Recent and Noteworthy Sequences in the OEIS®

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Abstract[1].

An illustrated account of some new and noteworthy additions to the *On-Line Encyclopedia of Integer Sequences*® (or *OEIS*®), concentrating on sequences that are associated with attractive unsolved problems.

1 Introduction.

The *On-Line Encyclopedia of Integer Sequences* (or *OEIS*) has existed in various forms since it was started by the author in 1964. Since 2009 it has been owned and maintained by *The OEIS Foundation* [14], and since November 2010 it has been on the web as a Wiki [15]. It presently contains about 220000 sequences. New sequences arrive every day. Some come with a complete analysis, giving formulas, asymptotic estimates, computer programs, references, etc. Some, on the other hand, are such that one says "That is a really lovely problem and I wish I had time to work on it". This paper will describe a dozen or so sequences of the latter type. Please update the corresponding entry in the OEIS if you make progress on any of them!

2 Toothpick structures and the snowflake sequence.

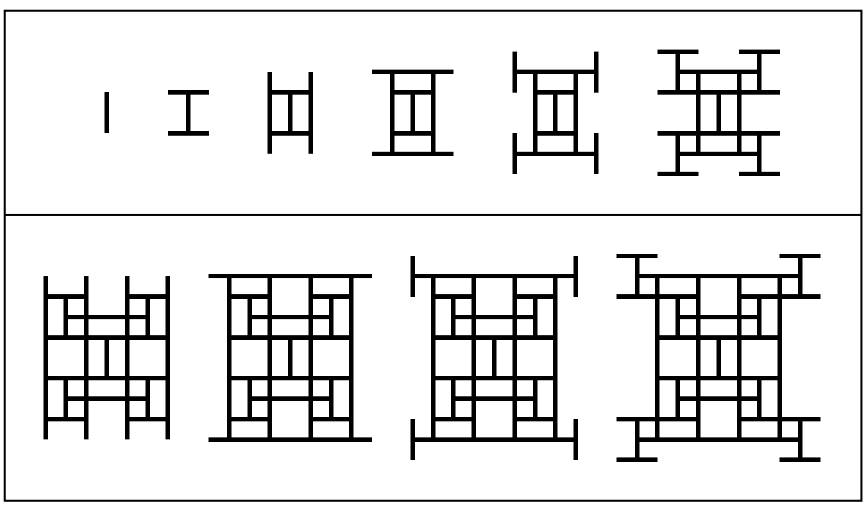

Figure 1: Beginning of the evolution of Omar Pol's toothpick structure. The numbers of toothpicks in stages 1 through 10 are $1, 3, 7, 11, 15, 23, 35, 43, 47, 55$ (A139250).

[1] Prepared for distribution at the AMS/MAA Joint Mathematics Meetings in San Diego, Jan. 2013. For a version with larger illustrations but without the text, see [20].

Omar E. Pol (in Buenos Aires) has contributed many interesting sequences to the OEIS. His "toothpick sequence" [16] was one that was impossible to ignore. Start with a large supply of toothpicks (line segments of length 1) and a large sheet of graph paper. At stage 1, place a single toothpick on the paper, aligned with the y-axis. At each subsequent stage, for every exposed toothpick end, place a perpendicular toothpick centered at that end. The resulting structure has a fractal-like appearance (see Fig. 1). The problem is to find $t(n)$, the number of toothpicks present after n stages. David Applegate, Omar Pol and I made a complete analysis [2], giving a recurrence, generating function and, finally, an explicit formula for $t(n)$. David Applegate has made an animation of the growth of the toothpick structure, which is well worth watching (it is one of the top links in A139250, labeled "Movie version").

That problem is completely solved. However, there are many variants [19] that have no known recurrences and are not at all well understood. Some of them can be seen in Applegate's animation, for example A160120, which is concerned with Y-shaped "toothpicks."

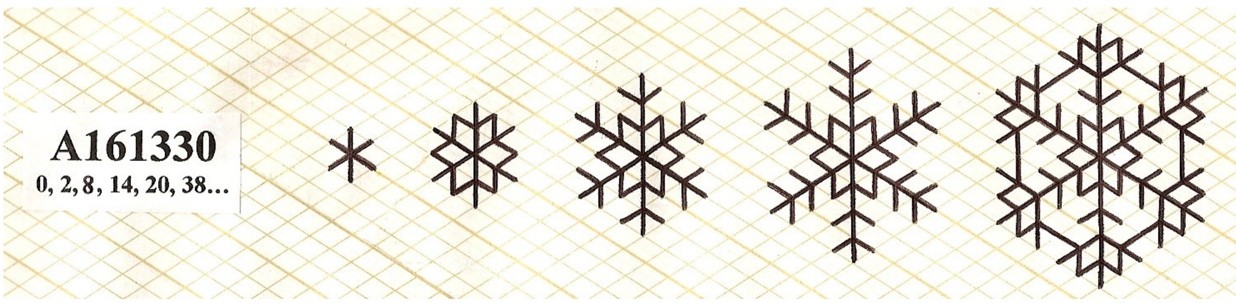

Figure 2: Omar E. Pol's illustration of the first five stages of the E-toothpick (or snowflake) sequence A161330. The first stage consists of two E-toothpicks back-to-back.

The following sequence, however, is not illustrated in the animation and has several attractive features: it has hardly been studied at all, no recurrence is known, and the pictures are appropriate for this season of the year (see Fig. 2). It is another of Omar Pol's sequences, and is based on E-shaped "toothpicks," which look like a bird's footprint:

We start with two E-toothpicks placed back-to-back on triangular graph paper, forming a six-pointed star (Fig. 2). The rule for adding new E-toothpicks is as follows. Each E has three ends, which initially are free. If two ends of two E's meet, those ends are no longer free. To go from stage n to stage $n+1$, we add an E-toothpick at each free end in the structure (extending that end in the direction it is pointing), provided no end of a new E touches an end of an existing E from stage n or earlier. (Two new E's are allowed to touch.) The sequence (A161330) gives the number of E-toothpicks in the structure after n stages:

$$0, 2, 8, 14, 20, 38, 44, 62, 80, 98, 128, 146, 176, 218, 224, 242, 260, \ldots .$$

Figure 3 shows the structure after 32 stages. A161331 (the first differences) gives the number added at the n-th stage. Is there a recurrence?

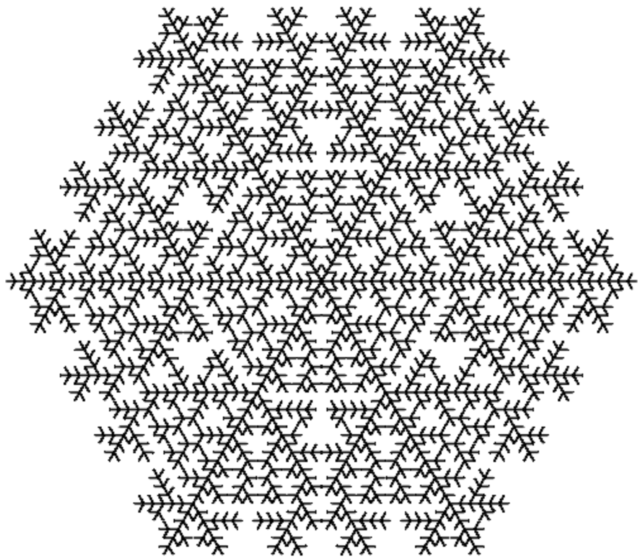

Figure 3: The E-toothpick (or snowflake) sequence A161330 after 32 stages, courtesy of David Applegate. The figure contains 1124 copies of the E-toothpick.

3 Alice Kleeva's figurate numbers.

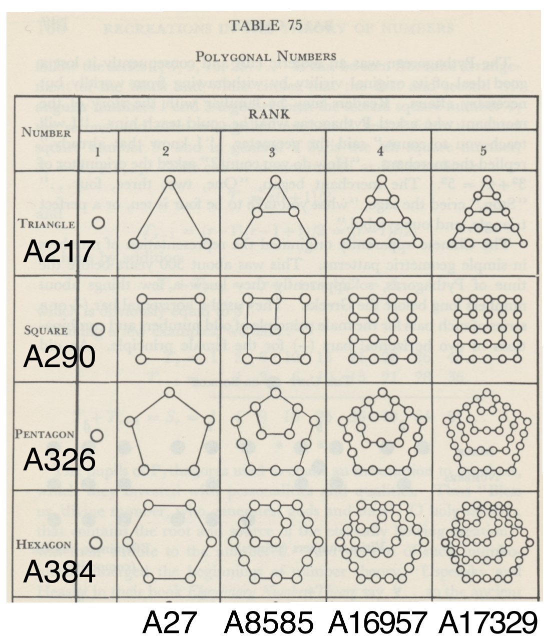

Figure 4: Some familiar figurate numbers and their A-numbers.

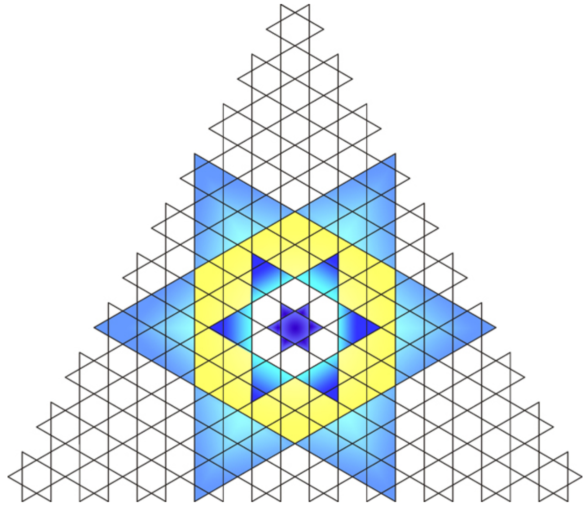

Figure 5: Alice V. Kleeva's figurate numbers A169720.

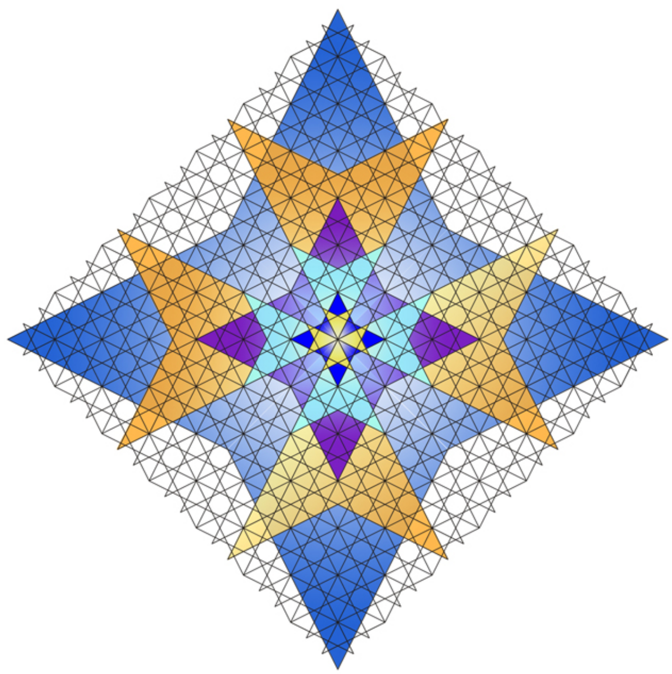

Figure 6: Alice V. Kleeva's figurate numbers A169721.

The *figurate* numbers (the numbers of points in various geometric figures—triangles, squares, pyramids, etc.) are among the most common of all sequences. A few examples are shown in Fig. 4, which is an annotated section of an illustration in Beiler's classic work [3]. Naturally the OEIS contains a large number of such sequences.

However, in 2010, Alice V. Kleeva, of the State Hermitage Museum in St. Petersburg, sent in eight new sequences of figurate numbers to the OEIS, accompanied by some quite spectacular illustrations (two of her drawings are shown in Figs. 5 and 6). The sequences are A169720–A169727, but we shall only discuss the first of them here.

The initial terms of A169720 are

$$1, 10, 55, 253, 1081, 4465, 18145, \ldots .$$

This is a subsequence of the triangular numbers, A000217, the n-th term being $(3\cdot2^n-2)\,(3\cdot2^n-1)/2$. It was clear that this sequence was somehow counting the points inside the nested figures in Fig. 5 (and that was how Alice Kleeva described it). However, it was not at all clear how to match the numbers with the drawings. The mystery was finally solved by Robert P. Munafo [13], who found the following explanation. Start with the familiar triangular grid of points—the dots in Fig. 7. Draw a series of nested Stars of David, in such a way that the tip of one star is just inside the center of an edge of the next star. In Fig. 7 the stars at generations 0, 1, 2, 3 are indicated by the symbols O, o, #, and x. The n-th star has $3 \cdot 2^n - 2$ points on its long edge, and sequence A169720 gives the total number of points in either of the two triangles whose union is the n-th star. For example, in Fig. 5, the blue-green Star of David has edge length 10, and so the total number of points in either of the two triangles is $10\cdot11/2 = 55$. Robert Munafo has found explanations for two other sequences from this family, but the rest (including A169721) are still somewhat of a mystery.

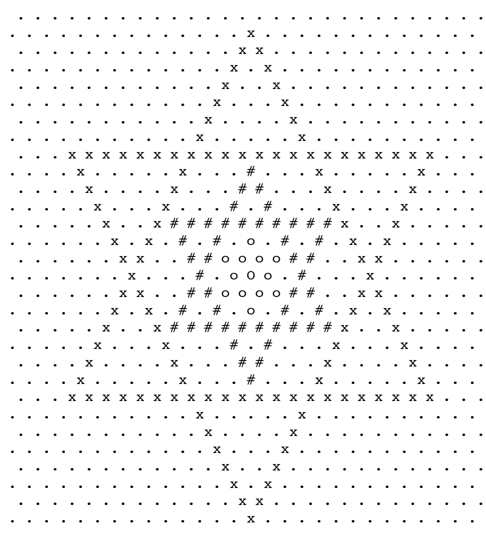

Figure 7: Robert P. Munafo's explanation of A169720.

4 Dissecting a rectangle into rectangles, etc.

There is something very appealing about dissection problems, and some of them—although not the ones to be mentioned here—even have applications in the real world [23]. One fairly new class of what might be called combinatorial dissection problems concerns the dissection of a rectangle or square into rectangles or squares of equal area. Not much is known about these sequences and they could all use more terms (and, in fact, a precise formulation, although their meaning is intuitively clear).

For example: in how many ways can one dissect a rectangle that is not a square into n rectangles of equal area, where dissections that differ by a rotation and/or reflection are regarded as distinct?

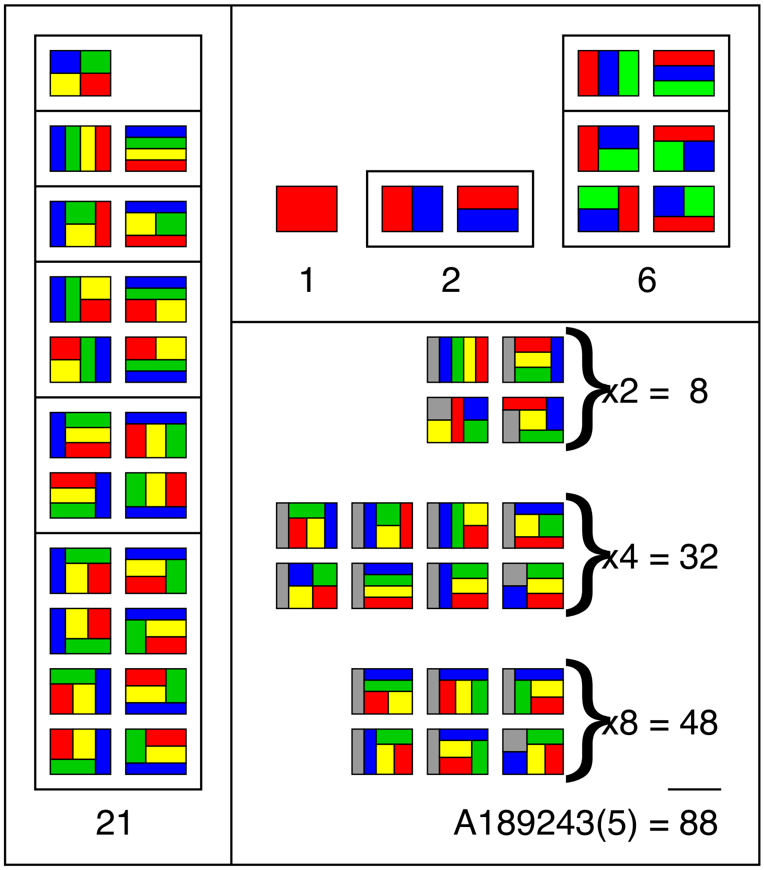

Figure 8: A189243 gives the number of ways to dissect a non-square rectangle into n rectangles of equal area. Only 18 of the 88 solutions are shown for $n = 5$, the others being obtained by rotations and reflections (and changing the aspect ratio in the case of rotations). Figure courtesy of Geoffrey H. Morley.

This is sequence A189243, submitted by Yi Yang [27], which begins[2]

$$1,\ 2,\ 6,\ 21,\ 88,\ 390,\ 1914\,,$$

The seventh term was only added in December 2012, by Geoffrey H. Morley. Figure 8 illustrates the first five terms.

If we regard dissections that differ only by rotations and reflections as equivalent, A189243 turns into A219861: $1, 2, 4, 11, 35, 130$, where only six terms are known.

A somewhat older sequence, A108066 [17] gives the number of inequivalent dissections of a *square* into n rectangles of equal area:

$$1,\ 1,\ 2,\ 6,\ 18,\ 65,\ 281,\ 1343,\ 6953,\ 38023\,.$$

In this problem, the first time that all n rectangles can be made to have different proportions (but equal areas) is when $n = 7$. Figure 9 shows an example found in 1971 by "Blanche Descartes" [7], [24]. A closely related sequence is A100664, the number of inequivalent dissections of a square into n rectangles of equal *perimeter*. Again only 10 terms are known.

[2]The absence of the ellipsis is a signal that no further terms are known.

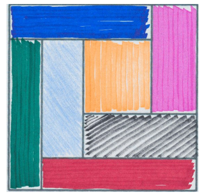

Figure 9: Blanche Descartes's dissection of a square into seven rectangles of equal area but different proportions (cf. A108066).

A still older problem in this class is to count the dissections of a square into n smaller squares of different sizes. This has a long history: the smallest example, found by A. J. W. Dujivestijn in 1978, is unique, and is a dissection of a square into 21 distinct squares. The corresponding sequence (A006983) therefore begins with 20 zeros and a one:

$$0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 1, 8, 12, 26, 160, 441, 1152, 3001, 7901\,.$$

Since this is a fairly well-known problem, we shall say no more about it here. See A006983 for numerous references and links to related sequences.

It has a somewhat different flavor, but what is probably the oldest and simplest dissection problem of all is still unsolved. It is well known that any polygon can be cut up into a finite number of pieces which can be re-arranged, without overlapping, to form a square of the same area. (The pieces allowed to be rotated or turned over; their boundaries must be simple curves.) The question is, what is the minimal number, $d(n)$ say, of pieces that are required for dissecting a regular n-gon ($n \geq 3$) into pieces that can be re-arranged to form a square? In the case $n = 3$, there is a famous 4-piece dissection, apparently first published by Dudeney in 1902 [8], [9], shown in Fig. 10. It seems unlikely that a three-piece dissection exists, but this seems to be an open question. In other words, is $d(3)$ really 4? No values of $d(n)$ are known for certain (except of course $d(4) = 1$). The best values presently known for $n = 3, \ldots, 10$, taken from Frederickson [9] and Theobald [22], are:

$$4?, 1, 6?, 5?, 7?, 5?, 9?, 7? \quad \text{(A110312)}.$$

This is most unsatisfactory: the normal rule is that every term in a sequence in the OEIS should be known to be correct. This sequence is quite an exception, the values shown being merely upper bounds.

Figure 10: A triangle can be cut into four pieces which can be rearranged to form a square. It is an open question to show this cannot be done using only three pieces (cf. A110312). Figure courtesy of Vinay A. Vaishampayan.

5 Dominoes.

How many different connected planar figures can be made from n 1×2 rectangles (or blank dominoes)? Consider each long edge of a domino to be divided into two length-1 segments. Two dominoes can share a short edge, or a long edge, or a short edge can meet a long edge as long as the shared portion is one of the two halves of the long edge. "Connected" means "edge-connected." The basic sequence in this family is A056786 [18], in which figures differing by a rotation and/or reflection are not considered different. (Figures differing in their internal arrangement do count as different.) Even twelve years later, only seven terms are known:

$$1, 4, 26, 255, 2874, 35520, 454491\,.$$

Figure 11 show the 26 figures that can be made with three dominoes.

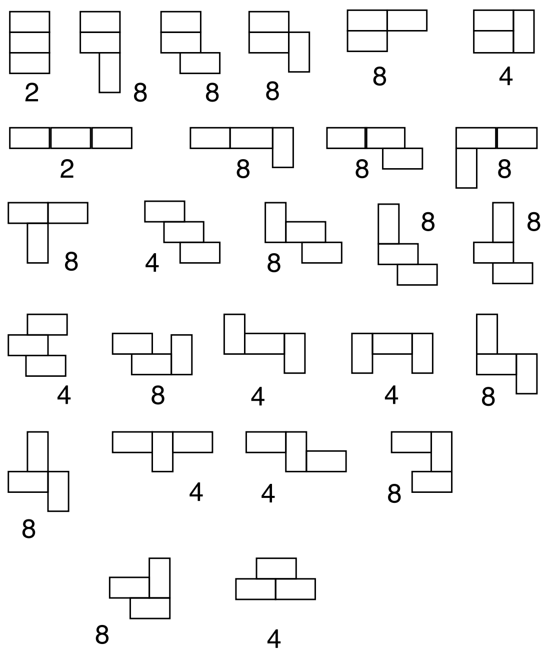

Figure 11: The 26 figures that can be formed with three dominoes (A056786). The figures in the top row all contain two dominoes that share a long edge (cf. A216583), and the two figures in the bottom row have a loop in their adjacency graph (cf. A216492). The figures are labeled with the numbers of their images under rotations and reflections (A216598).

Recently, César Eliud Lozada [11] submitted A216492 which imposes two additional restrictions on the figures: two dominoes may not share a long edge, and the adjacency graph of the dominoes must be a tree. This excludes the figures in the top and bottom rows of Fig. 11, so the third term is now 18. The known values are

$$1, 3, 18, 139, 1286, 12715, 130875\,.$$

If we simply exclude figures that share a long edge, but allow loops in the adjacency graph, we get A216583:

$$1, 3, 20, 171, 1733, 18962\,.$$

(Only the six figures in the top row of Fig. 11 must be excluded.) Finally, in the original question, if we count figures that differ by a rotation or reflection as different, we get A216598, where now only three terms are presently known:

$$2,\ 16,\ 164\,.$$

The figures in Fig. 11 have been labeled with the numbers of distinct images that are obtained by rotations and reflections; these add up to 164. As in all these problems, it would be nice to have more terms and some insight into the rate of growth of these sequences.

6 Meanders on a square grid.

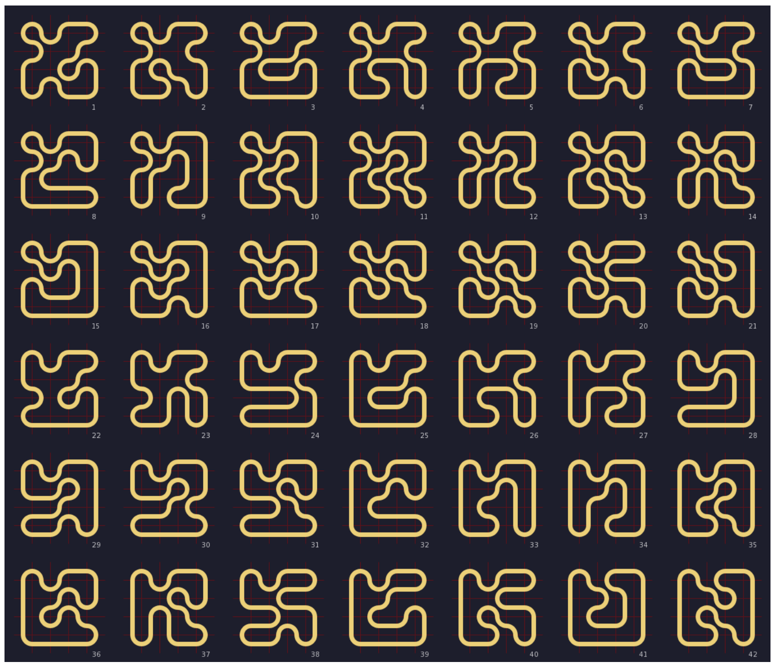

Figure 12: The 42 non-self-intersecting closed paths that visit every cell of a 4×4 grid at least once and do not cross any edge more than once (cf. A200000). Figure courtesy of Jonathan Wild.

The classical meander sequences are A005316, which gives the number of different possibilities when a river crosses a road n times, and its subsequence A005315, giving the number of possibilities when a closed loop crosses a straight line $2n$ times. Instead of these, in this and the following section we will describe two new meander problems.

As the OEIS neared 200000 sequences, it was decided to reserve sequence number A200000 for an especially noteworthy sequence. This number was finally awarded to Jonathan Wild for his sequence [26] that gives the number of non-self-intersecting closed paths that visit every cell of an $n \times n$ grid at least once and do not cross any edge more than once. The sequence begins

$$1,\ 1,\ 0,\ 4,\ 42,\ 9050,\ 6965359,\ 26721852461,\ ?,\ 31194475941824888769\,.$$

The 42 solutions for $n = 5$ are shown in Fig. 12. The eighth and tenth terms were found by Alex Chernov; the ninth and subsequent terms are not known.

7 Meanders from circular arcs.

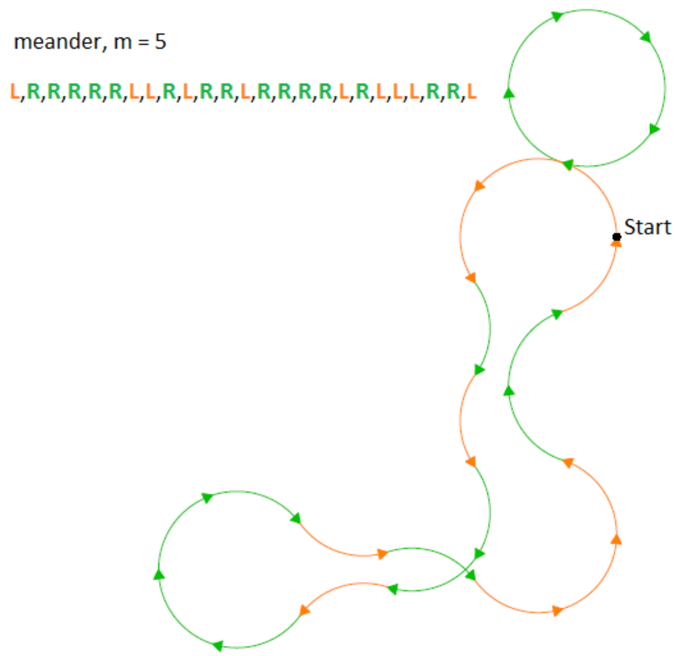

Figure 13: A meander constructed from 25 circular arcs of angle $2\pi/5$, one of the 13504 meanders counted by $T(5,4,1)$ (cf. A197654). Figure courtesy of Susanne Wienand.

In A197654, Susanne Wienand [25] introduced a different class of meanders. These are closed curves formed from circular arcs of unit length and angle $2\pi/m$, for some integer $m \geq 1$ (in A197654, m is taken to be 5). The meander is described by a string S of L's and R's, starting with L, where L indicates an arc which turns left, R an arc which turns right, and where S contains $m(k+1)$ L's and $m(n-k)$ R's, for some $k \geq 0$. This ensures that the curve (which is therefore made up of $m(k+1)$ consecutive arcs) is closed. There is one further condition. After i steps of the construction ($0 \leq i < m(k+1)$), the curve will be pointing in one of m possible directions: it is required that each possible direction occur equally often. Let $T(m,n,k)$ denote the number of such meanders. A197654 contains the triangle of numbers $T(5,n,k)$, the initial values of which are:

$n\backslash k$	0	1	2	3	4
0	1				
1	5	1			
2	31	62	1		
3	121	1215	363	1	
4	341	13504	20256	1364	1

Figure 13 show an example with $m = 5$, $n = 4$ and $k = 1$ (this is one of the 13504 meanders counted by $T(5,4,1)$). The associated string S is $LRRRRRLLRLRRLRRRRLRLLLRRL$. There are conjectured recurrences for $T(m,n,k)$ (see A197654), but so far they are unproved.

8 Duraid Madina's braid sequence.

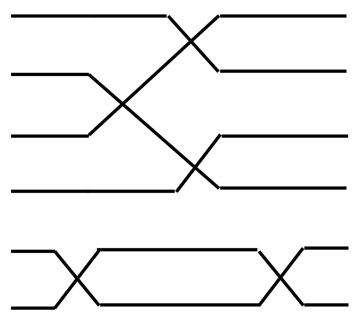

Figure 14: Illustrating A200919: five crossings are enough to ensure that every pair out of six wires are adjacent. (the top and bottom lines are considered to be adjacent).

Consider n lines running from left to right across the page, each line being adjacent to its two neighbors, with the top line considered adjacent to the bottom line. Duraid Madina [12] has considered the question of finding the minimal number of crossings needed for any two lines to be adjacent (see Fig. 14). He computed the first nine terms, which are

$$0, 0, 0, 1, 3, 5, 9, 13, 19, 25, 34$$

(A200919). Not much is known about this problem, although Madina has some conjectures about subsequent terms. (This set of lines is similar to what is technically called a "braid," although here it does not matter which line is on top at a crossing.)

The sequence arose from an engineering problem. Duraid Madina says: "A200919 is an abstraction of a practical problem that I first came across in 2008 in connection with the design of wires in semiconductors, where the goal was to 'mix' bundles of wires as cheaply as possible so as to amortize noise occurring on any wire with its neighboring wires, where the cost to be minimized was the number of wire crossings."

9 Reed Kelley's sequence.

The 14-th century Indian mathematician Narayana studied what is now called "Narayana's cows" sequence, defined by $a(n) = a(n-1) + a(n-3)$, with $a(0) = a(1) = a(2) = 1$; see A000930 for details. An interesting variant of this sequence was recently introduced into the OEIS by Reed Kelly [10], with definition

$$a(n) \;=\; \frac{a(n-1) + a(n-3)}{\gcd\{\, a(n-1), a(n-3)\,\}}, \tag{1}$$

and initial values 1, 1, 1:

$$1, 1, 1, 2, 3, 4, 3, 2, 3, 2, 2, 5, 7, 9, 14, 3, 4, 9, 4, 2, 11, 15, 17, 28, 43, 60, 22, \ldots, \tag{2}$$

(A214551). A graph of the first 5000 terms is shown in Fig. 15. Examination of the first 300000 terms suggests that the sequence is dominated by a term that grows like $e^{0.11n}$, but it is hard to make any more precise statement than that, as the graph is quite wobbly. Essentially nothing has been proved about this sequence. A214323 gives the successive gcd's used in the construction, and A219898 gives the high-water marks.

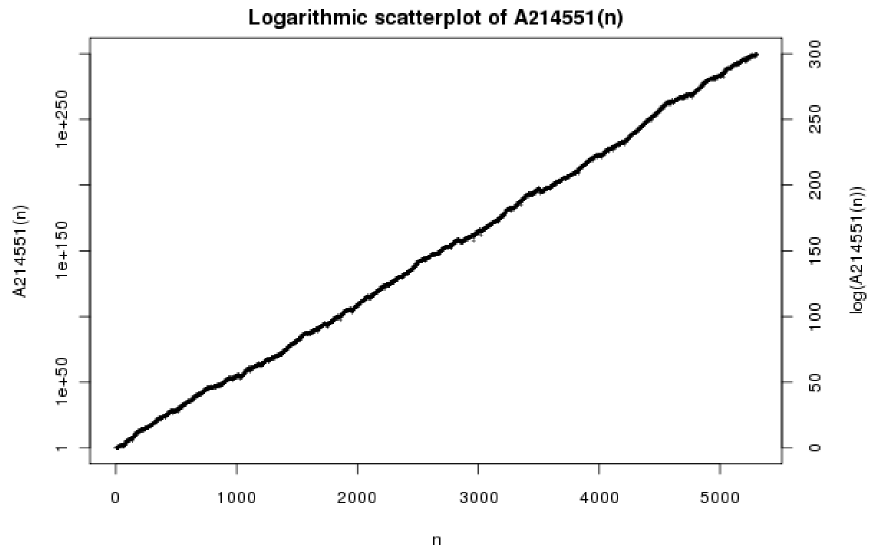

Figure 15: Log-plot of 5000 terms of Reed Kelly's sequence A214551, defined by the recurrence $a(0) = a(1) = a(2) = 1$, $a(n) = (a(n-1) + a(n-3))/\gcd\{a(n-1), a(n-3)\}$. Although the graph is clearly increasing, there are pronounced irregularities. So far nothing has been proved about the rate of growth of this sequence.

10 Words with no final repeats.

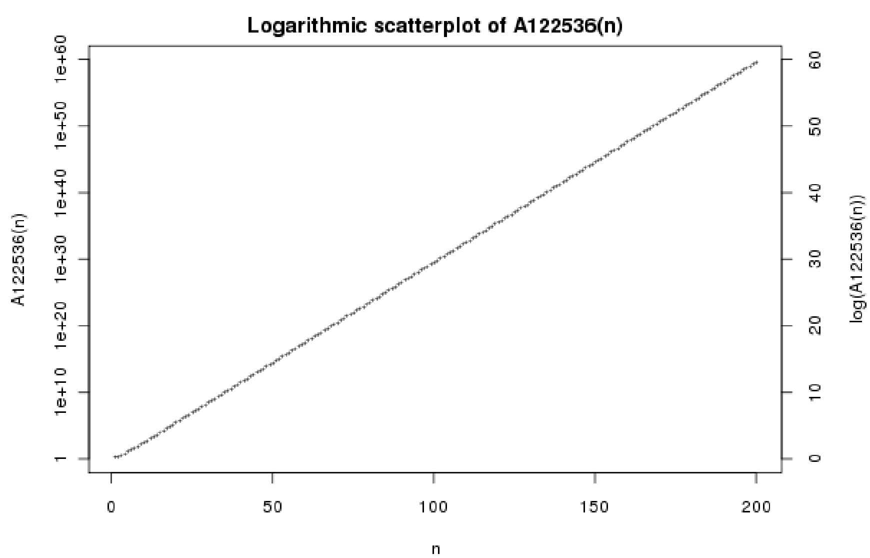

Figure 16: Log-plot of the 200 known values of A122536, the number of binary sequences of length n with no final repeats (or curling number 1). Is there an explicit formula?

The curling number of a binary word S is the exponent of the largest suffix that is a pure power. For example, $S = 0110101 = 011(01)^2$ has curling number 2, $S = 1001 = 100(1)^1$ has curling number 1, and

$$S = 0011011011 = 00110110(1)^2 = 0(011)^3$$

has curling number 3 (since we must maximize the exponent of the suffix). Let $c(n,k)$ denote the number of binary sequences of length n with curling number k. Benjamin Chaffin, John P. Linderman, Allan R. Wilks and I [5] have made an extensive study of $c(n,k)$, as part of our (so far unsuccessfull) attack on the "Curling Number Conjecture." We have learned enough about this problem to be able to compute the first 200 terms of $c(n,1)$. This is the number of binary sequences of length n that have no final repetitions, A122536, of which the first 30 terms were contributed to the OEIS in 2006 by Guy P. Srinivasan [21]. The first few terms are

$$2, 2, 4, 6, 12, 20, 40, 74, 148, 286, 572, 1124, 2248, 4460, 8920, \ldots,$$

and Fig. 16 shows a graph of the known terms. This graph is much smoother than that in Fig. 15, and suggests that

$$\lim_{n\to\infty} \frac{\log c(n,1)}{n} \;=\; 0.693147\cdots. \tag{3}$$

We know that $c(2k+1,1) = 2c(2k,1)$ and we can express $c(2k,1)$ in terms of $2c(2k-1,1)$ and certain other combinatorial quantities. We were hoping to find an explicit expression for $c(n,1)$, perhaps similar to that for the number of primitive binary sequences (those that are not pure powers), A027375, which is

$$\sum_{d|n} \mu\left(\frac{n}{d}\right) 2^d, \tag{4}$$

where μ is the Möbius function. However, even with 200 terms to work on, we have not found a formula.

11 Martin Gardner's minimal no-3-in-line problem.

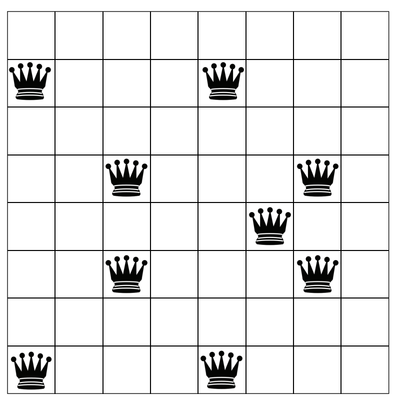

Figure 17: 9 queens on a chessboard, no 3 in a line, such that adding one more queen produces 3 in a line; 9 is minimal (A219760). Figure courtesy of Gregory S. Warrington.

A recent paper by Alec S. Cooper, Oleg Pikhurko, John R. Schmitt and Gregory S. Warrington [6] considers a question raised by Martin Gardner: what is the minimal number of queens that can be placed on an $n \times n$ chessboard, no three in a line, such that adding one more queen on any vacant square will produce three in a line? The known values (A219760) are

$$1, 4, 4, 4, 6, 6, 8, 9, 10, 10, 12, 12\,.$$

The authors show that the n-th term is at least n, and give bounds on the next few terms. Figure 17 shows a 9-queen solution for the case $n = 8$.

(There are many classical versions of the no-3-in-line problem—see for example A000769, or the Index to the OEIS.)

12 Circulant determinant equals number.

Look at this determinant:

$$\begin{vmatrix} 2 & 4 & 7 \\ 7 & 2 & 4 \\ 4 & 7 & 2 \end{vmatrix} = 247\,.$$

What other numbers have the property that the circulant determinant formed from their digits is equal to the number? The question was studied by N. I. Belukhov [4] and the sequence (A219324) was recently submitted by Max Alekseyev [1]:

1, 2, 3, 4, 5, 6, 7, 8, 9, 247, 370, 378, 407, 481, 518, 592, 629, 1360, 3075, 26027, … .

Figure 18 shows a larger example. 47 terms are presently known. Belukhov found a construction for these numbers (see A219324), but it does not explain all the known values.

$$\begin{vmatrix} 4 & 5 & 6 & 7 & 9 & 0 & 1 & 2 & 3 \\ 3 & 4 & 5 & 6 & 7 & 9 & 0 & 1 & 2 \\ 2 & 3 & 4 & 5 & 6 & 7 & 9 & 0 & 1 \\ 1 & 2 & 3 & 4 & 5 & 6 & 7 & 9 & 0 \\ 0 & 1 & 2 & 3 & 4 & 5 & 6 & 7 & 9 \\ 9 & 0 & 1 & 2 & 3 & 4 & 5 & 6 & 7 \\ 7 & 9 & 0 & 1 & 2 & 3 & 4 & 5 & 6 \\ 6 & 7 & 9 & 0 & 1 & 2 & 3 & 4 & 5 \\ 5 & 6 & 7 & 9 & 0 & 1 & 2 & 3 & 4 \end{vmatrix} = 456790123\,.$$

Figure 18: 456790123 is equal to the circulant determinant formed from its digits (A219324). 247 is the smallest nontrivial number with this property.

13 Acknowledgments.

Thanks to all the people whose figures were used in this article (their names are given in the figure captions). Thanks also to all the correspondents who suggested sequences that might have been included. The following is a partial list of some of these. They are all worth further investigation:
A209401: The number of noncommutative rings with n elements.
A216377: The leading digit in the base n representation of $n!$.
A216999: The number of integers obtainable from 1 in n steps using addition, multiplication and subtraction.
A217032: The minimal number of steps to reach $n!$ starting from 1 and using addition, multiplication and subtraction.
A218831: $a(n)$ is the smallest k such that there are no primes in the interval $[nk, (n+1)k]$, or 0 if no such k exists. $a(1) = 0$ is Bertrand's postulate. It appears that $a(n) = 0$ just for $n = 1, 2, 3, 5, 9, 14$.

A218904: Sum of the sizes of the kernels of all integer partitions of n. (The kernel of a partition is the intersection of the Ferrers diagram and its transpose.)

Many more examples could be listed. If problems like this appeal to you, please consider becoming an associate editor[3] of the OEIS: you get to see the new sequences as they arrive, and they often contain lovely problems. There are no formal duties, everything is voluntary, and we badly need more editors to cope with the ever-increasing flow of submissions. Last but not least, please make a donation to the OEIS Foundation [14] to help keep the OEIS running!

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