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1. Introduction

The Employment Cost Index (ECI) is a principle economic indicator. It is one of the outputs produced by the National Compensation Survey (NCS) and conducted by the Bureau of Labor Statistics (BLS). The ECI provides quarterly and annual measures of the rate of change in compensation per hour worked including wages, salaries, and employer costs of employee benefits, free from the effect of employment shifts among occupations and industries. The estimates are based on a sample of about 54,000 occupational observations, selected using a three-stage stratified design with probability proportionate to employment at each stage. The three stages are: areas, establishments, and occupations. More details on the sample are provided in Ernst et. al. (2002).

Index estimates are calculated using change in Laspeyres Index. Variance estimates are calculated using the Balanced Repeated Replication (BRR) method. Many series are produced which break the data down by ownership (private and government), industry (construction, manufacturing, transportation, communications, retail, insurance, finance, health services, business services, education services, etc.), occupational group (white-collar, blue-collar, and service), bargaining status (union and nonunion), region (northeast, south, mid-west, and west), and area (metropolitan and other). Each series is published in three categories: wages, benefits, and compensation, the combination of wages and benefits. Series are published both with and without seasonal adjustments. In our study, we focus on the variance estimates of quarterly change for non-seasonally adjusted series from 1997 onward.

This paper presents analysis of the volatility of variance estimates in section 2, discussion of the appropriateness of smoothing

in section 3, the application of exponential smoothing procedures for ECI standard error estimates in section 4 and comparison of the results of the exponential smoothers with those from the five-year moving average currently used in section 5.

2. Analysis of the Volatility in Variance Estimates

Standard error estimates vary greatly over time both within and among series. In addition to differences in magnitude, many series also show seasonality, trends, and correlation with other series.

2.1 Magnitude

Among series, average magnitude of standard error estimates ranges from 0.09 to 1.48 ('All State and Local Government Wages,' 'Private Industry Banking Savings and Loans Wages'). Among the wages, benefits, and compensation categories, standard error estimates tend to be higher in benefits series. The average magnitude is below 0.3 for 80% of compensation series and 78% of wages series but only 30% of benefits series. Generally, standard errors at or below 0.3 are desirable so that changes of one-half of a percentage point in the index estimates are significant at the 90% confidence level.

Within series, the coefficient of variation of the standard error estimates ranges from 24% to 130% ('Private Industry Non-Union Goods-Producing Industries Wages,' 'Private Industry Banking Savings and Loans Wages'). For most series the coefficient of variation of the standard error estimates is between 35% and 65%.

Outliers are frequent among the standard error estimates and can have a large effect on the average magnitude. 93% of wages series, 91% of benefits series, and 86% of compensation series have at least one value more than two standard deviations above the mean over the 34-quarter period. Figure 3: Wages Series 2: 'All Civilian, Excluding Sales,' shows one such outlier.

2.2 Correlated Groups

Standard error estimates show remarkable correlation among series. Many series, even series with very different magnitudes, have correlations close to 1. Furthermore, there are several groups of series with particularly high correlations between group members. These groups tend to share one or more common characteristics. The four prominent groups in wages series can be described as: non-manufacturing series including sales, excluding sales series, government series, and goods-producing series. The four most prominent groups in benefits can be described as: non-manufacturing series, government series, service occupations series, and goods-producing blue-collar series.

Figure 1: Diagraph of Correlations between Wages Series

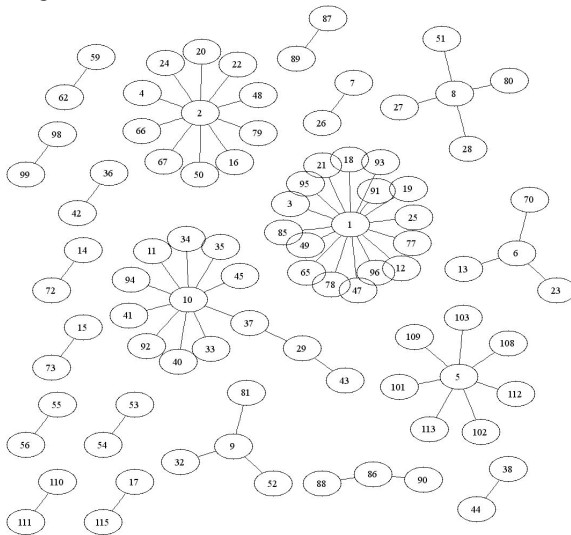


Figure 1: ‘Diagraph of Correlations between Wage Series’ illustrates the high correlation groups in wages series. Each oval, numbered 1 to 115, represents a wage series. Connecting lines between ovals indicate that the correlation between those series is greater than 0.8. Lines showing correlations between group members have been omitted for clarity.

These high correlation groups are partly due to the nature of the relationships between series. Several series are subsets of other series. Several series overlap over large portions of the observations. For example, the non-manufacturing group for wages is comprised of the 16 series listed below. Each of these series has a correlation of at least $r=0.9$ with each other

series in the group. Each series also has substantial areas of overlap with other series in the group.

Non-Manufacturing Group

Series Name	% of all observations	Avg SE
All Civilian	100%	0.151
Civilian White Collar	58%	0.220
Civ. Service Producing	78%	0.203
Civ. Non-Manufacturing	84%	0.187
All Private Industry	92%	0.185
Private White Collar	53%	0.282
Private Sales	9%	1.367
Priv. Service Producing	70%	0.270
Private Service Producing White Collar	47%	0.345
Private Finance, Insurance, Real Estate	13%	1.448
Priv. Non-Manufacturing	76%	0.245
Priv. Non-Manufacturing White Collar	48%	0.336
Private Non-Union	78%	0.198
Private Non-Union Non-Manufacturing	66%	0.252
Private Non-Union Service Producing	62%	0.268
Priv. Metropolitan Areas	79%	0.190

Notice that the two smallest series by percentage of observations, ‘Private Sales Occupations’ and ‘Private Finance, Insurance and Real Estate,’ are the series with the highest standard errors. The magnitude of the standard error estimates in these series is much higher than would be expected from size differences alone. The variances of the observations in these two smaller series are influential in the standard errors trends of all series containing them. The volatility in these series is due to high percentages of workers receiving incentive based pay in these series. Series which exclude many incentive pay workers, such as ‘excluding sales’ series, tend to be much less volatile and better reflect changes in fixed pay in the industry. More details are available in Barkume and Moehrl (2001).

2.3 Trends

Trends in standard errors may originate from several possible sources including: changes in industry behavior, changes in collection and design, and properties of the estimator. One

important aspect of the estimator is the use of cost weights. The cost weights used in the index are the product of the cost weight of the previous quarter and the estimate of quarterly change. Since cost weights are chained to the base period, variability may be increasing each quarter from the base period. Several changes have been made to ECI collection and design since 1997 that would be expected to produce a trend in standard errors. The most noticeable of these changes are a change to an area based design and a significant increase over time in the number of observations collected. Both changes were made gradually over a five year period. The extent to which the effects of these changes cancel each other is difficult to measure. More information is available in Paben (2001). Several series appear to have decreasing standard errors but a decreasing trend is not always apparent or significant. Figure 5: Benefits Series 32: 'Private Industry Service Occupations' shows one such series.

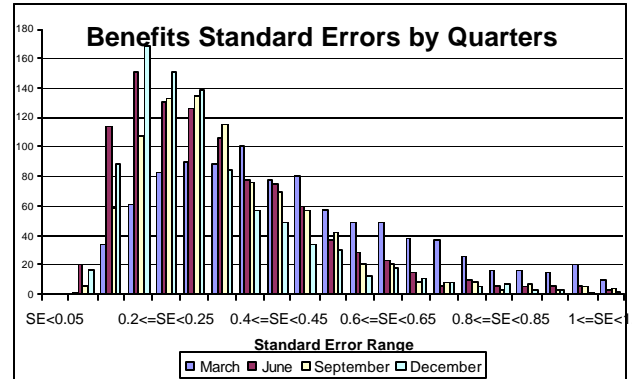
The standard error estimates can be considered as a non-stationary time series. Dependence on previous values can be seen in trends in the series. Differences in the distributions at different quarters are apparent when the quarters are graphed separately (Figure 2). Certain quarters have markedly different peaks and spreads. In individual series, this difference is evidenced by many large values for autocorrelation lag 4, indicating seasonality. Series with no seasonality are expected to have zero autocorrelation lag 4, with standard deviation of 0.17. Eighty-four series have autocorrelation lag 4 greater than two standard deviations above zero: 11 in wages, 54 in benefits, and 19 in compensation.

2.4 Seasonality

It is well known that many of the ECI index estimates are seasonal. The standard error estimates of many series also show strong seasonality. This is a common feature in macro time series (Jaditz 2000). The different peaks and spreads in the distributions of quarterly data in Figure 2 clearly illustrate a seasonal pattern. Seasonality is also apparent in the graphs of many series such as wages series 113: 'State and Local Government Elementary and Secondary Schools' (Figure 4). Using ANOVA difference of means to test for seasonality, many series showed strong seasonality. This test is not well-suited to time series data since the assumption of independent observations is violated; however,

extremely high magnitudes of the F statistics, suggest seasonality is present. The X-11 program uses probabilities less than one in one thousand to suggest seasonality (Dagum 1999). For this data set, $p=0.001$ corresponds to $F=7$. Forty-five series have F statistics greater than seven (9 wages, 26 benefits, 10 compensation).

Figure 2: Distribution of Standard Errors



In general, standard error estimates from the March quarter tended to be higher than other quarters. This makes sense intuitively; benefit providers normally make changes to contracts and benefit plans at the start of the year and, consequently, standard errors tend to be higher in this quarter. In education series, standard error estimates from the September quarter tended to be higher than other quarters reflecting this industry's tendency to make changes to contracts and benefit plans at the start of the academic year in August.

The properties of the data support three general types of trends: a trended, non-seasonal pattern, a trended and seasonal pattern, and a non-trended, seasonal pattern.

3. Appropriateness of Smoothing Variance Estimates

The original, unsmoothed standard error estimate is best suited for forming a confidence interval around the index estimate. However, with moderate smoothing, confidence interval coverage remains essentially the same. In Valliant's simulation study done with variance estimates in the Consumer Price Index, confidence interval coverage for variance estimates smoothed using loess and supersmoother methods was almost equal to the coverage of the point variance estimates (Valliant 1992).

Smoothing the standard error estimates was undertaken to reduce the volatility of the standard error estimates and to better reflect the movements and values of the true sampling variance. It is important for the smoothed standard error estimates to reflect the properties found in the original un-smoothed standard error estimates, particularly seasonality and trends.

The volatility in the standard error estimates comes from three sources: volatility in the true sampling variance, properties of the estimator, and variance in the BRR variance estimates. Ideally, a smoother would filter out the noise from variance of the variance estimates while preserving the patterns and trends of the true sampling variance and preserving enough noise from other sources to retain adequate confidence interval coverage. The volatility in the true sampling variance and from the estimator is important to reflect possible deviation of the actual percent change from the estimated value caused by the sampling errors.

Some methods commonly used for smoothing variances are generalized variance functions (GVFs), scatterplot smoothers, and weighted moving averages. Some concerns with GVFs and any modeling approaches for variances include the volatility of the data, the relatively short time frame of the data, and changes in collection and design. These features may make selecting an appropriate model extremely difficult. In addition, GVFs are not practical for estimating variance of a Laspeyres Index because the number of parameters to be estimated increases each quarter from the base period (Valliant 1992). General scatterplot smoothers such as Loess and supersmoother, used by Valliant on CPI standard error estimates (1992), are appropriate for this type of data but considering evidence of seasonality, we may be able to better represent the true values and the movement of the true values with weighted moving averages. Weighted moving averages are appropriate for non-stationary time series, and can be used to reduce volatility while retaining trends and seasonality.

4. Development of Exponential Smoothing Procedures for ECI Standard Errors

The distribution of the standard error estimates is strongly skewed right. To account for this, we transformed the data using natural logarithms.

$$X_t = \ln(Y_t)$$

The distribution of the logs of the standard error estimates is roughly normal. Outlier treatment was done on the logged estimates using Winsorization. Any values more than two standard deviations from the series mean were replaced with the value at the second standard deviation.

Although we do not want to use models directly on the data, the three types of models provide a good starting point and guide for developing smoothers. The three models are a trended, non-seasonal pattern, a trended and seasonal pattern, and a non-trended, seasonal pattern. Using the information in these three models, three smoothers can be generated to fit each type, which we will call S_R (regular), S_H (hybrid), and S_S (seasonal), respectively.

The seasonal, non-trended series (S_S), corresponds to the once differenced, moving average process model. This model assumes

$$X_{t+4} - X_t = Z_{t+4} - qZ_t$$

where X_t is the process modeled at time t , Z_t is a noise process satisfying

$$E[Z_t] = 0, E[Z_t^2] = \sigma_z^2, \text{ and}$$

$$E[Z_t Z_{t-k}] = 0 \quad \forall k > 0.$$

The assumption can be rewritten as

$$Z_{t+4} = X_{t+4} - X_t + qZ_t$$

and used to substitute for Z_t

$$Z_{t+4} = X_{t+4} - X_t + q(X_t - X_{t-4} + qZ_{t-4})$$

and again to substitute for $Z_{t-4}, Z_{t-8}, Z_{t-12}$, etc., finally obtaining an infinite sum which simplifies to:

$$Z_{t+4} = X_{t+4} - (1-q)[X_t + qX_{t-4} + q^2X_{t-8} + \dots]$$

This can be rearranged as a formula for projecting:

$$X_{t+4} = Z_{t+4} + (1-q)[X_t + qX_{t-4} + q^2X_{t-8} + \dots]$$

The following correction can be used to apply this formula to a finite set of points

$$X_{t+4} = Z_{t+4} +$$

$$\frac{(1-q)}{(1-q^n)} [X_t + qX_{t-4} + q^2X_{t-8} + \dots + q^{n-1}X_{t-4n}]$$

Equivalently, this can be written as a weighted mean.

$$X_{t+4} = Z_{t+4} +$$

$$\frac{X_t + qX_{t-4} + q^2X_{t-8} + \dots + q^{n-1}X_{t-4n}}{1 + q + q^2 + \dots + q^{n-1}}$$

This formula projects X_{t+4} using previous points; however, since

$$E[X_{t+4}] = E[X_t + Z_{t+4} - qZ_t] = X_t,$$

the same formula can be used to smooth the current point X_t . Using this formula for the current time is preferable since it uses all available data.

$$\hat{X}_t = Z_{t+4} + \frac{X_t + qX_{t-4} + q^2X_{t-8} + \dots + q^{n-1}X_{t-4n}}{1 + q + q^2 + \dots + q^{n-1}}$$

When transforming the smoothed data from logs back to its original form, an adjustment must be

made for Z_{t+4} since $E[e^Z] = e^{\frac{1}{2}s^2}$

Z_t can be estimated by rearranging the forecast equation as

$$Z_{t+4} = X_{t+4} - \frac{X_t + qX_{t-4} + q^2X_{t-8} + \dots + q^{n-1}X_{t-4n}}{1 + q + q^2 + \dots + q^{n-1}}$$

and the variance of Z_t , s^2 , can be calculated from this series and used to adjust the smoothed, transformed data.

The final smoother is of the form

$$\hat{Y}_t = e^{-\frac{1}{2}s^2} e^{\frac{X_t + qX_{t-4} + q^2X_{t-8} + \dots + q^{n-1}X_{t-4n}}{1 + q + q^2 + \dots + q^{n-1}}} \quad (S_S)$$

A similar weighted average, can be developed for trended, non-seasonal series using the assumption $X_{t+1} - X_t = Z_{t+1} - qZ_t$. The resulting smoother is of the form

$$\hat{Y}_t = e^{-\frac{1}{2}s^2} e^{\frac{X_t + qX_{t-1} + q^2X_{t-2} + \dots + q^{n-1}X_{t-n}}{1 + q + q^2 + \dots + q^{n-1}}} \quad (S_R)$$

And a hybrid can be used for series that exhibit some seasonality and some trend. The hybrid (S_H) is the geometric mean of the seasonal smoother (S_S) and a modified version of the non-seasonal smoother (S_R) using all seventeen data points in the time interval used in S_S instead of only the most recent five data points used in S_R . The hybrid smoother (S_H) appears to be the most widely applicable and frequently gives the best results.

Choosing Parameters

Two parameters need to be chosen for these models: q and n . Research suggests that parameters between $q = 0.75$, for fast smoothing done on shorter time series, and $q = 0.9$, for normal smoothing, are most effective (Gardner 1985). Since all parameters within this range seem to give acceptable results, we chose to use $q = 0.8$ for all series. We chose $n = 4$. Any points beyond $n = 4$ would each contribute to less than 10% of the value. This parameter value has an added benefit for series with ambiguous evidence of seasonality. Since S_R includes five data points, two from the present quarter and one from each other quarter, S_R will show some seasonality for seasonal series. Parameters do not need to be recalculated for each series; $q = 0.8$ and $n = 4$ should give acceptable results for any series.

5. Comparison of Smoothers

5.1 Some Instances

The following graphs show the results of four smoothers: the five-year moving average, currently in use in the ECI program, and smoothers S_R , S_H , and S_S discussed above. To compare the smoothers, four statistics were calculated: relative Mean Square Error (relMSE), relative Mean Error (relME), Autocorrelation Lag 4 of the errors (AC4), and Autocorrelation Lag 1 of the errors (AC1). These statistics are computed using the following formulas:

$$relMSE = \frac{1}{n} \sum \left(\frac{\hat{y}_t - y_t}{y_t} \right)^2,$$

$$relME = \frac{1}{n} \sum \frac{\hat{y}_t - y_t}{y_t}$$

$$AC4 = \frac{\sum (e_t - \bar{e})(e_{t+4} - \bar{e})}{\sum (e_t - \bar{e})^2} \quad (\text{standardized}),$$

$$AC1 = \frac{\sum (e_t - \bar{e})(e_{t+1} - \bar{e})}{\sum (e_t - \bar{e})^2} \quad (\text{standardized}),$$

$$\text{where } e_t = \hat{y}_t - y_t, \quad \bar{e} = \frac{1}{n} \sum e_t$$

These statistics describe proximity to the original series and the amounts of seasonality

and trend not included in the smoothed series. $relMSE$ and $relME$ provide a measure of the proximity of the original and the smoothed series. $AC4$ and $AC1$ provide a measure of the amount of seasonality and trend left out of the smoothed series. Differences in $AC4$ or $AC1$ values for the errors of smoothers in a particular series indicate which smoothers are more and less effective at including seasonality or trend. $relME$ can also be used to find the average change in confidence interval coverage. For example, if the original standard error estimate was correct, then $relME + 1 = 0.78 = \frac{1.282}{1.645}$ would indicate that the 90% CI ($z=1.645$) formed using the smoothed estimate would actually be an 80% CI ($z=1.282$) using the original estimate.

Figure 3: Standard Errors and Smoothed Standard Errors

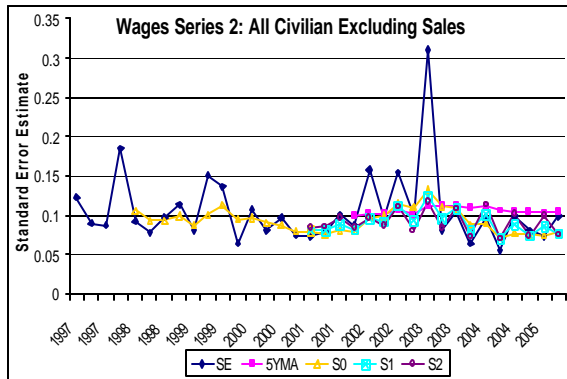


Figure 3: Standard Errors and Smoothed Standard Errors of Wages Series 2: ‘All Civilian Excluding Sales’ illustrates how each smoother handles outliers. The five year moving average rises only slightly in the quarter with the outlier but remains inflated, noticeably higher than the original standard error estimates for the next several years. S_R , S_H , and S_S all dampen the effect of the outlier in the quarter in which it occurs but are not greatly influenced by it in the following quarters.

Wages Series 2: All Civilian Excluding Sales

	S_R	S_H	S_S	5YMA
relMSE	0.08182	0.07354	0.07241	0.1618
relME	-0.0640	-0.0758	-0.0827	0.12970
AC4	1.1230	0.6340	0.2963	0.9587
AC1	-0.9209	-0.1145	0.4883	-0.1190

For the Civilian Excluding Sales Series, the high values for $AC4$ indicate that S_R and

$5YMA$ did not sufficiently capture seasonality. The two remaining smoothers, S_H and S_S are very similar in $relMSE$ and $relME$ which, if the original standard error estimate was correct, would correspond to about 86% CIs. There is a tradeoff between seasonality and trend in these two smoothers; S_S captures more seasonality but less trend and S_H captures less seasonality but more trend. Both S_H and S_S appear to be appropriate smoothers for this series.

Figure 4: Standard Errors and Smoothed Standard Errors

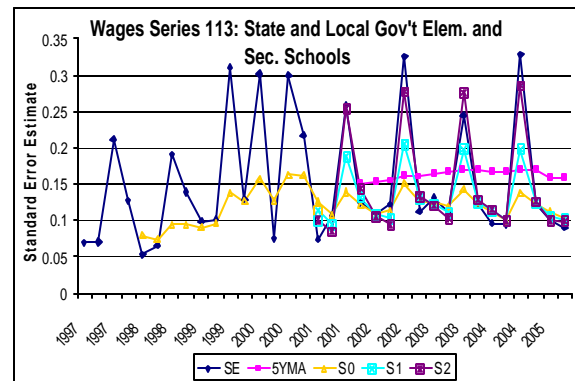


Figure 4: Standard Errors and Smoothed Standard Errors of Wages Series 113: ‘State and Local Government Elementary and Secondary Schools’ shows how each smoother handles seasonality. In the five year moving average, seasonality is entirely smoothed out. The resulting smoothed values are in the middle of the data as a whole but are never near the original estimate at a specific point in time. S_R , S_H , and S_S are better at capturing the magnitudes of the standard errors and, to increasing degrees, capture the strong seasonality present in this series. The exponential smoothers, particularly S_S , are overall much closer to and better reflect the movements of the original estimates.

Wages Series 113: State and Local Government Elementary and Secondary Schools

	S_R	S_H	S_S	5YMA
relMSE	0.07577	0.03785	0.01677	0.2488
relME	0.00218	-0.0150	-0.0070	0.2928
AC4	2.205	1.507	0.1394	2.171
AC1	-0.7091	-0.4552	0.4654	-0.5171

For this series, trends in relMSE and AC4 clearly indicate S_S as the optimal smoother for this series. RelMSE and AC4 values both decrease as the amount of seasonality included in the smoother increases. AC1 and relME values are all reasonably small, except for the relME of the five year moving average. Considering the seasonality apparent in this series, S_S appears to be the only appropriate smoother for this series.

Figure 5: Standard Errors and Smoothed Standard Errors

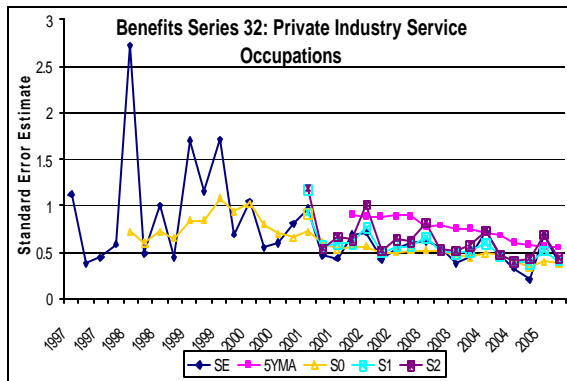


Figure 5: Standard Errors and Smoothed Standard Errors of Benefits Series 32: ‘Private Industry Service Occupations’ shows how each smoother handles a decreasing trend. The five year moving average decreases steadily, clarifying the movement in the original estimates but the values are noticeably higher than the original estimates throughout the entire time period. The three exponential smoothers remain close in value to the original estimates. S_S decreases jaggedly and tends to over-emphasize seasonal peaks. S_R decreases smoothly and steadily and tends to under-estimate seasonal peaks. S_H captures the correct magnitude for the seasonal peaks and overall, is remarkably close to the original estimates.

Benefits Series 32: Private Industry Service Occupations

	S_R	S_H	S_S	5YMA
relMSE	0.1085	0.1249	0.1730	0.5676
relME	0.0898	0.1717	0.2581	0.5662
AC4	0.8957	0.4685	0.4472	1.089
AC1	-0.3271	-0.3724	-0.5046	-0.5393

For this series, except for the five year moving average which is farthest from the

original series, relMSE and relME are increasing with the amount of seasonality included in the smoothers. This is due to the fact that the seasonal smoothers use data from a longer time span. AC1 values are all reasonably small. The high values for AC4 for S_R and 5YMA indicate that these smoothers were less effective at capturing seasonality. There is a tradeoff between proximity and seasonality; S_R is closer to the original series but leaves out more seasonality, and S_S is farther from the original series but includes more seasonality. S_H appears to be the best smoother for this series.

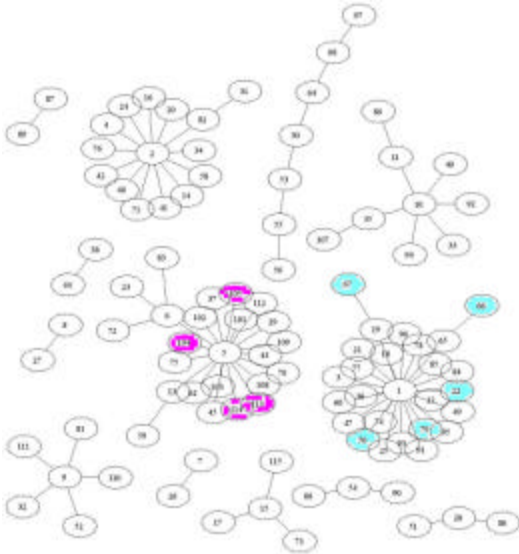
5.2 Overall Results

All three smoothers are very easy to compute and perform far better than the 5YMA. In wages series, S_S is clearly the best smoother for about a tenth of the series and S_R seems to be the best smoother for about a tenth of the series but for most series, selecting a smoother requires making some tradeoffs between desirable traits. S_H is often good at capturing the desirable traits in both S_R and S_S and is an effective smoother for almost every wages series. In benefits series, S_S is clearly the best smoother for about a quarter of the series, S_H is clearly the best smoother for about a tenth of the series. Of the remaining benefits series, most involve trade-offs between smoothers. S_S is adequate for almost every benefits series and S_R is almost always the worst smoother. Overall, using S_H for all wages series and S_S for all benefits series seems adequate.

Figure 6: Diagram of Correlations between Wages Series Smoothed with S_H shows the effects of the hybrid smoother on high correlation groups. Similar to figure 1, each oval, numbered 1 to 115, represents a series smoothed using the hybrid smoother and a connecting lines between ovals indicate that the correlation between those series is greater than 0.8. Notice that three of the four prominent groups increased in size and several of the smaller groups linked to form larger groups. After smoothing, several government series (pink) formerly not in any group were annexed into the government group, which suggests that some noise has been removed and underlying patterns are more visible. The changes in the excluding and non-excluding sales series also suggest that noise has

been removed from these series. Sales occupations were excluded because they add a great deal of volatility and noise to the estimates. Without this noise, series including sales would be expected to be similar to the corresponding excluding sales series. After smoothing, portions of the non-manufacturing including sales and excluding sales groups merged. Excluding sales series that merged with the non-manufacturing series including sales are shown in blue. Both the increase in group size and nature of the groups added indicate that the hybrid smoother was effective in removing noise and clarifying the underlying patterns in series.

Figure 6: Diagraph of Correlations between Wages Series Smoothed with S_H



6. Summary

Many ECI standard error estimate series exhibit frequent outliers, trends, and seasonality. Exponential smoothers are effective in reducing the noise in the estimates without jeopardizing confidence interval coverage. Exponential smoothers are also theoretically sound, easily calculated, and effective in preserving the trends and seasonality.

7. Topics for Further Investigation

NCS is in the process of implementing two new procedures: Fay's Method for BRR and Series excluding workers with incentive pay which will eventually replace the 'excluding sales' series. We would like to analyze the

effects of these changes on the standard error estimates.

In addition, we would like to investigate if improvements are possible on these smoothers. The hybrid smoother could be adapted to better suit individual series using a weighted geometric average with weights dependent on a measure of seasonality and parameters could be chosen specifically for each series. It would be interesting to measure the effects of these adaptations to the smoothers developed in this paper.

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