ESTIMATORS FOR AVERAGE HOURLY EARNINGS AND AVERAGE WEEKLY HOURS FOR THE CURRENT EMPLOYMENT STATISTICS SURVEY

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I. Introduction

In this paper the results of a theoretical and empirical investigation of different estimators for average hourly earnings, average weekly hours, and the respective monthly changes are presented. The study considers imputation under both situations of low and high non response rates.

The investigations began in connection with the revision program for the Bureau of Labor Statistics' Current Employment Statistics (CES) survey. This is a longitudinal survey of approximately 400,000 establishments that provides monthly estimates of, among other parameters, total employment, average weekly hours and average hourly earnings for production workers, for industry groups and for total private industry. In this program, population employment counts are obtained once a year from Unemployment Insurance administrative records. The CES frame is updated annually, based on population information up to the first quarter of the year. At this time, estimation from the second quarter forward is revised, post-stratified to the March population employment. March is referred to as the "benchmark month," and the March population employment is called "benchmark employment." Note that population counts for the number of production workers, their hours, and their earnings are not available.

The ten estimators considered along with some of their theoretical properties are presented in Section II. In Section III the details of the empirical investigation are explained, and the recommendations are given in Section IV.

II. Theoretical Investigation

1. Definitions and notation

Before discussing the estimators the following notation are needed.

$PW_k(i)$ = a random variable denoting the number of **production workers** for establishment i during month k,

$PR_k(i)$ = a random variable denoting the weekly **payroll** for production workers for establishment i during month k,

$WH_k(i)$ = a random variable denoting the weekly production worker **hours** for establishment i during month k,

$Y_k(i)$ = a random variable denoting all **employment** for establishment i at month k,

$w_k(i)$ = the sampling weight associated with establishment i at month k,

S_k = the set of establishments in the sample at time k, for k=0,1,2,…, and k= 0 denotes the benchmark month for **employment**,

$S_{k-1}S_k$ = $S_{k-1} \cap S_k$ set of establishments that responded in both time periods k-1 and k, (referred to as a matched sample)

N = number of establishments in the population (It will be assumed that the number of establishments in the population is fixed from month to month.)

Let A denote a subset of the **population**, such as an industry class. We are interested in an estimator for the average weekly hours and average hourly earnings for the set A during time k. This is defined as,

Average weekly hours for A:

$$AWH_k(A) = \frac{\sum_{i \in A} WH_k(i)}{\sum_{i \in A} PW_k(i)} = \frac{WH_k(A)}{PW_k(A)}$$

Average hourly earnings for A:

$$AHE_k(A) = \frac{\sum_{i \in A} PR_k(i)}{\sum_{i \in A} WH_k(i)} = \frac{PR_k(A)}{WH_k(A)} \qquad (1)$$

2. Estimators

The investigation for average weekly hours, AWH, and average hourly earnings, AHE, differs from the employment estimation in two main areas. One, in employment estimation, we estimate a population total, whereas here we are estimating a ratio of totals. Second, in employment we have a true population count, a benchmark at certain time intervals. For our variables, which are total weekly payroll, total weekly hours and total number of production workers there are no benchmarks. However, since our variables and total employment are highly correlated, it may be possible to model our variables on total employment and make use of the benchmark employment.

Nine estimators are developed and compared with the current estimator. The nine estimators can be categorized into one of two types. One type uses information only from the current time period and is the ratio of two Horvitz-Thompson estimators for total. The second considers more than the current time period, and is a regression type estimator for all months but the benchmark month. The benchmark month is estimated either by modeling or by a Horvitz-Thompson type estimator.

We will start with the first type, which contains only one estimator, the ratio of two Horvitz -Thompson estimators for total. That is:

$$A\hat{W}H_k(A) = \frac{\sum_{i \in S(A)} WH_k(i)\, w_k(i)}{\sum_{i \in S(A)} PW_k(i)\, w_k(i)}$$

and **(EST1)**

$$A\hat{H}E_k(A) = \frac{\sum_{i \in S(A)} PR_k(i)\, w_k(i)}{\sum_{i \in S(A)} WH_k(i)\, w_k(i)}$$

where $S(A)$ represents a random sample from A.

Since it is known that payroll, hours, and production workers from the previous time period are each correlated with the corresponding variables in the current time period, this estimator does not make use of all the information available, including benchmark employment.

All the estimators in the second type are developed from a modeling point of view, even though they can be obtained from a strictly probabilistic view point. With the use of models the assumptions are clearly visible, and if appropriate data sets are available the models can be tested. In fact, in the early eighties, West (1982), we were exploring models for the employment variable using the universe data base. We found that the most promising model was the simple proportional regression model:

$$E(Y_k(i) | \underline{Y_{k-1}} = \underline{y_{k-1}}) = b_k \; y_{k-1}(i) \tag{2}$$

$$\operatorname{cov}(Y_k(i), Y_k(j) | \underline{Y_{k-1}} = \underline{y_{k-1}}) = \begin{cases} s^2 Y_{k-1}(i) & if \quad i = j \\ 0 & if \quad i \neq j \end{cases}$$

For a variable that does not have a benchmark, such as payroll, $PR_k(i)$, or hours, $WH_k(i)$, it is possible to add to the above two equations a link with the benchmark employment. Thus for the benchmark month, payroll is studied under the following model:

$$E(PR_0(i) | \underline{Y_0} = \underline{y_0}) = b_{PR} \; y_0(i) \tag{3}$$

$$\operatorname{cov}(PR_0(i), PR_0(j) | \underline{Y_0} = \underline{y_0}) = \begin{cases} s^2 Y_0(i) & if \quad i = j \\ 0 & if \quad i \neq j \end{cases}$$

This relation is a way of representing the assumption that payroll is roughly proportional to employment. The same model is considered for relating production workers and hours to benchmark employment. Using these relationships, the new variables are all estimated for the benchmark month. For example, an estimator for total payroll at the benchmark month is:

$$P\hat{R}_0(A) = \hat{b}_{PR} Y_0(A),$$

where (4)

$$\hat{b}_{PR} = \sum_{j \in S_0} pr_0(j) w_0(j) \Big/ \sum_{j \in S_0} y_0(j) w_0(j)$$

For the subsequent months:

$$P\hat{R}_k(A) = \hat{b}_k P\hat{R}_{k-1}(A),$$

where (5)

$$\hat{b}_k = \sum_{j \in S_{k-1} \cap S_k} pr_k(j) w_k(j) \Big/ \sum_{j \in S_{k-1} \cap S_k} pr_{k-1}(j) w_{k-1}(j)$$

$$for \; k = 1,2,3,$$

Note that the estimator in (5) has the same form as the current employment estimator (link relative estimator) except here the regression coefficient reflects the sample design by the incorporation of the sampling weights.

Similar formulas exist for hours, $W\hat{H}_k(A)$, and production workers, $P\hat{W}_k(A)$. Thus estimators for average weekly hours, $A\hat{\hat{W}}H_k(A)$, and average hourly earnings, $A\hat{\hat{H}}E_k(A)$, are:

$$A\hat{\hat{W}}H_k(A) = W\hat{H}_k(A) \big/ P\hat{W}_k(A)$$

and **(EST2)**

$$A\hat{\hat{H}}E_k(A) = P\hat{R}_k(A) \big/ W\hat{H}_k(A)$$

$for \; k = 1,2,3,..$

$$A\hat{\hat{W}}H_0(A) = W\hat{H}_0(A) \big/ P\hat{W}_0(A)$$

$$A\hat{\hat{H}}E_0(A) = P\hat{R}_0(A) \big/ W\hat{H}_0(A)$$

where for time period 0 the totals are estimated as in equation (4).

The next two estimators differ from EST2 only for time period, k =0. For the third estimator, **EST3**, for k =0, the ratio of Horvitz-Thompson estimators is used, rather than using the model that relates the new variables to employment, as in (4).

Note that although EST2 and EST3 are defined differently for k =0, they can produce the same result. Since in EST2 both the numerator and denominator are functions of benchmark employment, it is possible for employment to cancel leaving simply the ratio of two Horvitz-Thompson estimators as in EST3. This happens when the "modeling cell" (formation of $\hat{b}$) is the same as the publication cell (the point where the ratio is formed). This was avoided as much as possible by selecting the modeling cells as small as possible. Thus, the estimates for the larger cells were obtained by estimating the numerator and denominator separately and then taking the ratio at the higher level.

The next estimator, **EST4,** differs from EST2 only in the alternative estimation procedures for k =0. EST4 uses a prediction approach which resulted in one of the top estimators in earlier studies, West(83,84). Specifically, the numerator and denominator of each estimator is written as the sum of the sampled units plus the sum of the non sampled units. In this case the effect of employment will not cancel out.

Recall that in estimator, EST2, the movement from one month to the next is done by moving the numerator and denominator and then forming the ratio. In the next estimator, **EST5**, the movement is done on the ratio. If in the underlying model, the conditional variance is proportional to the prior months value squared, then the resulting estimator is the sum of the ratios, as opposed to the EST2 which is the ratio of sums. **EST6** is the current estimator, which is an ad hoc estimator that has produced reasonable results, for the most part, over the years, but has no statistical theory underlying it. For details on these estimators see West, Kratzke, Grden, (1997).

The next two estimators are link relative type estimators, only now the current month is linked to the 0^{th} month, rather than to the prior month as in the case of EST2 through EST5. For lack of space the formulas will not be given, but they will be summarized.

EST7 and EST8: These two estimators are similar to the link relative except "linking" is to the 0^{th} month rather than the $(k-1)^{th}$ month. EST7 is most like the link relative in that it does not use imputation and uses matched samples. EST8 uses imputation and does not require matched samples Both these estimators use benchmark employment in modeling for the 0^{th} month estimators. (Note that the totals in EST8 are of the same form as the proposed employment estimator.)

EST9 and EST10: For both these estimators it is assumed that total payroll for production workers is known for the 0^{th} month. EST9 is similar to EST8, and EST10 is similar to EST2, with the difference being that payroll in the 0^{th} month does not have to be modeled.

Note that only EST1, EST8 and EST9 use an explicit imputation method, which will be discussed in the next section. In the situation of 100% response rate EST2 = EST4 = EST7 = EST8.

III. Empirical investigation

1. Population

We used 13 months, from March 1994 through March 1995, of national CES data, which is the only data base that has wages for production workers. A frame was created which consisted of CES reporters with private ownership and with non-missing data for the variables of interest: number of production workers, weekly payroll, weekly hours, and employment. This frame consists of 173,434 establishments.

2. Design of the sample

A national sample (26,308) was taken from the frame according to the selection weights specified for the employment simulation study. The allocation was done by Allocation Industry Cell/Size.

3. Nonrespondents and imputation methods

We are concerned only with the first closing monthly estimates. The expected response rate for establishments making up the first estimates was suggested as 80%. However the current response rate is more around 60%. We decided to consider both scenarios. Note that we are concerned only with unit nonresponse. From previous investigations, it is clear that the nonrespondents are not missing at random, so we devised the following scheme to select the nonrespondents.

Each observation on the frame has a closing code which indicates the time in which the establishment reported to the survey. The closing codes have values from 1 to 4, which indicate the month that the data were reported after the reference month. Initially, in the sample the nonrespondents are considered to be the reporters with closing codes 3 or 4. Across industries and time, there is a range from 3 to 17 percent of reporters whose closing codes are 3 or 4, which actually gives us a better than 80% response rate overall. In order to see how the best two estimators and the current estimator withstand high nonresponse rates, the study was also done with nonrespondents considered to be the reporters with closing codes 2, 3 or 4. Across industries and time, there is a range from 14 to 47 percent of reporters whose closing codes are 2, 3 or 4, which actually gives us a fairly low overall response rate.

Explicit imputation is done only for estimators, EST1, EST8, and EST9. The imputation methods are as follows: for the first month essentially a mean imputation is used, and a ratio imputation for the subsequent months. For a subsequent month, the imputed value for nonrespondent j is obtained by multiplying the previous month's value for establishment j by the ratio of the sum of values of current month to sum of values of previous month in a matched sample. Matched samples are done by industry/size class. This method had top ratings in earlier studies, West, Butani, Witt (1991), using population data for wage and employment. In that study it turned out that whether the imputation was done on wage or wage over employment the results were similar. In our method, it is assumed that an editing procedure will be run after all data are imputed. This is a good procedure to follow for any imputation method that is used.

4. Comparison at the 3 digit SIC level of the first five estimators

EST1 through EST5 were tested on the construction and durable goods industries at the 3-digit SIC level. The two most promising estimators were EST1 and EST2.

5. Comparison of EST1, EST2, and the current estimator, EST6, when publishing estimates for the major industry divisions, and for total private industry.

For each estimator, estimates were made for the nine major industry divisions, and for total private industry for thirteen months for level and twelve months for monthly change. The error is defined as the estimate minus the true. We looked at the sums of errors and absolute errors (as well as relative errors). The three estimators were compared in a number of ways. Since they led us to the same conclusion we will just exhibit the summary tables for the absolute errors. The absolute errors for each major industry estimate are summed over the thirteen (or twelve) months and the nine major industries. The results are shown in Table I. for both small and large nonresponse rates. Here we see the pattern: EST1 is best for level and EST2 is best for change The absolute error for total private industry is summed over the months and is shown in Table II. Here in the higher nonresponse rate the pattern emerges again: EST1 best for level, EST2 best for change.

6. Observations

a. The size of the response rate has less of an effect on EST1 than EST2 when it relates to level, but more of an effect for change. EST6 is not as good as either of the other two.

b. As theory would predict, EST2 did very well for estimating the monthly change, quite a bit better than EST1. Since EST2 makes use of last month's information and is based on a matched sample, it should do better for change. However EST2 did not do as well as EST1 for level. What is particularly troubling about the EST2 level estimate is that it is made up of a ratio, where both the numerator and denominator are each formed by ratio adjusting an initial value, which is not the true value (as in the employment estimator). The estimate is the ratio of two parts where each part is made up of an initial estimate multiplied by a string of ratios, the length of which, depends on the distance from the "benchmark". The properties of the variance of this estimator is not clear. Even if the variance of the estimators for the numerator and denominator increased the further away you were from the benchmark month, the variance of the ratio would not necessarily be monotone.

c. Between the top two estimators, the Horvitz-Thompson estimator, EST1, with ratio imputation for the nonrespondents is the best choice if level is the most important issue. Note that using ratio imputation is essentially using EST2 for the nonrespondents. On the other hand if change is the major issue than EST2 would be a better choice. However EST1 and EST2 do not differ by much, and in light of the objection with EST2 stated previously, EST1 with ratio imputation, would be the preferred method at this stage. However, we will look at the additional estimators in the next section and estimate the variances and bias of the top two estimators.

d. In EST2, the model based estimators, Link Relative, in the numerator and in the denominator were better than the corresponding Horvitz-Thompson estimators in EST3. Specifically, for the ratio, say AWE = PR / WH, most often the Link Relative estimator did better at estimating PR and WH than did the Horvitz-Thompson. However. for the ratio, AWE, the opposite was true, Horvitz-Thompson did better than Link Relative. In West et al (1997), it is shown theoretically under what conditions the numerator and denominator each can be better estimated by a specific estimator over an alternative estimator, but for the resulting ratio the reverse is true.

7. Comparison of additional estimators

Estimators EST7 through EST10 were added to the empirical study for the major industry estimates with the result that they did not do as well as EST1 and EST2. As expected, EST7 and EST8, which link to the 0^{th} month which is not a benchmark, did not do as well as EST2, which links to the prior month. This was especially true for monthly change, which is the most important parameter.

One estimator, EST10, is worth mentioning; it has the same form as the link relative estimator, EST2, except for the benchmark month, where production worker payroll, PR, is assumed known. Note that EST9 and EST10 can not be applied in practice, since we do not have production worker wages for the population. The closest that exists is on the data base that gives us the population employment for the 0^{th} month, which also has a quarterly wage for all employees. With this in mind, we decided to see how well we could do if in fact we did have the true wages for production workers at the benchmark month. Note we still do not have true total hours. Thus, for average hourly earnings for the benchmark month we have true total payroll for the numerator and an estimate from our model for total hours. Average weekly hours would not change at all. In Table III., EST1, EST2, EST8, EST7, and this hypothetical estimator, EST10, are compared. For each estimator, estimates were made for the nine major industry divisions, and for total private industry for thirteen months for level and twelve months for monthly change. In Table III. the absolute errors are summed over the thirteen (or twelve) months for each industry.

We arrive at the same conclusion as before that EST1 does well for level and EST2 does well for change. What may seem surprising is that knowing true production worker payroll for benchmark month did not do better than our original way of modeling on benchmark employment. Reflecting on this, it is not surprising. When both numerator and denominator are highly correlated random variables we do better than with a fixed number and a random variable. Note that the estimators that require matched samples, EST2, EST7, and EST10, all do better for change than the estimators that use imputation, EST1 and EST8.

8. Mean Square Error Estimation

One last comparison between EST1 and EST2 was made. A mean square error was computed over thirty samples from which we could obtain estimates of variance and bias. Specifically, thirty samples were randomly drawn from our population. For each sample, estimates were computed for the four parameters using EST1 and EST2. For each estimator, the mean square error, MSE, was computed for each parameter over the thirty samples. The mean square error was computed for each month and the average formed over the thirteen (or twelve) months. The results, shown in Table IV., support the same conclusion as before. Note that the results in these Tables are computed using all the establishments that had a closing code of 2, 3 or 4 as the nonrespondents.

IV. Recommendations

EST1, which is the ratio of Horvitz-Thompson estimators, is best for level, and EST2, which is the ratio of Link Relative estimators, is best for change. Since EST1 and EST2 do not differ by much, our recommendation would be to use EST1 with ratio imputation. The main problem with EST2 is that we are starting out at the “benchmark” month with an estimate and we are continually ratio adjusting it from one month to the next, so that if we have a bad estimate initially the level will continue to be bad for subsequent months. This is not a problem with monthly change. Thus, until we have a benchmark for AWH and AHE, we recommend EST1 over EST2, or any EST2 derivative, such as the ratio to 0^{th} month (which is EST2 with 100% response rate). EST1 could be thought of as a composite of the Horvitz-Thompson and Link Relative estimators, since the imputation used for the Horvitz-Thompson is the same model underlying the Link Relative estimiator.

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Table I. Sum Of Absolute Errors For Major Industry Estimates Over Time And Major Industries

	Small NonResponse Rate			Larger NonResponse Rate		
	EST1	**EST2**	**EST6**	**EST1**	**EST2**	**EST6**
Level AWH	**12.229**	15.676	15.871	**22.450**	40.803	40.708
Level AHE	**07.350**	10.533	08.927	**8.993**	18.028	16.876
Change AWH	10.923	**08.857**	10.841	29.366	**20.489**	26.951
Change AHE	03.576	**02.854**	05.073	06.070	**04.229**	07.994

Table II. Sum Over Time Of Absolute Errors For Total Industry Level

	Small NonResponse Rate			Larger NonResponse Rate		
	EST1	**EST2**	**EST6**	**EST1**	**EST2**	**EST6**
Level AWH	0.414	**0.363**	0.950	**0.709**	0.919	2.431
Level AHE	0.287	**0.269**	0.391	**0.429**	1.098	1.573
Change AWH	0.382	**0.321**	0.618	1.005	**0.554**	2.460
Change AHE	0.135	**0.089**	0.348	0.258	**0.146**	1.414

Table III. Sum of monthly absolute errors for major industry estimates over time.

LEVEL - Average Hourly Earnings

INDUSTRY	**EST1**	**EST2**	**EST8**	**EST7**	**EST10**
FIRE	0.76	**0.36**	1.20	0.59	5.71
Construction	**0.68**	1.13	0.80	1.23	0.76
Durable	0.95	3.27	2.63	3.50	**0.71**
Mining	2.70	9.72	**2.50**	4.05	3.29
Nondurable	0.75	0.48	**0.36**	0.50	2.39
Retail	**0.45**	0.52	0.59	0.74	1.25
Services	**0.50**	0.79	0.85	0.91	0.55
Transportation	1.03	1.03	1.15	**1.00**	1.15
Wholesale	1.16	**0.74**	1.28	0.95	1.24

CHANGE - Average Hourly Earnings

INDUSTRY	**EST1**	**EST2**	**EST8**	**EST7**	**EST10**
FIRE	0.32	**0.26**	0.30	0.46	**0.26**
Construction	0.52	0.42	0.52	0.46	**0.41**
Durable	0.56	0.28	0.57	**0.19**	0.25
Mining	1.34	**1.30**	1.47	1.77	1.72
Nondurable	0.46	**0.14**	0.47	0.24	0**.14**
Retail	0.49	**0.**31	0.51	0.41	0**.29**
Services	0.22	**0.15**	0.24	0.28	0.16
Transportation	1.30	0.85	1.31	0.84	**0.82**
Wholesale	0.86	**0.51**	0.88	0.60	**0.51**

Table IV. Average MSE, $\overline{MSE}$, and Average Bias, $\overline{B}$, for EST1 and EST2

(Computed for each month using 30 sample replicates and then averaged over the 3/94 to 3/95 time period)

EARNINGS	**EST1**				**EST2**			
	LEVEL		**CHANGE**		**LEVEL**		**CHANGE**	
INDUSTRY	$\overline{MSE}$	$\overline{B}$	$\overline{MSE}$	$\overline{B}$	$\overline{MSE}$	$\overline{B}$	$\overline{MSE}$	$\overline{B}$
FIRE	0.008	0.021	0.003	-0.005	0.008	0.008	0.001	0.000
construction	0.011	-0.011	0.004	0.010	0.016	-0.043	0.004	0.008
Durable	0.009	0.018	0.010	-0.030	0.069	0.258	0.001	-0.005
Mining	0.071	-0.088	0.042	-0.051	0.131	-0.254	0.020	-0.034
Nondurable	0.005	0.016	0.004	-0.016	0.004	-0.012	0.000	-0.004
Retail	0.003	-0.014	0.003	0.015	0.002	-0.016	0.001	0.002

Services	0.002	-0.002	0.001	0.003	0.004	0.043	0.000	-0.003
Transportation	0.018	0.008	0.018	-0.008	0.017	0.003	0.012	0.008
Wholesale	0.014	0.030	0.005	-0.011	0.039	0.139	0.003	-0.003
Total Private	0.002	0.005	0.002	-0.012	0.007	0.082	0.000	-0.001