

A Note on the Effects of Extreme Price Values on Price Indexes

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1. Introduction

Many economists have come to favor the “superlative” Fisher and Törnqvist price indexes over the more traditional Laspeyres formula (see, for example, Diewert 1996, Aizcorbe and Jackman 1993). The US Bureau of Labor Statistics is now planning the release of a new price index series targeting the Törnqvist formula. The choice between the Fisher and Törnqvist formulas may be based on a variety of factors, including other price index formulas in use by the producer and sensitivity to extreme values. In this note, we compare the two formulas with respect to the latter criterion.

Extreme-valued price ratios often occur as a result of deep discounts or “free” promotional goods or services. Such outliers can be either large or small, depending on whether the discounted price appears in the numerator or denominator of the price ratio. Less often, extremely high prices appear with converse effects. The Laspeyres formula is sometimes attacked as sensitive to extreme values because it is based on an arithmetic mean of the price ratios. We will see, however, that such sensitivity depends on the direction of the outlying value (high or low) as well as on the weights used in the selected mean.

In the next section, we consider the effect of an extreme value on the unweighted arithmetic, harmonic, and geometric means. Section 3 contains a discussion of the corresponding effects on the Fisher and Törnqvist index formulas under differing assumptions regarding the correlation between the expenditure-share weights and the prices. This correlation is related to the “elasticity of substitution,” i.e., the extent to which consumers shift their purchases toward lower priced items when relative prices change. We present an empirical example in Section 4 and summarize our conclusions in Section 5.

2. Effects of Extreme Values on Unweighted Means

We use the following simple model to examine the effects of an extreme value on three types of unweighted means. Let x_1, \dots, x_n be a collection of nonnegative values, where $x_i = \mu$ for $i = 1, \dots, n-1$, while $x_n = y\mu$ for some number very large or small positive y ; i.e., x_n is an outlier in the collection. We define the unweighted arithmetic, harmonic, and

geometric means, respectively, as follows:

$$A = \frac{1}{n} \sum_{i=1}^n x_i,$$

$$H = \left(\frac{1}{n} \sum_{i=1}^n \frac{1}{x_i} \right)^{-1},$$

and

$$G = \prod_{i=1}^n x_i^{1/n}.$$

For $M \in \{A, H, G\}$, let

$$f_M = \frac{M}{\mu}.$$

It is easy to see that

$$f_A = \frac{n-1+y}{n},$$

$$f_H = \frac{n}{n-1-y^{-1}},$$

and

$$f_G = y^{1/n}.$$

We first consider the rate at which the various means approach μ as n approaches infinity. For fixed y , we have

$$|1 - f_A| = \left| \frac{1-y}{n} \right| = O\left(\frac{1}{n}\right)$$

as $n \rightarrow \infty$. For the harmonic mean also,

$$|1 - f_H| = \left| \frac{1-y^{-1}}{n-(1-y^{-1})} \right| = O\left(\frac{1}{n}\right)$$

as $n \rightarrow \infty$, and, similarly,

$$\begin{aligned} |1 - f_G| &= \left| 1 - e^{\frac{1}{n} \ln y} \right| \\ &= \left| -\frac{1}{n} \ln y - O\left(\frac{1}{n^2}\right) \right| \\ &= O\left(\frac{1}{n}\right) \end{aligned}$$

as $n \rightarrow \infty$. Thus, as n becomes large, all three of the means approach μ at approximately the same rate.

Suppose now that n is fixed. We show that the behavior of the various means differs as y approaches zero or becomes large. First note that

$$\lim_{y \rightarrow \infty} f_A(y) = \infty, \quad \text{while} \quad \lim_{y \rightarrow 0} f_A(y) = \frac{n-1}{n},$$

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indicating that A is much less sensitive to low outliers than to high outliers. Conversely,

$$\lim_{y \rightarrow \infty} f_H(y) = \frac{n}{n-1}, \quad \text{while} \quad \lim_{y \rightarrow 0} f_H(y) = 0.$$

We may therefore conclude that H is quite robust to high outliers but sensitive to low ones. For the geometric mean, we have

$$\lim_{y \rightarrow \infty} f_G(y) = \infty, \quad \text{while} \quad \lim_{y \rightarrow 0} f_G(y) = 0.$$

These limits suggest that, unweighted, G is about equally sensitive to low outliers as to high outliers. The geometric mean, however, is less sensitive than the arithmetic mean to high outliers and less sensitive than the harmonic mean to low outliers. Note that

$$f_A(y) = \frac{n-1+y}{n} = \Omega(y)$$

as $y \rightarrow \infty$, whereas

$$f_G(y) = O\left(y^{1/n}\right)$$

as $y \rightarrow \infty$, where $n \geq 1$. Thus, although the geometric and arithmetic means both approach infinity as y becomes large, G grows much more slowly than does A , given reasonably large n . Similarly, as $y \rightarrow 0$, we have

$$f_H(y) = O(y),$$

whereas

$$f_G(y) = \Omega\left(y^{1/n}\right),$$

indicating that, as y becomes small, H approaches zero more quickly than does G .

3. Effects of Extreme Values on Price Indexes

The results in Section 2 may lead one to conclude that price index formulas based on the geometric mean are, overall, the most robust formulas available; at the very least, they represent a sensible choice when both high and low outliers are expected to occur. In most applications, however, price indexes are not computed as unweighted means. In this section, we examine the effect of expenditure-share weights on the Laspeyres, Paasche, Fisher, and Törnqvist indexes, with special emphasis on the latter two. We begin by stating the index formulas.

The Laspeyres index measuring price change between time periods 1 and 2 is defined as

$$L = \frac{\sum_{j=1}^N q_{j1} p_{j2}}{\sum_{j=1}^N q_{j1} p_{j1}} = \sum_{j=1}^N w_{j1} \left(\frac{p_{j2}}{p_{j1}} \right),$$

where p_{jt} denotes the price of item j at time $t \in \{1, 2\}$, q_{jt} denotes the quantity of item j purchased

at time t , $w_{jt} = p_{jt} q_{jt} / \sum_{k=1}^N p_{kt} q_{kt}$, and N denotes the number of items in the target population. The weight w_{jt} is the *expenditure share* for item j in period t ; the price ratios p_{j2}/p_{j1} are often called *price relatives*. Clearly L is the arithmetic mean of the price relatives with weights representing first period expenditure shares. The Paasche index is a harmonic mean of the price relatives, with second period expenditure-share weights:

$$P = \frac{\sum_{j=1}^N q_{j2} p_{j2}}{\sum_{j=1}^N q_{j2} p_{j1}} = \frac{1}{\sum_{j=1}^N w_{j2} (p_{j2}/p_{j1})^{-1}}.$$

The Fisher index is simply defined as $F = \sqrt{LP}$, while the Törnqvist is a geometric mean of the price relatives with weights representing the averages of the period 1 and period 2 expenditure shares; i.e.,

$$T = \prod_{j=1}^N \left(\frac{p_{j2}}{p_{j1}} \right)^{w_{j,1,2}},$$

where $w_{j,1,2} = (w_{j1} + w_{j2})/2$.

To examine the effects of an outlier on the indexes described above, suppose we have a collection of n items priced in time periods 1 and 2. Suppose further that, for $j = 1, \dots, n$, we have $p_{j1} = q_{j1} = 1$ and that, for $j = 1, \dots, n-1$, we also have $p_{j2} = 1$, while $p_{n2} = y$. (That is, we assume for simplicity that the μ from the previous section is 1.) Let

$$\tau = 1 - \frac{\ln(p_{j2} q_{j2}) - \ln(p_{j1} q_{j1})}{\ln(p_{j2}) - \ln(p_{j1})},$$

and assume that τ is constant across all items j . Then

$$p_{j2} q_{j2} = p_{j1} q_{j1} \left(\frac{p_{j2}}{p_{j1}} \right)^{1-\tau}$$

for $j = 1, \dots, n$. Given $0 \leq \tau \leq 1$, higher values of τ indicate less impact of price change (represented by the price relatives) on second period item-level expenditure levels. For $j = 1, \dots, n-1$, we have $q_{j2} = q_{j1} = 1$; and

$$q_{n2} = y^{-\tau}.$$

The resulting first and second period expenditure-share weights are as follows:

$$w_{j1} = \frac{1}{n}, \quad j = 1, \dots, n;$$

$$w_{j2} = \frac{1}{n-1+y^{1-\tau}}, \quad j = 1, \dots, n-1;$$

and

$$w_{n2} = \frac{y^{1-\tau}}{n-1+y^{1-\tau}}.$$

The “average weights,” used in the Törnqvist index, are

$$w_{j,1,2} = \frac{1}{2} \left(\frac{1}{n} + \frac{1}{n-1+y^{1-\tau}} \right), \quad j = 1, \dots, n-1,$$

and

$$w_{n,1,2} = \frac{1}{2} \left(\frac{1}{n} + \frac{y^{1-\tau}}{n-1+y^{1-\tau}} \right).$$

When τ is small (low or zero elasticity) and y is large,

$$w_{n1} < w_{n,1,2}, \quad (3.1)$$

so the Laspeyres index gives less weight to high outliers than does the Törnqvist. Similarly, when τ and y are both small,

$$w_{n2} < w_{n,1,2}, \quad (3.2)$$

indicating that the Paasche index gives less weight to low outliers than does the Törnqvist. Under conditions of low elasticity, we therefore observe the following phenomena: although the Laspeyres index, based on the arithmetic mean, is sensitive to high outliers, it assigns them weights that are low relative to the Törnqvist weights, while the Paasche index, a harmonic mean, assigns lower weights to low outliers. The weights in the Laspeyres and Paasche indexes can therefore be expected to compensate, at least partially, for the sensitivity of the arithmetic and harmonic means to high and low outliers, respectively.

Under this simple model, the values of the Laspeyres, Paasche, Fisher, and Törnqvist indexes are as follows:

$$L(n, y) = \frac{n-1+y}{n};$$

$$P(n, y, \tau) = \frac{n-1+y^{1-\tau}}{n-1+y^{-\tau}};$$

$$F(n, y, \tau) = \left[\left(\frac{n-1+y}{n} \right) \left(\frac{n-1+y^{1-\tau}}{n-1+y^{-\tau}} \right) \right]^{1/2};$$

and

$$T(n, y, \tau) = \exp \left[\frac{1}{2} \left(\frac{1}{n} + \frac{y^{1-\tau}}{n-1+y^{1-\tau}} \right) \ln y \right].$$

Both the Fisher and Törnqvist indexes are known as superlative indexes, because economic theory suggests that they approximate a “true cost of living index” under relatively weak assumptions regarding economic conditions (Diewert 1987). We therefore focus on the relative robustness of $F(n, y, \tau)$ and $T(n, y, \tau)$ under the assumptions $\tau = 1$ and $\tau = 0$. The value $\tau = 1$ indicates that, to some degree, consumers shift their purchases toward items (or item categories) whose relative prices have decreased between periods 1 and 2,

while $\tau = 0$ represents the case of little or no change in buying behavior in response to price change.

First consider the case $\tau = 1$, a value of τ representing the assumption that consumers alter the quantities of the items they purchase so as to maintain the same share of expenditure on each item—a situation corresponding to a fairly high level of elasticity. In this case, we have, for fixed n and large y ,

$$\begin{aligned} & F(n, y, 1) \\ &= \left[\left(\frac{n-1+y}{n} \right) \left(\frac{n}{n-1+y^{-1}} \right) \right]^{1/2} \\ &= \left[\left(\frac{n-1+y}{n-1+y^{-1}} \right) \right]^{1/2} \\ &\approx \left(1 + \frac{y}{n} \right)^{1/2}, \end{aligned} \quad (3.3)$$

while

$$T(n, y, 1) = y^{1/n}. \quad (3.4)$$

So, for reasonably large n , T is more robust than F in the presence of high outliers. For the case of low outliers, we have

$$\begin{aligned} F(n, y, 1) &= \left[\left(\frac{n-1+y}{n-1+y^{-1}} \right) \right]^{1/2} \\ &= O(y^{1/2}), \end{aligned} \quad (3.5)$$

and

$$T(n, y, 1) = \Omega(y^{1/n}) \quad (3.6)$$

for fixed n as y approaches 0. Under our simple model, we may therefore conclude that, with regard to robustness, conditions of high elasticity favor the Törnqvist index over the Fisher.

With $\tau = 0$, we have

$$F(n, y, 0) = \left(\frac{n-1+y}{n} \right),$$

and

$$T(n, y, 0) = \exp \left[\frac{1}{2} \left(\frac{1}{n} + \frac{y}{n-1+y} \right) \ln y \right].$$

(Observe that $F(n, y, 0) = L(n, y, 0) = P(n, y, 0)$.) For fixed n and large y ,

$$F(n, y, 0) \approx 1 + \frac{y}{n} = \Omega(y), \quad (3.7)$$

while

$$T(n, y, 0) \approx y^{(n+1)/2n}. \quad (3.8)$$

As a rough “rule of thumb,” the above approximations suggest that T is likely to outperform F

whenever we have outliers as large as n^2 . The relative robustness of T and F thus depends on the relative values of y and n , which may, in turn, depend on the aggregation level being considered. Expressions (3.4) and (3.8) also indicate that, for large values of n , T is much more robust to high outliers under high elasticity than it is under low elasticity. For low outliers, however, the elasticity assumption has less impact on T . With n fixed and y small, we have

$$F(n, y, 0) = \Omega\left(\frac{n-1}{n}\right), \quad (3.9)$$

and

$$T(n, y, 0) = O\left(y^{1/2n}\right), \quad (3.10)$$

revealing that, under conditions of low elasticity, T is more sensitive to low outliers than is F . Equations (3.5), (3.6), (3.9), and (3.10) suggest that T is somewhat more robust to low outliers for $\tau = 0$ than for $\tau = 1$, while F is much more robust.

The above results lead us to conclude that, under conditions of low elasticity, the Fisher index may often be more robust to outliers than the Törnqvist: the Fisher is more robust to low outliers and, when n is sufficiently large relative to any prices in the data set, the Fisher is also more robust to high outliers. Conditions of high elasticity (τ close to 1) render both indexes more robust to extremely high values. Under conditions of high elasticity, the Törnqvist is preferable to the Fisher, as it is less sensitive to both high and low outliers.

The numerical examples shown in the Appendix illustrate these conclusions. Tables 1 through 4 give values of the Fisher and Törnqvist indexes under the single-outlier scenario described above. (Note that these are not random values produced by a Monte Carlo simulation but simply the values of the functions $F(n, y, \tau)$ and $T(n, y, \tau)$ for the given parameters.) Table 1 gives index values under the assumption that $\tau = 1$ (high elasticity). Under this assumption, the Törnqvist is more robust than the Fisher to both high outliers (shown) and low outliers. Tables 2 through 4 show values of the indexes under the assumption that $\tau = 0$. The bold numbers in Tables 2 and 3 highlight points at which y becomes large enough, relative to n , to render the Törnqvist better than the Fisher for approximating the mean 1 in the presence of a high outlier. As expected, the “turning points” occur as y approaches n^2 . For the parameter values in Table 4, the Fisher always outperforms the Törnqvist; the table entries only illustrate the degree to which the Fisher does better in the presence of low outliers under low elasticity. Under low elasticity, both indexes are more sensitive to high outliers and less sensitive to low outliers than they are under high

elasticity. The former tendency can be seen by comparing the last two columns of Table 1 ($n = 30$) with the corresponding columns of Table 3.

4. An Empirical Example

To illustrate the practical effects of the results discussed above, the Appendix provides graphs of air travel price index series for the various index formulas. For these series, the apparent elasticity of substitution is low—in some cases, even negative. The series therefore exemplify only the behavior of the different indexes under conditions of low elasticity (τ close to 0). It is important to realize that the elasticity *reflected in the sample* data is the quantity that affects the performance of the indexes; this elasticity may differ from that of the underlying population (see Dorfman et al. 1999).

The air travel price index series shown in Figures 1 through 6 are based on data from the Bureau of Transportation Statistics’ quarterly Origin and Destination (O&D) Survey. The sample for the O&D Survey comprises about 10% of all passenger itineraries having some US component (i.e., itineraries that include at least one flight arriving at or departing from a US airport) and includes about two to four million itineraries per quarter. The index series shown are based only on sample itineraries flown on domestic carriers. For a description of the index estimation methodology, see Lent and Dorfman (2001a).

The figures show the Laspeyres, Paasche, Fisher, and Törnqvist index series for various classes of service and for all classes combined. In all figures, the Paasche series runs close to the Laspeyres series or even (for business class service) above the Laspeyres, indicating low or negative elasticity of substitution. (Lent and Dorfman 2001b describe a method of estimating the elasticity of substitution; elasticity estimates computed by their method run close to 0 for these data.) In examining Figures 1 through 6, it is important to note that the “class of service” variable in the O&D Survey was redefined and standardized in 1997-98. We therefore expect some unusual data values to affect the index series during this period; indeed, many of the series display a visible break between the fourth quarter of 1997 and the first quarter of 1998. These breaks may be exacerbated by the fact that a lower percentage of the O&D Survey observations were “matched” across time during 1997-98 (see Lent and Dorfman 2001a for a description of the across-time matching method), resulting in lower than usual effective sample sizes.

Figures 1 and 2 show the series for all classes of service combined and for restricted coach class (by far the largest class), respectively. The se-

ries in Figure 2 behave in “typical” fashion: the Laspeyres series runs slightly above the others, displaying a slight upward drift, while the Paasche shows a similar downward drift, and the two superlative series run between them, closely tracking each other. This type of behavior results from the large number of observations and the fact that the 1997-98 break has relatively little impact on these series. Figure 1 is similar to Figure 2, except for the noticeably larger effect of the 1997-98 change, which lifts the Törnqvist series slightly above the others. Recall that, under conditions of low elasticity, the Törnqvist index is often more sensitive to outliers than is the Fisher.

Index series for other classes of service (categories comprising fewer observations) appear in Figures 3 through 6. For the unrestricted first and restricted first class indexes (Figures 3 and 4), the Laspeyres series runs very slightly above the Paasche, indicating low but positive elasticity. For the unrestricted first class series, the 1997-98 break sends the Törnqvist above the other series, while the Törnqvist for restricted first class is “bumped down” and runs well below the others for 1998 and subsequent years. In both cases, the Törnqvist continues to roughly parallel the Fisher after the break, indicating that unusual data values generated the level shifts. Note also that the Törnqvist’s upward shift for unrestricted first class is noticeably less severe than its downward shift for restricted first class, perhaps due to its greater robustness to high outliers than to low ones.

The business class index series (Figures 5 and 6) display the relatively rare phenomenon of negative elasticity. The Paasche series runs above the Laspeyres, indicating that consumers are shifting their purchases toward *higher* priced services as relative prices change. We emphasize that sample survey data may not always reflect true population elasticity; in this case, the “class of service” categories are coarsely defined, and many different types of restrictions may apply to tickets in the “restricted” categories. Elasticity estimates based on these data do not reflect substitution *within* these categories (for the same route and carrier) and may therefore suffer a downward bias. On the other hand, since business class service is typically paid for by a third party (i.e., the passenger’s employer), very low elasticity is expected. Some business class passengers may even choose higher priced tickets assuming that “you get what you pay for,” and such behavior could also explain the negative elasticity indicated. Under negative elasticity, quantities purchased are positively correlated with price change, and this correlation may cause expenditure shares to increase dramatically when prices increase. The

Törnqvist index, whose weights are “average” expenditure shares, therefore assigns large weights to some high price ratios. Apart from the negative elasticity, the movements of the business class series appear similar to that of the first class series, i.e., the Törnqvist index is shifted up or down during the 1997-98 period, while the other series are less affected by the unusual values.

5. Conclusions

Practitioners may often consider robustness to outliers an important criterion in selecting a price index formula, especially for item categories such as airfares, in which extreme prices may regularly result from “frequent flyer” awards and other price discriminatory discounts. Although price index formulas based on different types of means inherit the relative robustness of these means, the weights applied in price index calculation also play a crucial role. We’ve seen that, under conditions of low elasticity of substitution, the high correlation between the weights and the price relatives may offset the sensitivity of the Laspeyres and Paasche indexes, making the Fisher a more attractive option than the Törnqvist. The choice between index formulas is therefore more complex than the mere selection of an arithmetic, harmonic, or geometric mean. It requires information on the elasticity of substitution reflected in the data as well as an estimate of the magnitude of outliers (high or low) that can be expected.

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Appendix: Tables and Figures

Table 1. Index Values Given a Single Outlier, $\mu = 1, \tau = 1$

Outlier <i>y</i>	n=10		n=30	
	F	T	F	T
100	3.30	1.58	2.07	1.17
200	4.57	1.70	2.76	1.19
300	5.56	1.77	3.31	1.21
400	6.39	1.82	3.78	1.22
500	7.13	1.86	4.20	1.23
600	7.80	1.90	4.58	1.24
700	8.42	1.93	4.93	1.24
800	8.99	1.95	5.26	1.25
900	9.53	1.97	5.56	1.25
1000	10.04	2.00	5.86	1.26

Table 3. Index Values Given a Single Outlier, $\mu = 1, \tau = 0$

Outlier	n=20		N=30	
	F	T	F	T
100	5.95	7.77	4.30	6.43
200	10.95	12.83	7.63	11.05
300	15.95	16.85	10.97	14.81
400	20.95	20.28	14.30	18.05
500	25.95	23.31	17.63	20.92
600	30.95	26.06	20.97	23.51
700	35.95	28.58	24.30	25.90
800	40.95	30.93	27.63	28.13
900	45.95	33.15	30.97	30.22
1000	50.95	35.24	34.30	32.19

Table 2. Index Values Given a Single Outlier, $\mu = 1, \tau = 0$

Outlier <i>y</i>	n=5		n=10	
	F	T	F	T
10	2.80	2.87	1.90	2.06
20	4.80	4.70	2.90	3.26
30	6.80	6.30	3.90	4.39
40	8.80	7.73	4.90	5.42
50	10.80	9.05	5.90	6.38
60	12.80	10.26	6.90	7.28
70	14.80	11.41	7.90	8.12
80	16.80	12.49	8.90	8.92
90	18.80	13.52	9.90	9.68
100	20.80	14.51	10.90	10.41

Table 4. Index Values Given a Single Outlier, $\mu = 1, \tau = 0$

Outlier	n=5		n=10	
	F	T	F	T
0.1000	0.820	0.772	0.910	0.880
0.0500	0.810	0.728	0.905	0.854
0.0333	0.807	0.702	0.903	0.838
0.0250	0.805	0.684	0.903	0.827
0.0200	0.804	0.670	0.902	0.819
0.0167	0.803	0.658	0.902	0.812
0.0143	0.803	0.649	0.901	0.806
0.0125	0.803	0.641	0.901	0.801
0.0111	0.802	0.634	0.901	0.796
0.0100	0.802	0.627	0.901	0.792

Figure 1
All Classes of Service
Quarterly Chained Preliminary Series, 95Q1=100

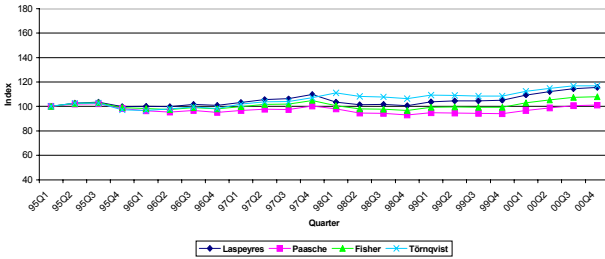


Figure 2
Restricted Coach Class
Quarterly Chained Preliminary Series, 95Q1=100

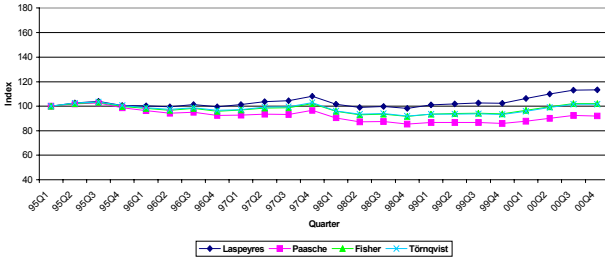


Figure 3
Unrestricted First Class
Quarterly Chained Preliminary Series, 95Q1=100

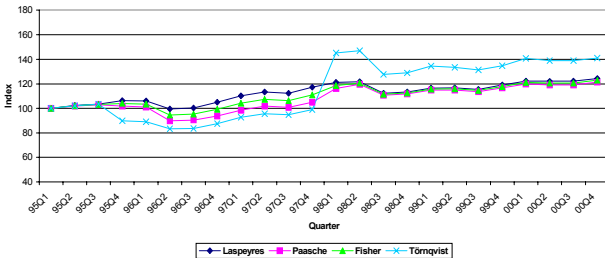


Figure 4
Restricted First Class
Quarterly Chained Preliminary Series, 95Q1=100

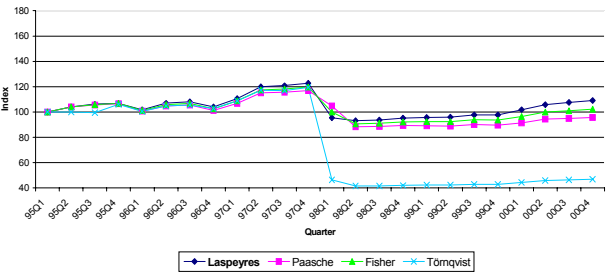


Figure 5
Unrestricted Business Class
Quarterly Chained Preliminary Series, 95Q1=100

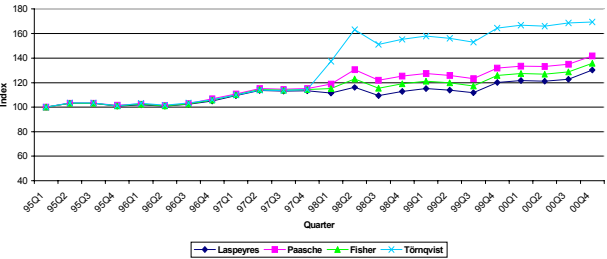


Figure 6
Restricted Business Class
Quarterly Chained Preliminary Series, 95Q1=100

