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Abstract

We re-estimate historical U.S. Producer Price Indexes (PPI) using the geometric Young formula at the elementary level. The geometric Young has better axiomatic properties than the modified Laspeyres, and may better approximate a feasible economic target. We find in most cases, indexes that use the geometric Young escalate between 0.1 and 0.3 percentage points less each year than those that use the modified Laspeyres. However, for wholesale and retail trade, as well as some other services, the differences are much larger. As a result, using the geometric Young at the elementary level lowers the PPI for Final Demand by 0.55 percentage points per year during the study period, a magnitude larger than what has been previously found for the U.S. Consumer Price Index.

Keywords: Inflation; Aggregation; Geometric Young index; Jevons index
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1 Introduction

This paper presents re-estimates of historical U.S. Producer Price Indexes (PPI) using the geometric Young formula at the elementary level. The U.S. Bureau of Labor Statistics (BLS) currently uses a modified Laspeyres formula (Bureau of Labor Statistics 2015), but statistical agencies in Italy, Chile, the Netherlands, among others, use the geometric Young (or something similar) for elementary PPIs (OECD 2011). In addition, since 1999, the BLS has used the geometric Young for most elementary indexes that comprise the U.S. Consumer Price Index (CPI) (Bureau of Labor Statistics 2019). While a number of preceding studies, e.g. Boskin, et al. (1996), analyzed differences between formulas for consumer prices, relatively little empirical research of this nature examines producer prices. PPIs are used widely in adjusting procurement contracts and as deflators for other economic time series, so formula choice is of broad significance. Unlike with consumer prices, use of the geometric Young may be less motivated by the issue of substitution bias (Waehrer 2000). However, compared to the modified Laspeyres, the geometric Young has better axiomatic properties (IMF 2004) and lower formula bias (McClelland 1996, Reinsdorf 1998), which are also relevant factors for PPIs. We discuss these considerations further in Section 2.

We use both the modified Laspeyres and geometric Young formulas to calculate approximately 7,000 elementary indexes per month covering January 2008 to December 2017. We then aggregate these into versions of 1,016 six-digit commodity (product-based) indexes, 759 six-digit industry indexes, and the headline PPIs for Final Demand and Intermediate Demand (FD-ID). For a more detailed description of PPI classifications, see Bureau of Labor Statistics (2015). In most cases, six-digit indexes using the geometric Young formula at the elementary level are between 0.1 and 0.3 percentage points per year lower than those that use the modified Laspeyres. However, for wholesale and retail trade, as well as some other services, the differences are much larger. As a result, re-estimating the PPI for Final Demand using the geometric Young at the elementary level lowers the index by 0.55 percentage points per year, a larger magnitude than what has been found previously for the U.S. CPI (Boskin, et al. 1996).

Formula choice has a greater impact on U.S. PPIs in part because the BLS uses gross margins (selling price minus acquisition price) to measure the prices received by firms that resell items, such as wholesalers and retailers. Excluding these categories, the geometric Young lowers the PPI for Final Demand by 0.24 percentage points per year, comparable to Boskin, et al.'s finding of 0.25 for the U.S. CPI. Changes in margins tend to be more highly dispersed, driving greater differences between index formulas. In addition, the geometric Young is sensitive to near-zero margins. However, our results change very little when we impose bounds on the most extreme price changes prior to index calculation.

2 Methods

A price index aggregates price changes for many items into a single summary measure. Producer price aggregation typically occurs in two stages. First, price changes within a narrowly defined grouping are combined to form an elementary index. Then, these elementary indexes are aggregated into broader measures like the headline PPI for Final Demand.

The PPI is currently based on a modified Laspeyres formula (Bureau of Labor Statistics 2015). More precisely, the target is known as the Lowe index. Let q_i^t and p_i^t denote quantity and price, respectively, for an item i in some period t. The Lowe index is then

$$I_{Lo}^t = \left(\frac{\sum_{i=1}^N q_i^b p_i^t}{\sum_{i=1}^N q_i^b p_i^0}\right) \times 100. \quad (1)$$

The index measures the change in expenditure on a fixed basket $\{q_1^b, \dots, q_N^b\}$ from the reference period 0 to the comparison period t. Period b is the base period from which quantity information is drawn. If the base and reference periods happen to be the same ($b = 0$), then the index coincides with the well-known Laspeyres formula, which we denote I_{Lasp}^t. If the base and comparison periods are the same ($b = t$), then it coincides with the Paasche formula, I_{Paas}^t. However, in practice, it is usually true that period b precedes period 0, particularly at the elementary level, where weighting information is not sampled as quickly or frequently as prices. Implementation of the Lowe index uses its expenditure share form, given by

$$I_{Lo}^t = \left(\sum_{i=1}^N s_i^{0b} \frac{p_i^t}{p_i^0}\right) \times 100, \quad (2)$$

where the $s_i^{0b} = p_i^0 q_i^b / \sum_{j=1}^N p_j^0 q_j^b$ are hybrid expenditure weights using period 0 prices and base period quantities, and p_i^t / p_i^0 is sometimes referred to as the long-term price relative.

At the upper level of aggregation, the PPI uses something close to Eq. 2. At the elementary level, however, shipments data are usually only available in dollar values (the products $p_i^b q_i^b$) rather than quantities (the q_i^b by themselves). As a consequence, the "modified Laspeyres" formula actually implemented is closer to the Young index, written

$$I_Y^t = \left(\sum_{i=1}^{N} s_i^b \frac{p_i^t}{p_i^0}\right) \times 100, \quad (3)$$

where $s_i^b = p_i^b q_i^b / \sum_{j=1}^{N} p_j^b q_j^b$ are the actual expenditure weights from the base period. If $b = 0$, then the Young, Laspeyres, and Lowe are all equivalent. The axiomatic shortcomings of the Young index have been documented in IMF (2004), and include failure of the time-reversal and transitivity tests, discussed in the next subsection. The geometric Young index, given in Eq. 4, is seen as a superior alternative because it satisfies these tests.

$$I_{GY}^t = \left(\prod_{i=1}^{N} \left(\frac{p_i^t}{p_i^0}\right)^{s_i^b}\right) \times 100 \quad (4)$$

This formula has also been called the geometric Lowe, weighted Jevons, and Cobb-Douglas price index. It combines the same price and expenditure information, but using a geometric mean instead of an arithmetic mean. The BLS uses a version of this formula for the majority of elementary CPIs, as do several other countries for their elementary level PPIs. To avoid confusion, we use "modified Laspeyres" to refer to the arithmetic form of the Young index given in Eq. 3, unless otherwise specified.

A corollary to Jensen's inequality implies the geometric mean will be less than or equal to the arithmetic mean when based on the same weights, and so we should generally expect index levels to be lower when using the geometric Young. The quantitative significance, however, is an empirical matter.

2.1 Axiomatic and Statistical Considerations

This subsection outlines how the geometric Young is superior to the modified Laspeyres from the axiomatic perspective. As way of background, several different sets of overlapping tests for index numbers comprise what is known as the axiomatic approaches (IMF, 2004). Historically, this approach has favored indexes with some sort of geometric mean, such as the Fisher and Tornqvist, both of which are infeasible for the BLS due to the detail and frequency of weight information required. IMF (2004) lists a set of 12 tests used to assess modified Laspeyres indexes whose weights correspond neither to the reference nor the current period.

The key axiomatic shortcoming of the modified Laspeyres is that, in general, it fails the time reversal test. Formally, a generic index $I(0, t)$ satisfies the time reversal test if the following holds:

$$I(0,t) = \frac{1}{I(t,0)} \quad or \quad I(0,t)I(t,0) = 1 \tag{5}$$

The idea of this test is that the index measurement should, in some sense, be independent of which period is regarded as the reference and which is regarded as the comparison. For example, if the index says the general price level doubled from period 0 to period *t*, then it should also say the price level fell by half if we instead treat *t* as the base period and 0 as the current. The modified Laspeyres fails because $I_Y(0,t) \geq 1/I_Y(t,0)$, with equality only in the unlikely case that all prices change by the same proportion. In other words, the modified Laspeyres leads to a higher measurement than if reference and comparison periods were reversed, which can be interpreted as an upward bias in the index. On the other hand, it is easily verified that the Geometric Young satisfies the time-reversal test.

The modified Laspeyres also fails the transitivity (or circularity) test, which says that when chained together, indexes over adjacent intervals should equal their direct counterpart.[‡] For time periods $0 < s < t$, it should be true that:

$$I(0,t) = I(0,s) \times I(s,t). \qquad (6)$$

The degree of failure, which is related to the idea of chain drift, depends on the particular patterns of the data. However, unlike time-reversal, intransitivity does not necessarily imply a bias in any particular direction, rather a sort of practical user-limitation. Ratios of modified Laspeyres indexes might not yield the intended comparison, as failure of Eq. 6 implies $I_Y(0,t)/I_Y(0,s) \neq I_Y(s,t)$. The Geometric Young satisfies the transitivity test, as will any geometric mean of price relatives with time-constant weights summing to one.

Reinsdorf (1998) describes another scenario in which the modified Laspeyres index systematically exceeds the Laspeyres indexes, which he refers to as "formula bias." In a statistical model of mean-reversion, meaning prices fluctuate around a common mean, he shows how the Young form of the modified Laspeyres index will exceed both the Lowe and Laspeyres in expectation. The intuition for his argument is that prices that are temporarily low in the reference period will receive excess weight in the Young formula. In a mean-reverting model, these are the prices expected to rise the most from 0 to t, leading to an inflation estimate that is biased upwards. He gives indirect evidence that this bias was empirically relevant for elementary indexes used in the CPI before formula changes that took place in the 1990s. He also discusses a geometric mean index as a solution. Assuming the elementary price

[‡] Note the transitivity test is not part of either the First or the Second Axiomatic Approaches described in IMF (2004), but it is included in the 12 tests used to compare the Lowe and arithmetic Young indexes.

data in the PPI follow a similar pattern, this argument supports use of the Geometric Young over the modified Laspeyres.

2.2 Economic Considerations

Despite the geometric Young having superior axiomatic properties to the modified Laspeyres, the economic approach to index numbers has led to questions about its appropriateness for output PPIs. The economic approach compares a formula against a theoretical target derived from a model of an optimizing agent (IMF, 2004). Fisher and Shell (1972) and Archibald (1977) propose a class of theoretical output price aggregators called Fixed Input Output Price Indexes (FIOPIs). A FIOPI measures the firm's hypothetical change in revenue between 0 and t if it had to keep inputs and technology fixed at some level. In this section, we argue that opposition to a geometric mean-type index on economic grounds (e.g., from Waehrer, 2000) is too narrowly focused on one index (the geometric Laspeyres, which is Eq. 4 with b=0) and one theoretical target (the reference-period FIOPI). We offer a plausible scenario in which a geometric Young index may better approximate a FIOPI based on average technology and inputs.

Let $\mathcal{S}$ denote the production possibilities set associated with a given level of technology and inputs. Suppose in periods $\tau = 0, t$, a representative producer facing prices $\boldsymbol{p}^{\tau} = (p_1^{\tau}, \dots, p_N^{\tau})$ chooses quantities $\boldsymbol{q} = (q_1, \dots, q_N)$ from $\mathcal{S}$ to maximize revenue, which maximizes profit since inputs are fixed. Define the maximized revenue as

$$R(\boldsymbol{p}^{\tau}, \mathcal{S}) = \max_{\boldsymbol{q} \in \mathcal{S}} \sum_{i=1}^{N} p_i^{\tau} q_i \,. \tag{7}$$

The class of FIOPIs is then given by

$$FIOPI(\mathcal{S}) = \frac{R(\boldsymbol{p}^t,\ \mathcal{S})}{R(\boldsymbol{p}^0,\ \mathcal{S})} \tag{8}$$

for a given $\mathcal{S}$. In general, FIOPIs are infeasible to implement in official statistics. Estimating a hypothetical revenue change under fixed inputs and technology would require specifying and estimating a model for every industry or commodity group—a challenge because shipments data, even if collected frequently enough, do not reflect producers reacting to price change in a vacuum.

Economic theory provides some observable bounds, however. Let $\mathcal{S}^0$ be the production possibilities set from the reference period, so that under the assumption of profit maximization, $R(\boldsymbol{p}^0,\ \mathcal{S}^0)$ equals the (in theory) observable revenue level $\sum_{i=1}^{N} p_i^0 q_i^0$. From Archibald (1977),

$$FIOPI(\mathcal{S}^0) = \frac{R(\boldsymbol{p}^t,\ \mathcal{S}^0)}{R(\boldsymbol{p}^0,\ \mathcal{S}^0)} \geq \frac{\sum_{i=1}^{N} p_i^t q_i^0}{\sum_{i=1}^{N} p_i^0 q_i^0} = I_{Lasp}^t. \tag{9}$$

The result follows because under profit maximization, $R(\boldsymbol{p}^t,\ \mathcal{S}^0) \geq \sum_{i=1}^{N} p_i^t q_i^0$. Similarly, I_{Paas}^t is an upper bound for $FIOPI(\mathcal{S}^t)$, where $\mathcal{S}^t$ is the production possibilities set from the comparison period. The intuition for these results is that all else equal, the optimizing firm would have an incentive to shift production towards items with higher relative prices. The geometric Laspeyres index, is necessarily less than or equal to I_{Lasp}^t. Waehrer (2000), therefore, opposes the geometric Laspeyres index for producer prices because it has greater bias for $FIOPI(\mathcal{S}^0)$ then the arithmetic Laspeyres.

However, this result for the geometric Laspeyres is not applicable to analysis of the geometric Young, which does not (to our knowledge) have a known relationship to potential targets like $FIOPI(\mathcal{S}^0)$ or even $FIOPI(\mathcal{S}^b)$. More broadly, the one-way bounds concerning

I^t_{Lasp} and I^t_{Paas} may be of limited empirical relevance because they relate to different FIOPIs. In particular, if $I^t_{Lasp} \geq I^t_{Paas}$, then we have

$$FIOPI(\mathcal{S}^0) \geq I^t_{Lasp} \geq I^t_{Paas} \geq FIOPI(\mathcal{S}^t). \tag{10}$$

While we lack data to evaluate the inequality $I^t_{Lasp} \geq I^t_{Paas}$ at the elementary level (since both indexes are infeasible), Weinhagen (2020) found $I^t_{Lasp} \geq I^t_{Paas}$ holds for broader industry and commodity aggregates. This inequality reflects negative correlation between market prices and quantities, implying that even if firms have a *ceteris paribus* incentive to substitute production towards higher relative output prices, this is outweighed in equilibrium by other factors like consumer demand or technology shocks. If this is the case, then targeting either $FIOPI(\mathcal{S}^0)$ or $FIOPI(\mathcal{S}^t)$ might be arbitrary, seeing how the gap between them is at least as large as $I^t_{Lasp} - I^t_{Paas}$.

Diewert (1983) offers an alternative FIOPI which, unlike $FIOPI(\mathcal{S}^0)$ or $FIOPI(\mathcal{S}^t)$, has two-way observable bounds. Let $\mathcal{S}^\alpha = \alpha\mathcal{S}^0 + (1-\alpha)\mathcal{S}^t$, denote a weighted average of the reference and comparison period production possibilities sets. Diewert showed there exists an α between zero and one such that either $I^t_{Lasp} \leq FIOPI(\mathcal{S}^\alpha) \leq I^t_{Paas}$ or $I^t_{Paas} \leq FIOPI(\mathcal{S}^\alpha) \leq I^t_{Lasp}$. If the difference between the I^t_{Lasp} and I^t_{Paas} is not too great, a symmetric average like the Fisher index, $I^t_{Fisher} = \sqrt{I^t_{Lasp} I^t_{paas}}$, is a good approximation to the "average" $FIOPI(\mathcal{S}^\alpha)$. The Tornqvist index has similar properties (Caves, et. al., 1982).

As stated in the previous subsection, neither the Tornqvist nor the Fisher are feasible for elementary PPI. Nevertheless, we describe a plausible scenario under which the geometric Young may be preferred to the modified Laspeyres from the standpoint of targeting an average

FIOPI. IMF (2004, Ch. 15, Sec. D.3) describes conditions, reasonable for elementary items, under which I_Y^t exceeds I_{Lasp}^t. The first is that the base period precedes the reference period, as with the U.S. PPI. The second is that prices are trending either up or down over the long-term, which is generally true for U.S. PPI data. Next is that changes in market quantities primarily reflect purchaser substitutions away from items with higher relative prices. This is true if $I_{Lasp}^t \geq I_{Paas}^t$, for which we interpret Weinhagen (2020) as indirect evidence. The last condition is that these substitutions are elastic, meaning revenues and prices move in opposite directions. Elastic substitution patterns are likely if the elementary product grouping contains highly similar varieties, as is the case with the retail sales data studied by Martin (2020).

It is always true that $I_Y^t \geq I_{GY}^t$, and under the scenario described above, we have that $I_Y^t \geq I_{Lasp}^t \geq I_{Fisher}^t$. This means that switching from the modified Laspeyres to the geometric Young moves the index in the direction of the Fisher index, which approximates $FIOPI(\mathcal{S}^\alpha)$. Of course, this does not guarantee that the geometric Young will have lower bias for $FIOPI(\mathcal{S}^\alpha)$, and the conditions in the preceding paragraph may not be appropriate for all sectors. Nevertheless, it is plausible that the geometric Young better approximates a more feasible economic target than the modified Laspeyres.

3 Application

3.1 Data

The previous section enumerated reasons for which the geometric Young (Eq. 4) should be preferred to the modified Laspeyres (Eq. 3) from the standpoint of price index theory. To better understand the practical implications of formula choice, we use both formulas with the PPI microdata to calculate approximately 7,000 elementary indexes per month covering January 2008 to December 2017. Roughly half of these measure output prices for industries, which are organized according to the North American Industry Classification System (NAICS). The other half measure prices for commodities (regardless of producing industry) according to an internal BLS classification system.

We then aggregate the elementary indexes to form versions of 1,016 six-digit commodity indexes, 759 six-digit industry indexes, and the headline PPIs for Final Demand and Intermediate Demand (FD-ID). Because the focus is on differences in elementary calculation, all indexes use same the Lowe formula (Eq. 2) at the upper levels. Furthermore, we recalculate indexes that use the modified Laspeyres formula at the elementary level, rather than comparing to the published PPIs, in order to better hold constant other components of methodology such as imputation and item structure changes which are harder to replicate in a research environment. In 98.5 percent of observations, monthly percent changes of the re-estimated six-digit commodity indexes fall within 0.1 percent of the actual indexes from production.

3.2 Results

As stated, we combine each set of elementary indexes into indexes covering 759 six-digit NAICS industries and 1,016 six-digit commodity groups. The average annual change across the six-digit commodity indexes calculated using the modified Laspeyres is 1.52 percent, versus 1.25 percent for the geometric Young, a difference of 0.27 percentage points. Across industries, the modified Laspeyres indexes average 1.70 percent, while the geometric Young indexes average 0.36 points lower at 1.34 percent. There is considerable heterogeneity across commodities and industries. Figure 1 plots the frequencies of annual percentage point differences for the six-digit commodity indexes. About two thirds of commodities show differences in the 0.0 to 0.3 percentage point range. Frequencies generally decline over higher values, but the right tail is long, with 79 commodities having differences exceeding 0.9 percentage points. As expected, the modified Laspeyres implies higher inflation than the geometric Young for about 95 percent of commodities.[§] Note that a geometric mean will generally result in lower index levels (i.e., reference period to comparison period measurements), but the comparison may not always hold for short-term percent changes or when the considered timeframe spans item rotations or weight updates.

Table 1 presents the average annual percent changes for seven broad commodity categories. In all but one category (Wholesale and Retail Trade), formulas give average percent changes of the same sign, and the average differences mainly fall in the 0.2 to 0.4 percentage point range. Notable exceptions include Construction, where the average difference is only 0.05 percentage points, and Wholesale and Retail Trade, where the difference (1.35 percentage points) is more than three times that of any other category. The formulas disagree in sign for only about 3.3 percent of six-digit commodities overall, but within Trade, they disagree in 32 percent of cases. Similarly, Table 2 gives the average annual percent changes for the six-digit industry indexes within broad NAICS categories. As with the commodities, most differences average well under one percentage point per year with the exception of Wholesale Trade,

[§] The distribution across six-digit industries is very similar, so we omit the corresponding histogram. Out of 759 industries, 520 have index differences in the 0.0 to 0.3 percentage point range, and 64 have differences exceeding 0.9 percentage points. The modified Laspeyres implies higher inflation than the Geometric Young for about 97 percent of industries.

Retail Trade, and Finance and Insurance, where the average differences are 1.14, 1.71, and 0.72 percentage points per year, respectively.

As a general principle, greater dispersion of the underlying elements (in this case, long-term price relatives) is associated with a greater difference between the arithmetic and geometric mean. We should then expect to see greater dispersion in industries like Trade. Because of periodic discontinuations, we can only recover the long-term relatives for items that are observed during the entire period between sample rotations, a group which we label "survivors". To check representativeness, we construct sets of industry indexes using only this subsample and present their average differences in column 4 of Table 3. The full sample differences from Table 2 have been copied to column 3 for comparison. Using the survivors only, the average differences are slightly greater in magnitude (0.42 versus 0.35 percentage points per year), but qualitatively similar to those based on the full-sample. Column 5 shows the average coefficients of variation within each NAICS category. Indeed, within the Trade, Financial Services, and Insurance industries, the long-term price relatives have coefficients of variation of 0.36 on average, versus 0.14 for all other industries.

BLS views firms that resell items as providers of services rather than goods. As such, the prices used for Trade are primarily gross margins (selling price minus acquisition price). Gross margins for retailers, for example, reflect the value added by the establishment for services such as marketing, storing, displaying goods, and making the goods easily available for customers to purchase. Several indexes within financial services also use measures that are similar to margins, like bid-ask spreads. Margin prices tend to be more volatile than selling prices alone. The BLS excludes zero or negative margins from calculation. Geometric mean indexes are still sensitive to margins that are close to zero, which can cause the long-term and month-to-month price relatives to be very small or very large (IMF 2004).

To assess potential sensitivity, we calculate the commodity indexes after imposing bounds of 0.05 and 20 on the monthly relatives, which matches the BLS procedure for the CPI. For example, if a relative is less than 0.05, we use the value 0.05 in its place. The results change very little. For the Trade category, the average percent changes for the modified Laspeyres and geometric Young increase by 0.025 and 0.04 percentage points, respectively, decreasing the gap between them by only 0.015 percentage points. Similar results hold for tighter bounds of [0.25, 4], which decrease the gap by an additional 0.026 percentage points. There are still influential outliers inside these bounds, but it does not appear that the most extreme price relatives are driving the formula differences.

Aggregation of the commodity indexes into the headline PPIs shows the importance of formula choice. Table 4 presents average annual percent changes for the FD-ID indexes over 2010-2017. We also calculate the indexes with and without Trade and Finance, which include margin prices to varying degrees. The Final Demand index escalates 0.55 percentage points per year less when the elementary indexes use the geometric Young formula. This magnitude is larger, but near the range found by similar studies of CPI elementary indexes, such as Boskin, et al. (1996), which found an all-items index difference of 0.25 percentage points, and Reinsdorf and Moulton (1996), which found a difference of 0.47 percentage points. In the case of the PPI for Final Demand, much of the difference is due to the Trade and Finance sectors, which collectively show 1.53 percentage points lower inflation using the geometric Young. Excluding these, the difference between Final Demand indexes is only 0.24 percentage points per year.

Figures 2 and 3 illustrate the role these industries play in driving the relative evolution of the alternative indexes. The Intermediate Demand indexes follow a similar pattern. The Processed and Unprocessed Goods indexes differ by 0.25 and 0.15 percentage points per year, respectively. As with Final Demand, the indexes covering Services for Intermediate Demand differ to a greater degree (0.54 percentage points per year), though this gap narrows considerably (to 0.17 percentage points per year) when excluding Trade and Finance.

4 Conclusion

The geometric Young formula has superior axiomatic properties to the modified Laspeyres, and it may better approximate a FIOPI based on an average production possibilities set. Our application to U.S. PPI data shows that these theoretical differences have economically significant consequences for elementary index aggregation. Using the geometric Young, for example, would lower the PPI for Final Demand by 0.55 percentage points per year. The effect on most industry and commodity PPIs is smaller—between 0.1 and 0.3 percentage points. For services like wholesale and retail trade, however, higher dispersion in margin prices leads to differences often exceeding one percentage point. Our main findings are little changed when bounding the price relatives, implying the formula differences are not primarily driven by outliers, e.g., values close to zero. The issue of margin prices is unique to PPIs and helps explain why we find greater differences between formulas than earlier studies found using consumer prices.

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Tables

Table 1: Commodity Averages by Category, 2008-17 (Annual Percent Change)

Category	Mod. Lasp.	Geo. Young	Lasp. – Geo.
Food	1.66	1.45	0.21
Energy	-0.02	-0.40	0.38
Goods Less Food & Energy	1.68	1.50	0.19
Wholesale and Retail Trade	1.09	-0.26	1.35
Transportation	2.04	1.64	0.40
Services Less Trade & Transp.	1.08	0.64	0.44
Construction	1.60	1.55	0.05

Note: Rows are averages of six-digit commodity indexes within specified category. Mod. Lasp. and Geo. Young refer to formulas used for elementary aggregation. Upper-level aggregation uses Lowe formula in all cases.

Table 2: Industry Averages by NAICS Category, 2008-17 (Annual Percent Change)

NAICS	Description	Mod. Lasp.	Geo. Young	Lasp. – Geo.
11	Ag., Forestry, Fishing and Hunting	1.63	1.47	0.16
21	Mining, Quarrying, and Oil and Gas	2.92	2.46	0.47
22	Utilities	1.17	0.64	0.53
23	Construction	2.10	1.86	0.23
31-33	Manufacturing	1.78	1.55	0.23
42	Wholesale Trade	1.96	0.82	1.14
44-45	Retail Trade	1.05	-0.66	1.71
48-49	Transportation and Warehousing	2.13	1.79	0.34
51	Information	-0.57	-1.00	0.43
52	Finance and Insurance	1.83	1.11	0.72
53	Real Estate and Rental and Leasing	1.02	0.67	0.35
54	Prof., Scientific, and Technical Services	1.57	1.40	0.17
56	Admin., Supp., Waste, & Rem. Services	1.10	0.97	0.14
61	Educational Services	1.23	0.96	0.27
62	Health Care and Social Assistance	1.47	1.29	0.19
71	Arts, Entertainment, and Recreation	2.19	1.80	0.39
72	Accommodation and Food Services	0.81	0.61	0.20
81	Other Services (ex. Public Admin.)	2.24	1.85	0.40

Note: Rows are averages of six-digit industry indexes within specified NAICS category. Mod. Lasp. and Geo. Young refer to formulas used for elementary aggregation. Upper-level aggregation uses Lowe formula in all cases.

Table 3: Industry Differences and Dispersion, 2008-17

NAICS	Description	Full Sample Lasp. – Geo.	Survivors Lasp. – Geo.	Survivors LTR C.V.
11	Ag., Forestry, Fishing and Hunting	0.16	0.48	0.17
21	Mining, Quarrying, and Oil and Gas	0.47	0.42	0.18
22	Utilities	0.53	0.84	0.28
23	Construction	0.23	0.23	0.11
31-33	Manufacturing	0.23	0.30	0.13
42	Wholesale Trade	1.14	1.41	0.47
44-45	Retail Trade	1.71	1.68	0.35
48-49	Transportation and Warehousing	0.34	0.35	0.16
51	Information	0.43	0.40	0.16
52	Finance and Insurance	0.72	1.26	0.33
53	Real Estate and Rental and Leasing	0.35	0.33	0.16
54	Prof., Scientific, and Technical Services	0.17	0.13	0.10
56	Admin., Supp., Waste, & Rem. Services	0.14	0.12	0.10
61	Educational Services	0.27	0.17	0.13
62	Health Care and Social Assistance	0.19	0.28	0.15
71	Arts, Entertainment, and Recreation	0.39	0.53	0.17
72	Accommodation and Food Services	0.20	0.41	0.21
81	Other Services (ex. Public Admin.)	0.40	0.41	0.22

Note: Rows are averages within specified NAICS category. Index differences are expressed as percentage points per year. “Survivors” refers to indexes calculated using only those items available during entire sample period. “LTR C.V.” denotes coefficient of variation for the long-term relatives. Lasp. and Geo. refer to formulas used for elementary aggregation. Upper-level aggregation uses Lowe formula in all cases.

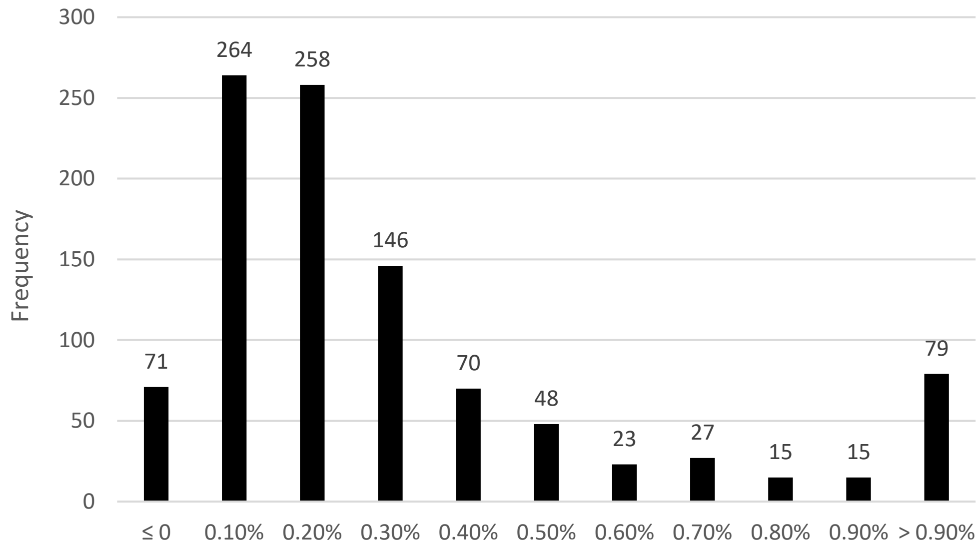
Table 4: PPI for Final and Intermediate Demand, 2010-17 (Annual Percent Change)

	Elementary Indexes		
Index	Mod. Lasp.	Geo. Young	Lasp. – Geo.
Final Demand	1.56	1.01	0.55
Less Trade and Finance	1.47	1.23	0.24
Trade and Finance	1.74	0.22	1.53
Intermediate Demand	--	--	--
Processed Goods	0.98	0.73	0.25
Unprocessed Goods	-1.25	-1.40	0.15
Services	2.02	1.48	0.54
Less Trade and Finance	1.38	1.21	0.17
Trade, and Finance	3.05	1.87	1.18

Note: Mod. Lasp. and Geo. Young refer to formulas used for elementary aggregation. Upper-level aggregation uses Lowe formula in all cases.

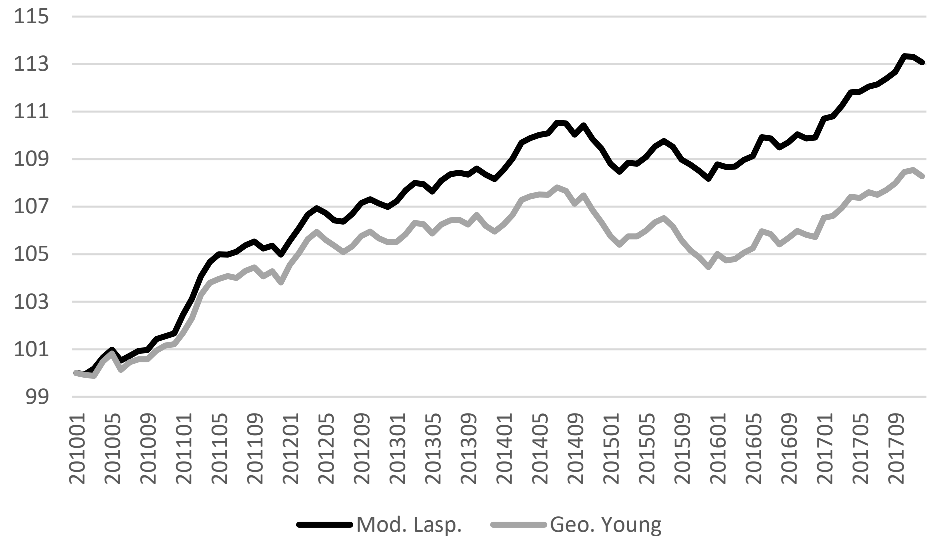
Figures

Figure 1: Difference between Modified Laspeyres and Geometric Young for Six-Digit Commodities



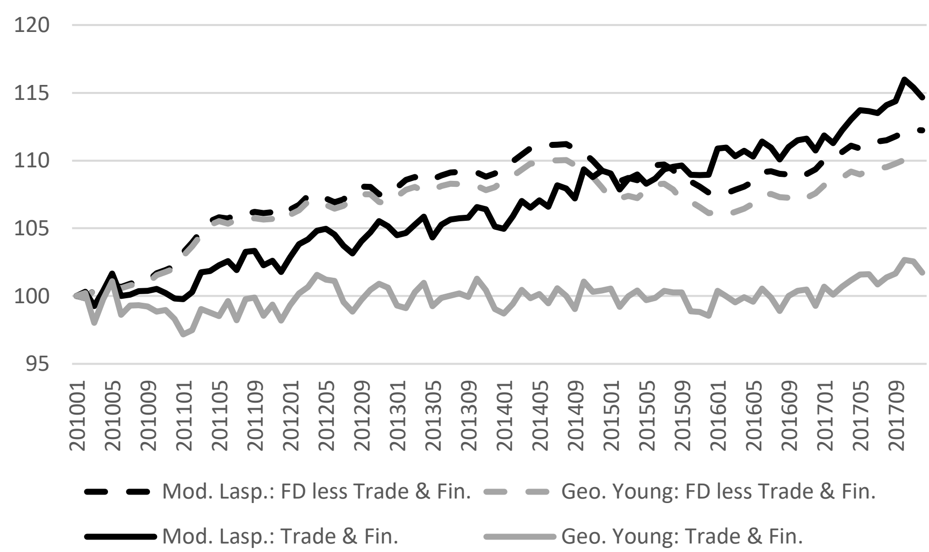
Note: Observations are differences in annual percent changes for 6-digit commodities. Modified Laspeyres and Geometric Young refer to formulas used for elementary aggregation. Upper-level aggregation uses Lowe formula in all cases.

Figure 2: PPI for Final Demand (Jan. 2010 = 100)



Note: Mod. Lasp. and Geo. Young refer to formulas used for elementary aggregation. Upper-level aggregation uses Lowe formula in all cases.

Figure 3: PPI for Final Demand with and without Trade and Finance (Jan. 2010 = 100)



Note: Mod. Lasp. and Geo. Young refer to formulas used for elementary aggregation. Upper-level aggregation uses Lowe formula in all cases.