i(\boldsymbol{p_t};\boldsymbol{\varphi}_t)$ the observed expenditure share, $t = 0, 1$, $s^*_{i1} = s_i(\boldsymbol{p_1};\boldsymbol{\varphi}_0)$ and $s^*_{i0} = s_i(\boldsymbol{p_0};\boldsymbol{\varphi}_1)$. Define the following Tornqvist style price indexes.

Definition E.1 *Tornqvist Price Index*

$$
\ln P_T = \sum_{i\in\mathcal{I}} \frac{1}{2}\left(s_{i0} + s_{i1}\right) \ln\left(\frac{p_{i1}}{p_{i0}}\right) \qquad (39)
$$

Definition E.2 *Reference taste Tornqvist index*

$$
\ln P_{T0} = \sum_{i\in\mathcal{I}} \frac{1}{2}\left(s_{i0} + s^*_{i1}\right) \ln\left(\frac{p_{i1}}{p_{i0}}\right) \qquad (40)
$$

Definition E.3 *Comparison period taste Tornqvist index*

$$
\ln P_{T1} = \sum_{i\in\mathcal{I}} \frac{1}{2}\left(s^*_{i0} + s_{i1}\right) \ln\left(\frac{p_{i1}}{p_{i0}}\right) \qquad (41)
$$

Proposition 1 *Under Assumption E.1,* $P_{T0} = \Phi(\boldsymbol{p}_0, \boldsymbol{p}_1; \boldsymbol{\varphi}_0)$ *and* $P_{T1} = \Phi(\boldsymbol{p}_0, \boldsymbol{p}_1; \boldsymbol{\varphi}_1)$.

The proof follows from substitution of Eq. 36 into Eq. 35.