ESTIMATION AND ANALYSIS OF VARIANCE COMPONENTS FOR THE REVISED CPI HOUSING SAMPLE

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In 1999, the Bureau of Labor Statistics (BLS) introduced a new Housing Sample for the Rent and Rental Equivalency (REQ) estimators in the U.S. Consumer Price Index (CPI). The Housing Sample consists of nearly 10,000 sampled segments, composed of U.S. Census blocks, allocated in 87 Primary Sampling Units (PSUs), and collected in six panels every six months. In this paper, we model the 6-month price relative for both Rent and REQ, and analyze a random effects model that treats PSU and Segment as two random effects. We look at three years of data (1999-2001). We use the Restricted Maximum Likelihood (REML) estimation method to produce the variance components. Then standard likelihood ratio test procedures are applied to determine the significance of the random effects in the model. Finally, the variance component results are compared to a set of variance components produced from the previous housing sample (1987-1998).

1. The Housing Sample Design

In January 1999, the BLS produced indexes for Rent at Primary Residence (Rent) and for Owner's Equivalent Rent at Primary Residence (REQ) using a completely new housing sample for the first time in ten years. A new Commodities and Services (C&S) sample was being rotated in, beginning in January, 1998, using a six-month rotation schedule that would be completed at the end of four years. The C&S sample design is separately derived from TPOPS (Telephone Point of Purchase Survey) frames and comprises 72.5%[1] of the CPI, as measured in expenditure shares. The remaining 27.5% is accounted for by the two Housing indexes (Rent and REQ). Both C&S and Housing are, however, sampled and priced in the same set of PSUs (i.e., cities).

The first stage of the overall design is the PSU sample selection. This stage is common to both Housing and C&S. The CPI survey is conducted in 87 PSUs. The 31 largest A-level PSUs are selected with certainty. The 56 smaller (B- and C-level) PSUs are then selected with probability proportional to size[2] (pps) within their respective regions: Northeast, Midwest, South or West. The two sets of Housing components of variance (one for Rent, one for REQ), will, however, be calculated at the All-US level, effectively treating PSU (using all 87 PSUs) as a random effect.

The second stage of sampling for Housing is segment selection, again by pps, with size here meaning total housing expenditures (for Rent and REQ within each segment). The segments, which are contiguous U.S. Census blocks, are then selected within fixed geographic strata within each PSU. Since we are producing All-US level components of variance, the fixed effect for the strata will be left out of the model. (The fixed effect for strata only has meaning within a given PSU.) From the nearly 10,000 sampled segments throughout the U.S., we utilize approximately 1500 for each monthly panel. The segments are assigned to six collection panels with each panel being collected two times a year. The segments constitute the second random effect, and thus the second component of variance for Rent and REQ. A residual (or within segment) component rounds out the set of variance components.

2. The Random Effects Model

The model we use is a two-way random effects model. The design is unbalanced. We let y_{ijk} be the observed unit price relative between time t and time

[1] Bureau of Labor Statistics, *CPI Detailed Report (Dec 2000)*, p. 16.

[2] Size here equals population.

t-6 (months), for housing unit i, PSU j, and Segment k.

$$Y_{ijk} = \mu + p_j + s_k + e_{ijk},$$

where μ is a fixed effect

$$p_j \sim N(0, \sigma^2_{\text{unit, psu}})$$
$$s_k \sim N(0, \sigma^2_{\text{unit, segment}})$$
$$e_{ijk} \sim N(0, \sigma^2_{\text{unit, error}})$$

with p, s, and e all independent of each other.

This random effects model allows us to investigate the covariance structure of a fuller model (say, to investigate stratum effects at the PSU level) to see whether these random effects need to be kept in the fuller model, so as to better accommodate normality issues. Normality assumptions are always a problem when working with the distribution of price relatives. These more refined and complicated covariance structures, where the effects are found to be significant [see below], can adjust and may correct some of these normality problems.

Table 1

RENT Variance Components 1999-2001

Time	**VC**	**Panel 1**	**PCT**	**Panel 2**	**PCT**	**Panel 3**	**PCT**	**Panel 4**	**PCT**	**Panel 5**	**PCT**	**Panel 6**	**PCT**
	PSU	0.0001	3%	**0.0000**	**1%**	0.0001	1%	**0.0000**	**0%**	0.0000	1%	0.0001	2%
Jan '99-	**SEG**	0.0005	12%	0.0002	5%	0.0003	7%	0.0004	12%	0.0004	11%	0.0004	11%
Jun '99	**RES**	0.0035	85%	0.0040	94%	0.0044	92%	0.0031	88%	0.0032	88%	0.0033	87%
	Tot	0.0042		0.0042		0.0048		0.0036		0.0037		0.0038	
	PSU	**0.0000**	**1%**	0.0001	1%	0.0001	1%	0.0001	1%	0.0003	4%	0.0001	2%
Jul '99-	**SEG**	**0.0002**	**3%**	0.0008	13%	0.0006	10%	0.0005	7%	0.0012	18%	0.0003	6%
Dec '99	**RES**	0.0054	96%	0.0055	86%	0.0050	89%	0.0059	92%	0.0050	78%	0.0049	92%
	Tot	0.0056		0.0064		0.0056		0.0064		0.0064		0.0053	
	PSU	**0.0000**	**0%**	**0.0000**	**0%**	0.0000	1%	**0.0001**	**1%**	**0.0000**	**0%**	0.0001	1%
Jan '00-	**SEG**	0.0002	3%	0.0002	3%	0.0002	3%	0.0008	6%	0.0021	24%	0.0005	8%
Jun '00	**RES**	0.0074	96%	0.0062	97%	0.0084	97%	0.0122	93%	0.0068	76%	0.0058	91%
	Tot	0.0077		0.0064		0.0087		0.0131		0.0089		0.0064	
	PSU	0.0002	1%	0.0000	1%	0.0001	1%	0.0002	2%	0.0002	2%	0.0002	3%
Jul '00-	**SEG**	0.0006	5%	0.0004	7%	0.0004	4%	0.0017	15%	0.0005	6%	0.0012	14%
Dec '00	**RES**	0.0100	93%	0.0060	93%	0.0096	96%	0.0090	83%	0.0082	93%	0.0072	83%
	Tot	0.0107		0.0065		0.0100		0.0109		0.0089		0.0087	
	PSU	0.0001	1%	0.0002	2%	0.0003	3%	0.0001	1%	0.0002	3%	0.0001	2%
Jan '01-	**SEG**	0.0003	5%	0.0008	9%	0.0007	9%	0.0005	4%	0.0005	7%	0.0009	12%
Jun '01	**RES**	0.0066	94%	0.0074	89%	0.0067	88%	0.0114	95%	0.0064	90%	0.0067	86%
	Tot	0.0070		0.0084		0.0077		0.0120		0.0072		0.0078	
	PSU	0.0002	2%	**0.0001**	**0%**	0.0001	1%	0.0001	1%	0.0002	2%	**0.0001**	**1%**
Jul '01-	**SEG**	0.0003	3%	0.0007	5%	**0.0001**	**2%**	0.0012	13%	0.0009	11%	0.0015	9%
Dec '01	**RES**	0.0095	94%	0.0129	94%	0.0088	97%	0.0078	86%	0.0075	87%	0.0148	90%
	Tot	0.0101		0.0137		0.0091		0.0090		0.0087		0.0164	

BOLD = NOT SIGNIF at an α = **.05** level

Table 2

REQ Variance Components 1999-2001

Time	VC	Panel 1	PCT	Panel 2	PCT	Panel 3	PCT	Panel 4	PCT	Panel 5	PCT	Panel 6	PCT
	PSU	0.0001	3%	0.0000	1%	0.0001	1%	**0.0000**	**0%**	0.0000	1%	0.0001	2%
Jan '99-	**SEG**	0.0003	7%	0.0005	9%	0.0004	9%	0.0004	11%	0.0003	8%	0.0006	14%
Jun '99	**RES**	0.0044	90%	0.0047	90%	0.0040	90%	0.0034	89%	0.0041	92%	0.0036	84%
	Tot	0.0048		0.0052		0.0044		0.0038		0.0045		0.0043	
	PSU	0.0001	1%	**0.0000**	**1%**	0.0001	2%	0.0001	1%	0.0002	3%	0.0001	2%
Jul '99-	**SEG**	**0.0000**	**1%**	0.0013	20%	0.0006	9%	0.0004	6%	0.0007	9%	0.0003	6%
Dec '99	**RES**	0.0059	98%	0.0052	79%	0.0061	89%	0.0065	93%	0.0061	88%	0.0047	92%
	Tot	0.0060		0.0065		0.0068		0.0070		0.0070		0.0051	
	PSU	**0.0000**	**0%**	0.0001	1%	**0.0000**	**0%**	0.0002	1%	0.0001	1%	0.0001	2%
Jan '00-	**SEG**	0.0003	3%	0.0002	3%	0.0017	16%	0.0008	5%	0.0005	8%	0.0007	9%
Jun '00	**RES**	0.0078	97%	0.0069	95%	0.0087	84%	0.0138	94%	0.0060	91%	0.0064	89%
	Tot	0.0081		0.0073		0.0103		0.0148		0.0066		0.0072	
	PSU	0.0001	1%	0.0000	1%	**0.0001**	**0%**	0.0002	1%	0.0002	2%	0.0002	2%
Jul '00-	**SEG**	0.0006	6%	0.0004	6%	0.0005	3%	0.0026	16%	0.0007	6%	0.0013	14%
Dec '00	**RES**	0.0097	93%	0.0066	93%	0.0139	96%	0.0135	83%	0.0099	92%	0.0079	84%
	Tot	0.0104		0.0070		0.0145		0.0163		0.0109		0.0093	
	PSU	0.0001	1%	0.0002	2%	0.0002	3%	0.0001	1%	0.0001	2%	0.0002	2%
Jan '01-	**SEG**	0.0008	10%	0.0013	15%	0.0007	7%	0.0005	3%	0.0008	10%	0.0009	9%
Jun '01	**RES**	0.0076	89%	0.0074	83%	0.0084	90%	0.0134	96%	0.0073	88%	0.0085	89%
	Tot	0.0086		0.0089		0.0093		0.0140		0.0083		0.0095	
	PSU	0.0002	2%	0.0003	2%	0.0003	3%	0.0001	1%	0.0003	3%	0.0002	1%
Jul '01-	**SEG**	0.0005	5%	0.0016	10%	0.0004	4%	0.0007	6%	0.0009	9%	0.0015	7%
Dec '01	**RES**	0.0096	93%	0.0146	88%	0.0095	93%	0.0102	92%	0.0087	88%	0.0203	92%
	Tot	0.0103		0.0166		0.0102		0.0110		0.0099		0.0220	

BOLD = NOT SIGNIF at an α = **.05** level

Our random variable is a six-month price change for Rent and REQ respectively (and separately). However, only rent prices are collected (observed). Then each rent price relative moves a given rental equivalent (REQ), except where there are rent controls or other special cases. The Rent and REQ prices then receive segment level weights (based on total housing expenditures, which are then proportioned by percent renter in the given segment), for use in their PSU-level price relative calculations. At the unit level, the level we are interested in, the weights drop off, since there will be the same one weight in the numerator of the unit price relative as in its denominator. Thus, we cannot expect much appreciable differences in the components of variance for REQ versus Rent in any given time frame.

The housing sample splits each PSU into six geographic strata within which, as was mentioned, the sample segments are drawn. For our component of variance analyses we can ignore this stratification. The housing sample units are then distributed into six panels with a current and a t-6 month rent price collected in each panel. And we will be interested in noting whether or not there are any apparent panel effects in our variance components. We then run All-US models over a 36-month period.

The methodology we have chosen for estimating the components of variance is unweighted Restricted Maximum Likelihood (REML). The REML method accommodates unbalanced designs and will not produce negative components of variance. REML estimation finds maximum likelihood estimates of variance components from the distribution of the residuals. Normality is a robust assumption under this structure. Moreover, REML estimation automatically corrects for fitting any fixed effect parameter, including μ. SAS uses REML as its default estimation methodology for linear mixed models.

3. Analysis of Results

For the three years of Rent and REQ data, the overall results are strikingly similar. But, as noted above, the REQ index is directly moved by the Rent price relatives, and since we are utilizing unit level housing units we do not apply weights, which are appreciably different between Rent and REQ (owned housing units for which a rental equivalency is estimated). Overall, in Rent, the PSU component of variance averaged 1.4% of the total variance, while in REQ the same component of variance averaged 1.5% of the total. Similarly, in Rent the Segment component of variance averaged 8.4% while in REQ the same component of variance averaged 8.3% of the total, leaving an average residual variance share of 90.2% for both Rent and REQ.

Sample sizes for these 36 (times 2) model runs ranged from 2300 to 4000 observations in Rent, and from 2500 to 4200 in REQ. Sample augmentation occurred across the time in these models. The sample sizes for REQ run slightly higher than for Rent due to the fact that certain "helper" segments are added to the sample and these "helper" segments only augment owners (i.e., REQ units) and not renters. The number of PSUs remained at 87 throughout. The number of segments per model averaged around 1200. Thus there were plenty of degrees of freedom in all of the models. Moreover, a check with CPI's production variances for Rent and REQ will show that these components of variances, viewed as total variances, are quite comparable in magnitude, with these model variance totals coming in half to three-quarters as big as their CPI counterparts.

In order to test the significance of these components of variance in the models, we had several choices. The most robust (and most conservative) results were found by relying on the REML model structure and directly applying a likelihood ratio test to the REDUCED versus FULL model. We first took out PSU and compared the -2 Res Log Likelihoods (which are provided in the SAS output), and then we took out Segment and compared its two –2 Res Log Likelihood results. We compared the fit of the two models (REDUCED versus FULL) under the null hypothesis H_o: $\sigma^2_{component} = 0$. The likelihood ratio statistic is then REDUCED – FULL, which has a chi-square distribution with 1 degree of freedom. The probability of the test statistic being greater than the $\chi^2(1)$ is its corresponding p-value (only divided by 2 to compensate back for the original –2 computation). To exemplify, in 200109, for Rent, the Segment component of variance is *not* significant at the $\alpha = .05$ level, whereas the 2000109 REQ Segment component of variance is significant.

RENT: Reduced – Full =
-7719.3 - -7721.3 = 2.0
$\Rightarrow$ P-value = 0.0786

REQ: Reduced – Full =
-8034.1 - -8045.1 = 11.0
$\Rightarrow$ P-value = 0.00046

Using this ratio likelihood test, we found nearly all the Segment variance components highly significant. Only two Rent and only one REQ Segment component of variance was *not* significant at the α = .05 level. Clearly, the segment effect is important and needed in the models. For the PSU components of variance, fully 75% of the Rent models showed PSU to be a significant effect, while 86% of the REQ models showed PSU to be significant at the α = .05 level. So, even though the percentage of the total variance was quite small (1.4% and 1.5%, respectively), the effect in the model was deemed significant the large majority of the times.

4. Comparative Results with 1991-1992 Variance Components

A comparison of the previous housing sample, in terms of variance components, revealed a smaller percentage number of significant PSU components (just only over half are significant at the $\alpha = .05$ level). The sample sizes were somewhat higher on average (although that could be attributed to the pre-augmentation stage of the newer housing sample) and the number of sampled segments were on average about 10% greater. The total variances in the models from this older sample were also, on average, larger but the percentage breakdowns were fairly similar. The new sample did seem to be lowering the overall variance. (This lowering can be seen in the production CPI variances too.)

Table 3

RENT 1991-1992

Time	VC	Panel 1	PCT	Panel 2	PCT	Panel 3	PCT	Panel 4	PCT	Panel 5	PCT	Panel 6	PCT
	PSU	0.0002	2%	**0.0001**	**1%**	0.0001	1%	0.0002	2%	**0.0000**	**0%**	**0.0000**	**0%**
Jan '91-	**SEG**	0.0007	6%	0.0017	15%	0.0012	11%	0.0006	6%	0.0013	12%	0.0009	7%
Jun '91	**RES**	0.0107	92%	0.0092	84%	0.0093	88%	0.0085	91%	0.0094	88%	0.0108	92%
	Tot	0.0116		0.0109		0.0106		0.0093		0.0107		0.0117	
	PSU	0.0001	1%	**0.0001**	**1%**	0.0002	1%	0.0002	1%	**0.0001**	**1%**	0.0001	1%
Jul '91-	**SEG**	0.0008	8%	0.0031	26%	0.0017	14%	**0.0003**	**1%**	0.0016	12%	0.0019	13%
Dec '91	**RES**	0.0086	91%	0.0085	73%	0.0101	84%	0.0184	98%	0.0114	87%	0.0124	86%
	Tot	0.0095		0.0116		0.0120		0.0188		0.0131		0.0144	
	PSU	**0.0001**	**1%**	**0.0001**	**1%**	0.0001	1%	0.0001	1%	0.0001	1%	**0.0001**	**0%**
Jan '92-	**SEG**	0.0017	14%	0.0026	23%	0.0015	13%	0.0013	13%	0.0008	11%	0.0008	8%
Jun '92	**RES**	0.0103	85%	0.0086	76%	0.0097	86%	0.0089	86%	0.0061	88%	0.0099	92%
	Tot	0.0121		0.0112		0.0112		0.0103		0.0069		0.0108	
	PSU	**0.0000**	**0%**	0.0001	1%	**0.0000**	**0%**	**0.0000**	**0%**	0.0004	5%	0.0001	1%
Jul '92-	**SEG**	0.0016	19%	0.0013	15%	**0.0007**	**4%**	0.0010	8%	0.0013	13%	0.0007	8%
Dec '92	**RES**	0.0066	80%	0.0074	84%	0.0153	95%	0.0111	91%	0.0081	82%	0.0080	92%
	Tot	0.0082		0.0088		0.0160		0.0121		0.0098		0.0088	

BOLD = NOT SIGNIF at an α = **.05** level

5. Conclusion

The use of segments in the sampling process has been shown to be a significant effect in almost all of these models. However, the small but generally significant PSU effect is the more interesting result. Even though all the PSUs are employed in these models and not broken out by A- and B- and C-size classes, it is still important to note that, at the All-US level, the PSU effect is significant and therefore remains important to the overall sampling structure.

6. Acknowledgements

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7. References

Bureau of Labor Statistics, *CPI Detailed Report (Dec 2000)*, Washington, DC, U.S. Government Printing Office.