Real-Time State-Space Method for Computing Filtered Estimates of Future Revisions of U.S. Quarterly GDP December 2007

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1. Introduction

U.S. gross domestic product (GDP) is not estimated directly from survey data like employment or retail sales, but is estimated by the Bureau of Economic Analysis (BEA) using observations on related variables. GDP measures economic activities in a quarter, and is, thus, released quarterly. Within five years, BEA releases initial, annual, and benchmark revisions of GDP for a given quarter. The initial estimates, termed advanced, preliminary, and final, are released one quarter after the quarter for which the estimates pertain. The next three annual estimates are released every July for quarters of the previous three calendar years. Finally, benchmark estimates are released about every five years, after new data from the Economic Census become available. Here, we limit the analysis to the first six, initial and annual, estimates of GDP, and consider the 6th estimate of GDP to be the final one, although, in practice, estimates are usually further revised due to changes in definitions or estimation methods, so that revisions are usually never truly final.

Quarterly GDP data may be indexed in two ways, by “historical” quarters to which they pertain or by “real-time” quarters in which they are released. In historical form, the data set is compact and generally has no missing values. In real-time form, the data set is expanded and sparse, with many missing values. The initial three releases are available delayed one quarter, and the three annual estimates are available every July, respectively, delayed 3-6, 7-10, and 11-14 quarters.

Due to delays in releases of final GDP, macroeconomic policymakers face the real-time difficulty of assessing the current state of the economy with incomplete statistical information, and, thus, are interested in obtaining timely “final GDP” data in order to design better macroeconomic policies. Since studies by Croushore and Stark (2001), there has been increasing interest in real-time estimation and forecasting of GDP and other important economic indicators (Chen and Zadrozny, 2002; Evans, 2005; Kishor and Koenig, 2005; Nunes, 2005; Jacobs and van Norden, 2007; and Zadrozny, 2007). Some of these studies focus on the use of preliminary and revised data in estimation and forecasting (Howrey, 1978, 1984; Conrad and Corrado, 1979; Mankiw and Shapiro, 1986; Sargent, 1989; Patterson, 1994, Fixler and Nalewaik, 2005). However, except in a parallel study (Zadrozny, 2007), data were used in both estimation and forecasting in historical form, although, in practice, GDP data become available in the real-time form.

This paper develops and illustrates with U.S. quarterly GDP a 2-step state-space method for estimating in any period the designated “final” GDP for that period. How final the GDP is depends on how many revisions the analysis includes. The state-space method proceeds in two steps. In the first step, the method estimates a vector autoregressive model of six initial and revised releases of GDP. In the second step, the method applies the missing-data Kalman filter (MDKF) to the estimated model to compute filtered estimates of final GDP, using information on all current and past observations of GDP. The second step estimation is a real-time filtering exercise, because in any quarter the final value of GDP for that quarter will be observed only in a future quarter. The accuracy of the filtered estimates, compared with eventually released final values, is measured by root-mean-squared error (RMSE). The results show that the forecasts of final GDP based on filtered estimates have lower RMSE than naïve forecasts based solely on initial releases.

The plan of the paper is as follows. Section 2 describes the data and the historical and real-time forms in which the data are indexed. Section 3 discusses the estimation strategy. Section 4 presents the estimation results. Section 5 concludes the paper.

2. Data Analysis

The data used in this study cover 112 quarters of U.S. real GDP from 1978:1 to 2005:4. There are 6 variables in the sample, 3 initial quarterly releases, termed advanced, preliminary, and final, and 3 annual revisions. The 3 initial estimates are released 1-3 months after a quarter ends, and, thus, are delayed by 1 quarter. The 3 annual revisions are released in succeeding Julys after a

[1]The paper reflects the authors’ views and does not necessarily reflect any views of the Bureau of Economic Analysis or the Bureau of Labor Statistics.

calendar year ends, and thus, are delayed, respectively, by 3-6, 7-10, and 11-14 quarters.

Quarterly GDP may be indexed in "historical" form according to the quarters to which the data pertain. In historical form, a data set is compact, has dimension Tx6, where T is the number of quarters in the sample, and has no missing values unless they are missing for reasons other than delays in releases. Table 1 shows initial quarterly releases and annual revisions of GDP in compact historical form. In Table 1, for i = 1, 2, 3, $y^{1}_{q,q+1}$ denotes advanced, preliminary, and final GDP estimates occurring in quarter q and observed in quarter q+1 and, for i = 4, 5, 6, $y^{i}_{q,t}$ denotes the 1st, 2nd, and 3rd annual GDP estimates occurring in quarter q and observed in quarter t > q. (Tables 1-2 and figures 1-2 are included at the end of the paper.)

GDP data may also be indexed in "real-time" form according to quarters in which data are released. In real-time from, the data set is expanded and sparse, with mostly missing observations due to delays. The real-time form has dimension Tx90, where 90 = 6·15 and 14 is the maximum number of quarters of delays in the releases. Table 2 shows initial releases and annual revisions of GDP in real-time form. In Table 2, for i = 1, 2, 3, $y^{i}_{t-1,t}$ denotes quarterly estimates occurring in quarter t-1 but observed in quarter t; for i = 4, 5, 6, $y^{i}_{q,t}$ denotes the 1st, 2nd, and 3rd annual revised estimates of GDP occurring in quarter q but observed in quarter t > q; and, NA means data are not available.

Prior to being used in estimation, data were normalized by subtracting sample means and dividing by standard deviations. Also, outliers defined as values more than 3 standard deviations from the mean were treated as missing values. A few outliers in each series are due to "jumps" induced by changes in "base years" of the GDP index. Figures 1-2 show, respectively, in historical form normalized percentage growth rates (quarter-to-quarter differences in logarithms), autocorrelations, and spectra of the 3 initial quarterly GDP values and the 3 annual revisions. The vertical line at 1998:4 separates the sample into two parts. Data up to 1998:4 are used for in-sample estimation, and data from 1998:4 to 2005:4 are used for computing and evaluating out-of-sample filtered estimates.

Figures 1-2 show that the GDP data in quarter-to-quarter percentage-growth form are stationary (with no trend, constant variances) and have no seasonality. Because only 1st-order autocorrelations appear significant, a VAR(1) model should satisfactorily account for contemporaneous, own-serial and cross-serial correlations in the data.

Table 3 gives means and standard deviations of initial estimates and annual revisions of GDP in percentage-growth-rate form. Compared with the initial quarterly estimates, the annual revisions have similar means but slightly smaller standard deviations.

Table 3: Sample Statistics of Initial Estimates and Annual Revisions of GDP in Quarterly Growth Rates, 1978:1 – 2005:4

Estimate of GDP	Mean	Stdv.
Advanced(y^1)	0.73E-02	0.72E-02
Preliminary(y^2)	0.72E-02	0.72E-02
Final(y^3)	0.73E-02	0.75E-02
1st Annual(y^4)	0.73E-02	0.67E-02
2nd Annual(y^5)	0.73E-02	0.68E-02
3rd Annual(y^6)	0.74E-02	0.64E-02

3. Estimation Strategy

The proposed state-space method for estimating final GDP proceeds in two steps. In the first step, a 6-variable VAR(1) model is estimated using maximum likelihood and the first 84 observations in historical form. The VAR(1) model is

(1) $y_t = Ay_{t-1} + \varepsilon_t$,

where $y_t = (y^1_t, \ldots, y^6_t)^T$, A is a 6x6 autoregressive coefficient matrix, and $\varepsilon_t = (\varepsilon^1_t, \ldots, \varepsilon^6_t) \sim NIID(0, \Sigma_\varepsilon)$.

We estimate 3 VAR models, an unrestricted VAR(1) model (UVAR1), a restricted VAR(1) model (RVAR1), and a restricted VAR(0) model (RVAR0). UVAR1 has 57 estimated parameters, 36 AR coefficients and 21 Cholesky factors of Σ_ε. RVAR1 model has 47 estimated parameters, 21 Cholesky factors of Σ_ε and 26 AR coefficients, after 10 insignificant AR parameters of UVAR1 are restricted to zero. The RVAR0 model has no lags and 21 parameters, Cholesky factors of Σ_ε. Thus, RVAR0 is simply $y_t = \varepsilon_t$, where $\varepsilon_t \sim NIID(0, \Sigma_\varepsilon)$.

We consider RVAR0 as a baseline model. Unlike UVAR1 and RVAR1, RVAR0 ignores any lags or persistence in the data generating process. One might think that RVAR0 provides no "leverage" for estimating yet-to-be-released future revisions of GDP, but this is not the case, because, by estimating Σ_ε, RVAR0 accounts for contemporaneous correlations among the estimates of GDP, which are the most significant correlations in the data. Thus, contemporaneous correlations combined with reporting delays provide sufficient leverage to estimate final GDP based on RVAR0. Nevertheless, Table 5 shows that RVAR0 yields the slightly larger RMSEs of the three considered models.

In step 2, the missing-data Kalman filter (MDKF) is applied to a state-space representation of each estimated model to obtain filtered estimates of final GDP. The representation used here allows for observation delays up to 14 quarters, which are accounted for by including in the state vector, x, the data vector, y, lagged up to 14 quarters. The state-space representation is

(2) $x_t = Fx_{t-1} + G\varepsilon_t,$

$$F = \begin{bmatrix} A_1 & 0 & 0 & . & . & . & 0 \\ I & 0 & 0 & . & . & . & 0 \\ 0 & I & 0 & . & . & . & . \\ . & . & I & . & . & . & . \\ . & . & . & . & . & . & . \\ . & . & . & . & . & . & 0 \\ 0 & . & . & . & 0 & I & 0 \end{bmatrix},$$

$$x_t = \begin{bmatrix} y_t \\ y_{t-1} \\ y_{t-2} \\ . \\ . \\ . \\ y_{t-14} \end{bmatrix}, \quad G = \begin{bmatrix} I \\ 0 \\ . \\ . \\ . \\ . \\ 0 \end{bmatrix},$$

where F, G, x, y and ε are, respectively, 90x90, 90x6, 90x1, 6x1, and 6x1, and 0 and I are 6x6. To simplify notation, we keep only indexes of quarters for which the data pertain, or $y_t = (y^1_t, y^2_t, \ldots, y^6_t)$, $x_t = (y^1_t, y^2_t, \ldots, y^6_t, \ldots, y^1_{t-14}, y^2_{t-14}, \ldots, y^6_{t-14})$, where superscript integers index individual variables.

In the second filtering step, x_t is both a data vector and a state vector. The data are expanded from the Tx6 historical form to the Tx90 real-time form. The first 84 out of 112 observation periods are used for estimating models and the remaining 28 observations are used for computing and evaluating filtered estimates. The filtered estimates are evaluated using root-mean-squared error (RMSE) measures of accuracy.

Numerous papers in the econometric literature have studied data revisions in terms of news, noise, and factors models (Howrey, 1978; Mankiw and Shapiro, 1986; Sargent, 1989; Patterson, 1994; Chen and Zadrozny, 2001; Fixler and Nalewaik, 2006; Jacobs and van Norden, 2006; Kishor and Koenig, 2007). Here, we consider nonstructural models with only possible zero restrictions on coefficients but not structural models with more general nonlinear restrictions on coefficients motivated by general economic or statistical reasoning. In principle, structural VARMA models could produce more accurate estimates of final GDP, although results in Table 5 indicate a narrow range for improving RMSE accuracy by using other restricted models, in particular, structural models.

We compute the filtered estimates as forecasts. For example, suppose t denotes quarter 1 in a year for which we want to compute the filtered estimate of final GDP. Let $y^6_{t|t} = E[y^6_t \mid I_t]$ denote the filtered estimate of y^6_t conditional of current information, I_t, which includes all current and past observations. We are interested in filtered estimates of final values of GDP, which are not released until July 3 years or 14 quarters later. Because $x_{90,t}$, element 90 of x_t, equals y^6_{t-14},

(3) $y^6_{t|t} = E(x_{90,t+14} \mid I_t).$

Thus, if t denotes the first quarter of a year, $y^6_{t|t}$ is computed as the 14-quarter-ahead forecast of element 90 of x_t. Similarly, true forecasts, $y^6_{s|t}$ for $s > t$, and smoothed estimates, $y^6_{s|t}$ for $t-14 \le s < t$, can be computed as forecasts of elements of x_t.

4. Estimation Results

Table 4 reports summary estimation statistics for the 3 VAR models. RVAR1's 47 parameters are estimated after setting to zero 10 previously estimated insignificant parameters of UVAR1. Resulting model fits, measured by R^2, are very close. Because RVAR0 has no lags, hence, no "explanatory" variables, its R^2s are zero. The last six rows report Q statistics and corresponding marginal significance levels, where Q_i, for i = 1, …, 6, reflect own-serial correlations of residuals of variable i at lags 1-10. Tables 4-5 that tabulate estimation results are included at the end of the paper.

We use information criteria to select a "best" model among the 3 VAR models. RVAR1 has the lowest Akaike information criteria (AIC) and the lowest Bayesian information criteria (BIC), which penalizes estimated parameters more strongly than AIC. Among the 3 models, RVAR0 has the highest AIC and BIC. Except for the preliminary GDP estimate, y^2, Q statistics for significant autocorrelations of residuals at lags 1-10, all have acceptable marginal significance levels or p values greater than 5%, indicating no need to extend UVAR1 and RVAR1 to higher-order lags.

The error between observed "final" GDP, y_t^6, and filtered final GDP k = 11, …, 14 quarters earlier, $\hat{y}_t^{6M}$, is

$$(4) \quad e^M_{y_t^6} = y_t^6 - \hat{y}_t^{6M} = y_t^6 - E(x_{6+6k,t+k} \,|\, I_t),$$

where M denotes the model being used in filtering. Filtering accuracy is measured by root mean-squared error,

$$(5) \quad RMSE^m_{y^6} = \sqrt{\textstyle\sum_{t=1}^{T} (e^M_{y_t^6})^2 / T}$$

$$= \sqrt{\textstyle\sum_{t=1}^{T} (y_t^6 - E(x_{6+6k,t+k} \,|\, I_t)^2 / T},$$

where T denotes the number of quarters used to compute RMSE.

To put the filtered estimates in perspective as forecasts, we consider Theil U statistics, which compare their accuracy with the most recently observed values of y_t^6 as "naïve" estimates of y_t^6. The Theil U statistic is the RMSE being considered divided by the RMSE of a naïve forecast,

$$\text{Theil U} = \frac{RMSE^M_{y^6}}{\frac{1}{T}\sqrt{\sum_{t=1}^{T}(y_t^6 - y_{t-\tau}^6)^2}},$$

where $t-\tau$ is the most recent observation of y^6 at time t. Theil U < 1 means that the filtered estimates of the considered model are more accurate than those of the naïve forecast.

Table 5 gives RMSEs of final GDP estimates based on the estimated VAR models. The top, middle, and bottom panels, respectively, contain RMSEs and Theil U statistics of UVAR1, RVAR1, and RVAR0. Column 1 gives the element of x_t for which the RMSE is computed. Because the 3rd annual GDP revisions are delayed 11-14 quarters, the filtered estimates are computed as 11–14 step-ahead forecasts, respectively, of elements 72, 78, 84 and 90 of x_t. Columns 2-3 give the number of steps ahead of the forecasts in quarters and the quarters for which the forecasts are made. Columns 4-5 give computed RMSEs and Theil U statistics of the filtered estimates for each quarter.

To interpret the results in Table 5, consider, as an example, row 4 of the middle panel. Recall that state equation (2) implies that a 14-quarter-ahead forecast made in quarter t of $x_{90,t}$ or $\hat{y}_{t-14}^6$ is the estimate made in quarter t of final GDP $\hat{y}_t^6$ for quarter t, using all available observations in quarter t. Thus, according to the computed RMSE, 14-quarter-ahead forecasts of $x_{90,t}$ made in quarter 1, but unobserved until 3 Julys later, are estimates of final GDP for those 1st quarters and have RMSE = .4826E-02 if RVAR1 is used for estimation. The Theil U statistic indicates that the filtered estimate for quarter 1 is 44.06% more accurate than the naïve forecast.

Table 5 shows average RMSE is slightly lower for RVAR1 than for UVAR1 or RVAR0. Average RMSE of RVAR0 is 7.34% higher than for UVAR1 and 7.54% higher than for RVAR1. Although, RVAR0 accounts for contemporaneous correlations among the GDP estimates, and for reporting delays, it does not account for significant lagged effects in the GDP data generating process. Similarly, the Theil Us show that UVAR1 and RVAR1 produce lower RMSEs than the naïve forecasts. The Theil Us of RVAR0 are larger than those of UVAR1 and RVAR1, but are less than one, except for the filtered estimates for quarter 3, indicating that filtered estimates based on RVAR0 model are more accurate than naïve estimates.

5. Conclusion

The paper has presented a 2-step state-space method for estimating final GDP based on an estimated VAR model, using data on 3 initial quarterly releases and 3 annual revisions of GDP, where the 6th estimate of GDP is considered "final." The filtered estimates of final GDP are based on three features in the data: historical contemporaneous correlations arising from the data generating process and captured by an estimated model; historical serial correlations arising from the data generating process and captured by an estimated model; and real-time observation delays built into the state-space representation of an estimated model.

The innovation of the method is that it enables one to compute filtered estimates of final GDP in "real time" as data become available. The results show that forecasts of final GDP based on filtered estimates, computed according to the proposed method, have lower RMSE than naïve forecasts of final GDP based solely on initial releases.

In the future, we plan to extend the analysis to include monthly variables correlated contemporaneously and at lags with initial, interim, and final GDP, in order to produce monthly estimates of quarterly GDP. Accordingly, the estimated model will operate at monthly intervals and filtered estimates of quarterly GDP, based on the estimated monthly model, will be produced at monthly intervals. Thus, the real-time analysis will address mixed monthly-quarterly observation frequencies as well as revisions and delays.

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Table 1: Quarterly Releases and Annual Revisions of GDP in Compact Historical Form

Quarter q	$y^1_{q,q+1}$	$y^2_{q,q+1}$	$y^3_{q,q+1}$	$y^4_{q,t}$	$y^5_{q,t}$	$y^6_{q,t}$
1	$y^1_{1,2}$	$y^2_{1,2}$	$y^3_{1,2}$	$y^4_{1,7}$	$y^5_{1,11}$	$y^6_{1,15}$
2	$y^1_{2,3}$	…	…	…	…	…
3	…	…	…	…	…	…
4	$y^1_{4,5}$	$y^2_{4,5}$	$y^3_{4,5}$	$y^4_{4,7}$	$y^5_{4,11}$	$y^6_{4,15}$
5	$y^1_{5,6}$	$y^2_{5,6}$	$y^3_{5,6}$	$y^4_{5,11}$	$y^5_{5,15}$	$y^6_{5,19}$
6	…	…	…	…	…	…
7	…	…	…	…	…	…
8	$y^1_{8,9}$	$y^2_{8,9}$	$y^3_{8,9}$	$y^4_{8,11}$	$y^5_{8,15}$	$y^6_{8,19}$
9	$y^1_{9,10}$	$y^2_{9,10}$	$y^3_{9,10}$	$y^4_{9,15}$	$y^5_{9,19}$	$y^6_{9,23}$
10	…	…	…	…	…	…
11	…	…	…	…	…	…
12	$y^1_{12,13}$	$y^2_{12,13}$	$y^3_{12,13}$	$y^4_{12,15}$	$y^5_{12,19}$	$y^6_{12,23}$

Table 2: Quarterly Releases and Annual Revisions of GDP in Expanded Real-Time Form

Quarter t	$y^1_{t-1,t}$	$y^2_{t-1,t}$	$y^3_{t-1,t}$	$y^4_{q,t}$				$y^5_{q,t}$				$y^6_{q,t}$			
1	NA	NA	NA	NA	NA	NA	NA	NA	NA	NA	NA	NA	NA	NA	NA
2	$y^1_{1,2}$	$y^2_{1,2}$	$y^3_{1,2}$	NA	…	…	…	…	…	…	…	…	…	…	NA
3	…	…	…	NA	…	…	…	…	…	…	…	…	…	…	NA
4	$y^1_{3,4}$	$y^2_{3,4}$	$y^3_{3,4}$	NA	…	…	…	…	…	…	…	…	…	…	NA
5	$y^1_{4,5}$	$y^2_{4,5}$	$y^3_{4,5}$	NA	…	…	…	…	…	…	…	…	…	…	NA
6	…	…	…	NA	…	…	…	…	…	…	…	…	…	…	NA
7	…	…	…	$y^4_{1,7}$	$y^4_{2,7}$	$y^4_{3,7}$	$y^4_{4,7}$	NA	…	…	…	…	…	…	NA
8	$y^1_{7,8}$	$y^2_{7,8}$	$y^3_{7,8}$	NA	…	…	…	…	…	…	…	…	…	…	NA
9	$y^1_{8,9}$	$y^2_{8,9}$	$y^3_{8,9}$	NA	…	…	…	…	…	…	…	…	…	…	NA
10	…	…	…	NA	…	…	…	…	…	…	…	…	…	…	NA
11	…	…	…	$y^4_{5,11}$	$y^4_{6,11}$	$y^4_{7,11}$	$y^4_{8,11}$	$y^5_{1,11}$	$y^5_{2,11}$	$y^5_{3,11}$	$y^5_{4,11}$	NA	…	…	NA
12	$y^1_{11,12}$	$y^2_{11,12}$	$y^3_{11,12}$	NA	…	…	…	…	…	…	…	…	…	…	NA
13	$y^1_{12,13}$	$y^2_{12,13}$	$y^3_{12,13}$	NA	…	…	…	…	…	…	…	…	…	…	NA
14	…	…	…	NA	…	…	…	…	…	…	…	…	…	…	NA
15	…	…	…	$y^4_{9,15}$	$y^4_{10,15}$	$y^4_{11,15}$	$y^4_{12,15}$	$y^5_{5,15}$	$y^5_{6,15}$	$y^5_{7,15}$	$y^5_{8,15}$	$y^6_{1,15}$	$y^6_{2,15}$	$y^6_{3,15}$	$y^6_{4,15}$
16	$y^1_{15,16}$	$y^2_{15,16}$	$y^3_{15,16}$	NA	…	…	…	…	…	…	…	…	…	…	NA

Figure 1: Historical Form Normalized Quarterly Growth Rates, Autocorrelations, and Spectra of 3 Quarterly Estimates of GDP, 1978:1 – 2005:4.

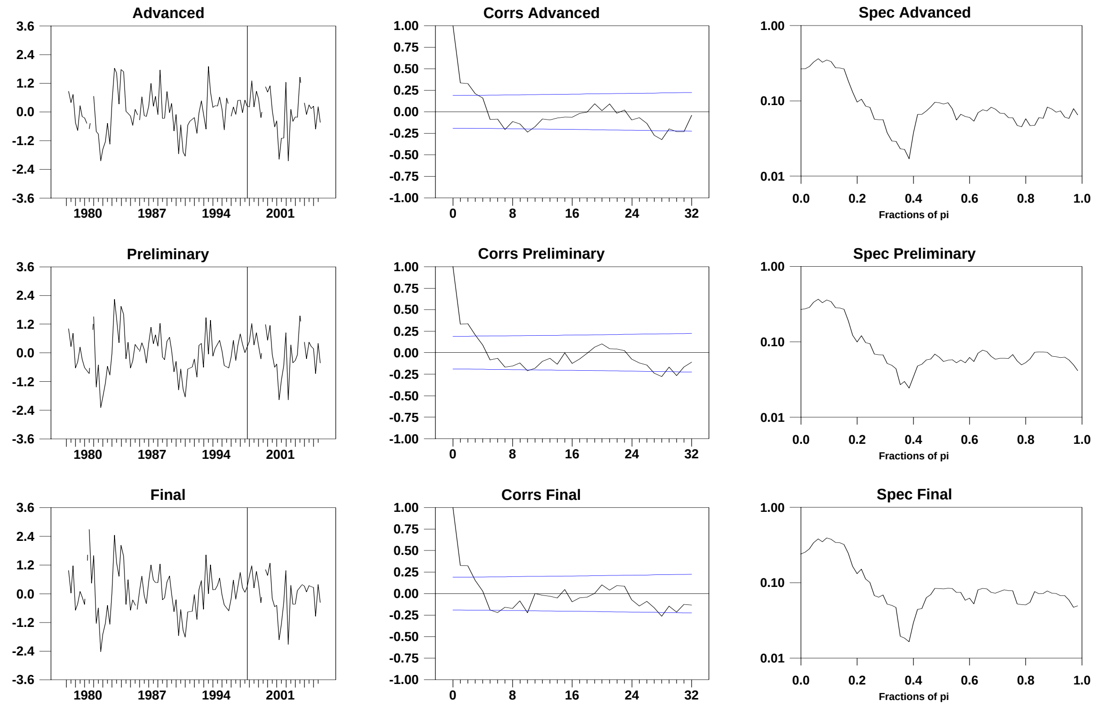

Figure 2: Historical Form Normalized Quarterly Growth Rates, Autocorrelations, and Spectra of 3 Annual Revisions of GDP, 1978:1 – 2005:4.

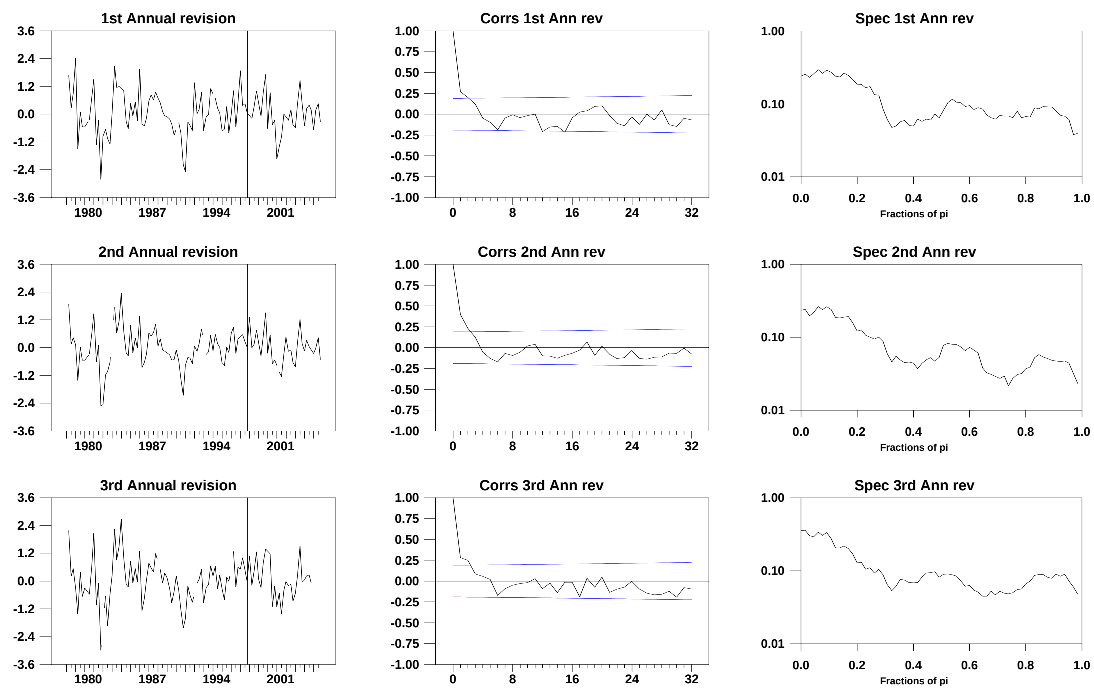

Table 4: Summary Model-Estimation Statistics of Unrestricted and Restricted VAR Models

Items	UVAR1	RVAR1	RVAR0
VAR order	1	1	0
$R^2(y^1)$	.3373	.3379	0
$R^2(y^2)$	.3037	.3082	0
$R^2(y^3)$	.2206	.2210	0
$R^2(y^4)$	.2590	.2539	0
$R^2(y^5)$	.3164	.3092	0
$R^2(y^6)$	.2710	.2696	0
No. est. params.	57	47	21
AIC	-239.3	-254.0	-188.6
BIC	-100.8	-139.8	-137.5
Q_1	14.00 (.5989)	13.35 (.6320)	46.94 (.0006)
Q_2	30.80 (.0143)	30.18 (.0171)	42.97 (.0003)
Q_3	7.790 (.9543)	8.983 (.9141)	5.489 (.9927)
Q_4	11.08 (.8048)	11.20 (.7968)	13.46 (.6385)
Q_5	17.24 (.3701)	17.03 (.3837)	27.94 (.0322)
Q_6	23.74 (.0953)	25.77 (.0574)	12.74 (.1274)

Table 5: RMSEs of Filtered Estimates of Final GDP Based on Estimated Unrestricted and Restricted VAR Models

RMSE of UVAR1 (57 parameters)				
Element of x_t	**Forecast steps ahead**	**Quarter**	**RMSE**	**Theil U**
72	11	4	.7802E-02	.8765
78	12	3	.7663E-02	.9444
84	13	2	.5633E-02	.7728
90	14	1	.4820E-02	.5588
Average	---	---	*.6480E-02*	*.7881*
Spread	---	---	*.2982E-02*	*.3856*
RMSE of RVAR1 (47 parameters)				
Element of x_t	**Forecast steps ahead**	**Occurs in**	**RMSE**	**Theil U**
72	11	4	.7690E-02	.8639
78	12	3	.7790E-02	.9600
84	13	2	.5583E-02	.7660
90	14	1	.4826E-02	.5594
Average	---	---	*.6472E-02*	*.7873*
Spread	---	---	*.2964E-02*	*.4006*
RMSE of RVAR0 (parameters = 21)				
Element of x_t	**Forecast steps ahead**	**Occurs in**	**RMSE**	**Theil U**
72	11	4	.8495E-02	.9543
78	12	3	.8562E-02	1.055
84	13	2	.5951E-01	.8166
90	14	1	.4954E-02	.5612
Average	---	---	*.6990E-02*	*.8467*
Spread	---	---	*.3608E-02*	*.4938*