

Graphical Enumeration and Stained Glass Windows, Additional Illustrations for Part 1, Rectangular Grids

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Abstract

An appendix to our paper *Graphical Enumeration and Stained Glass Windows, 1: Rectangular Grids* with further illustrations. Because of space limitations we were unable to include many of our favorite illustrations in the paper itself.

1 Introduction

This is an appendix to our paper *Graphical Enumeration and Stained Glass Windows, 1: Rectangular Grids* [1], with further illustrations. Because of space limitations we were unable to include many of our favorite illustrations in the paper itself. The section numbers here match those in the paper.

There are a great many additional illustrations in the relevant entries in the *On-Line Encyclopedia of Integer Sequences* [2], and we will indicate the A-numbers of the relevant sequences.

The coloring algorithms that we used are described in Section 10 of [1]. The “random coloring” algorithm (§10.3) was used in Figures 1, 2, 3, 5, 6, 10, 11, 12, 13, 14, 15, 17, and the “number-of-sides” coloring (§10.1) in Figures 4, 7, 8, 9, 16, 18. In Figure 2 the boundaries between the cells have been slightly enhanced, in an attempt to emphasize the stained-glass effect.

We recommend looking at these pictures with a good pdf viewer, since they can be enlarged several times without losing resolution.

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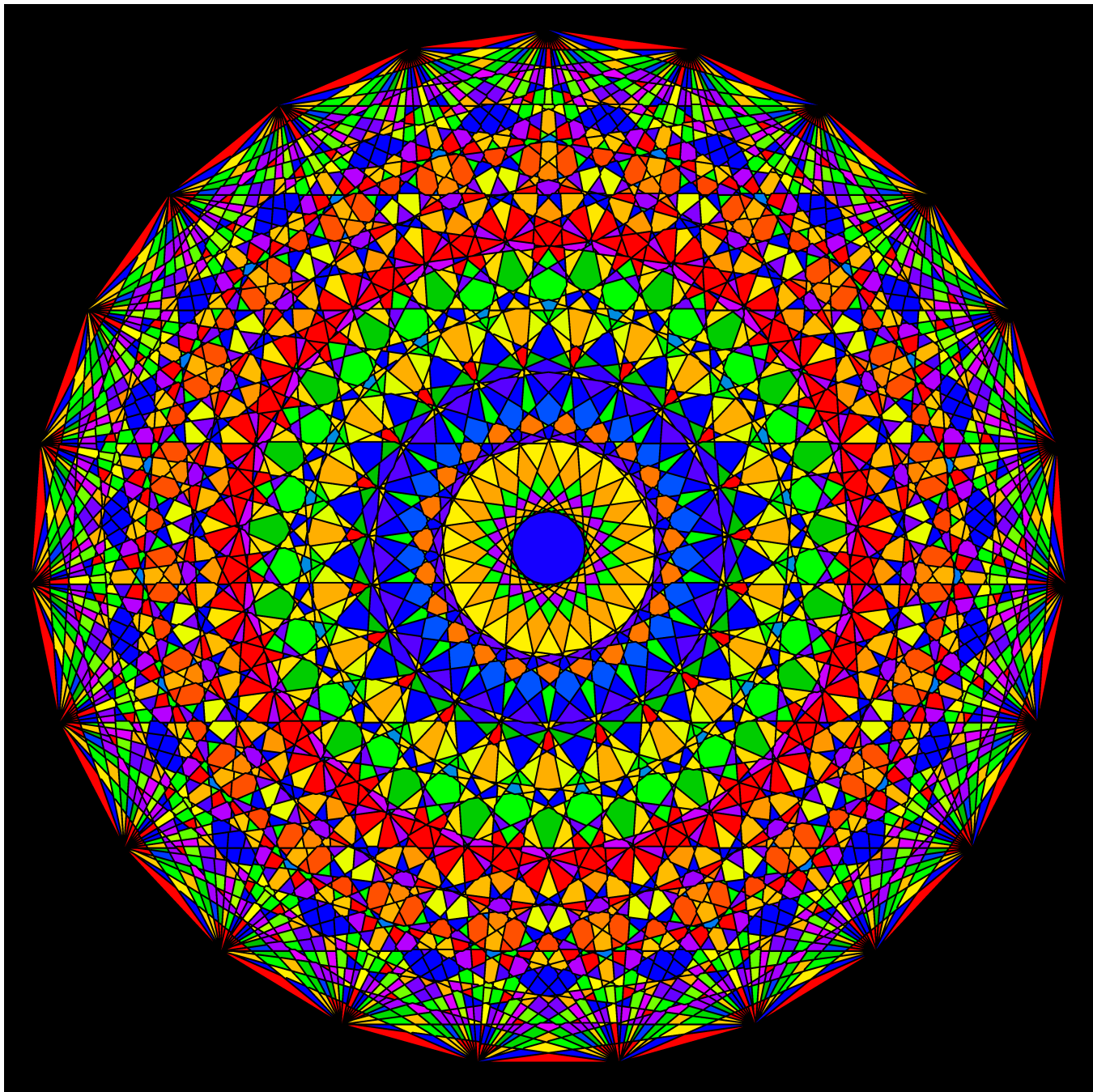


Figure 1: The complete graph K_{23} using “random coloring” (§10.3 of [1]). This is a higher-quality version of Fig. 1 in [1]. Sequence [A007678](#) has additional drawings of K_n .

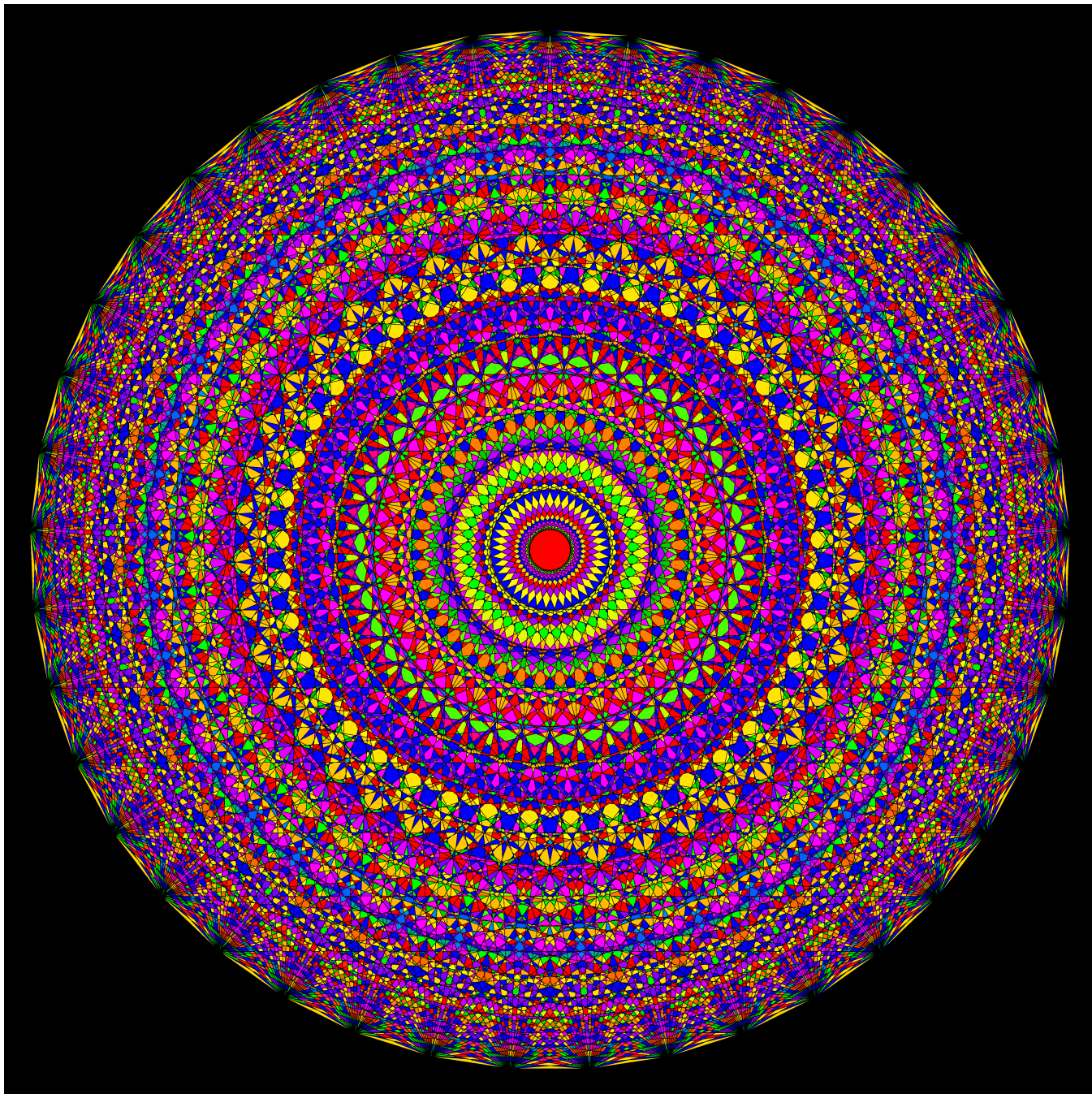


Figure 2: The complete graph K_{41} (random coloring, enhanced borders).

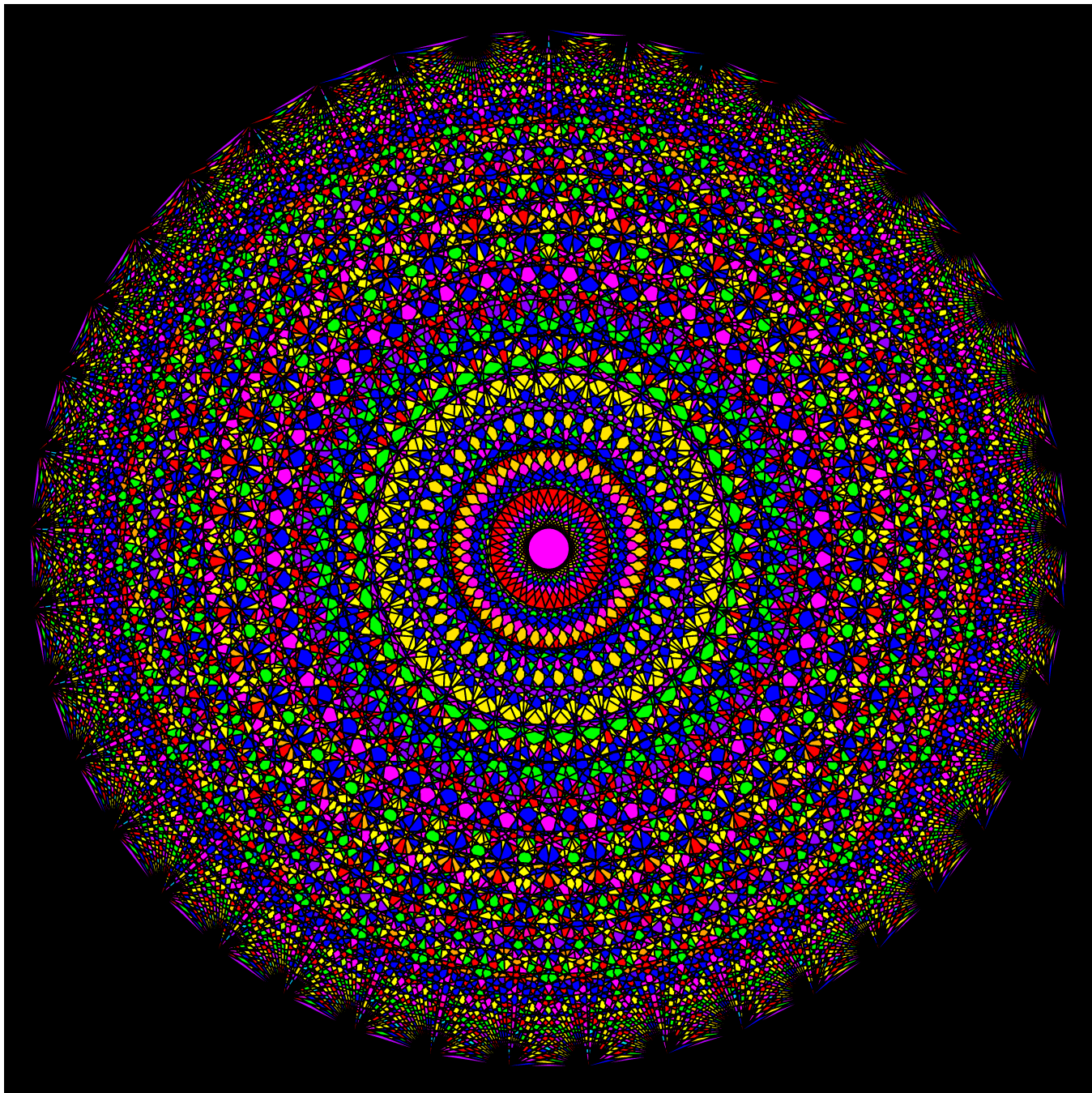


Figure 3: The complete graph K_{41} (random coloring).

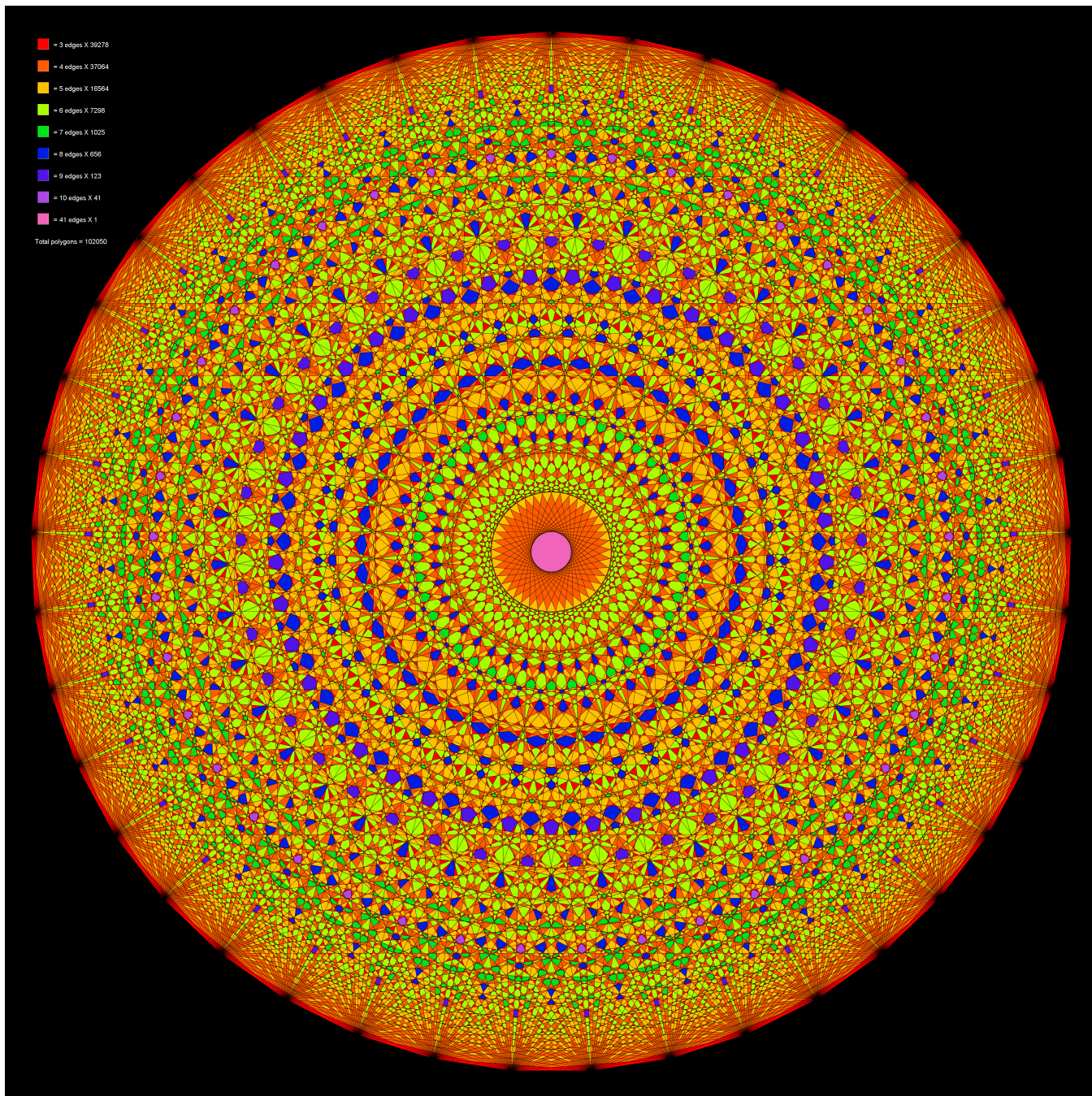


Figure 4: The complete graph K_{41} (number-of-sides coloring).

2 $BC(1, n)$: $1 \times n$ rectangular windows

The graph $BC(1, n)$ is horizontal, $BC(n, 1)$ is vertical, but of course as graphs they are identical. We show $BC(n, 1)$ in preference to $BC(1, n)$, in order to pay homage to the stained glass in the Sainte-Chapelle in Paris. Part 1 of the paper shows $BC(n, 1)$ for $n = 1, 2, 3, 4$. Here we show $BC(5, 1)$, $BC(6, 1)$, and $BC(7, 1)$. Sequence [A331452](#) has drawings of $BC(n, 1)$ for $1 \leq n \leq 15$.

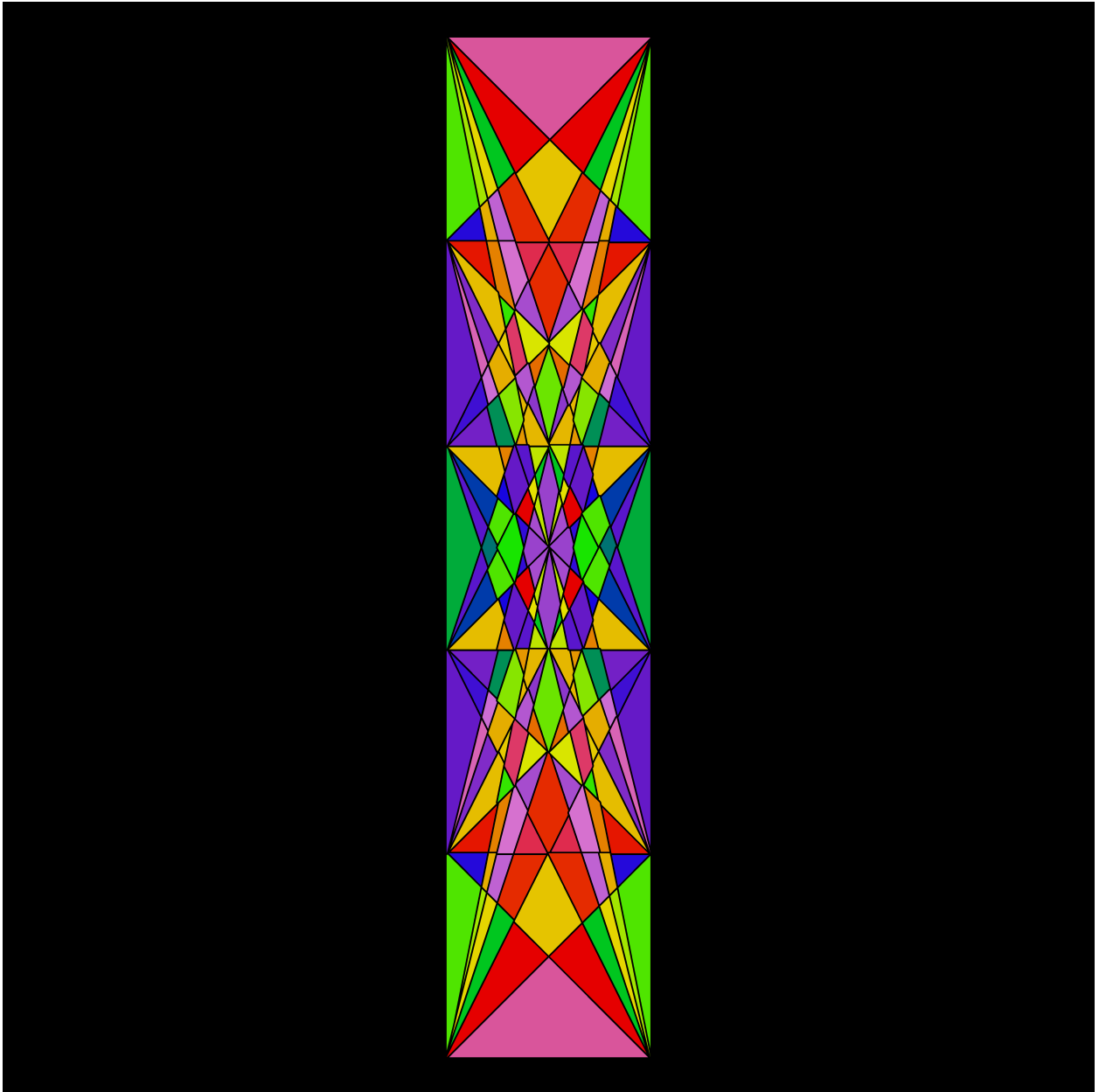


Figure 5: The graph $BC(5, 1)$.

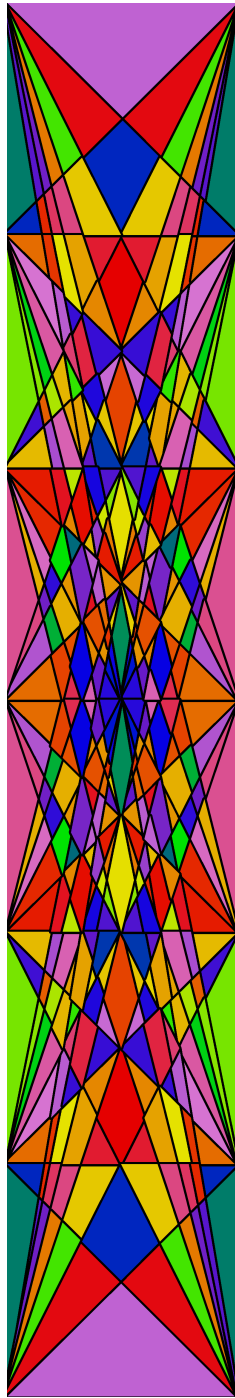


Figure 6: The graph $BC(6,1)$.

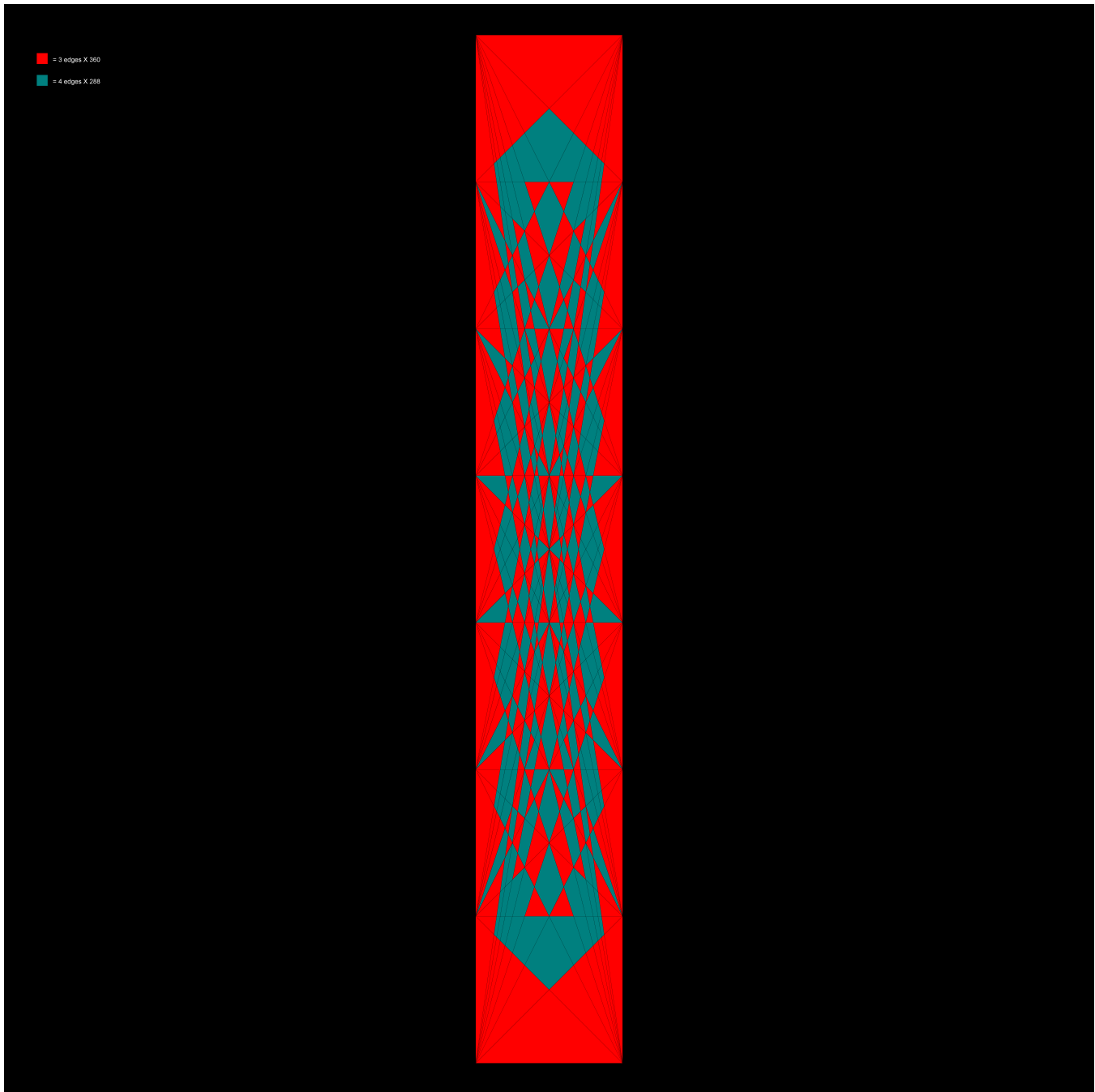


Figure 7: The graph $BC(7,1)$. The 360 triangular cells are colored red, the 288 quadrilateral cells are sea-green (cf. [A324042](#), [A324043](#)).

6 $BC(m, n)$: $m \times n$ rectangular windows

Part 1 shows $BC(n, 2)$, $n = 1 \dots 4$, and $BC(3, 3)$. Sequences [A331452](#), [A335701](#), ... have further drawings.

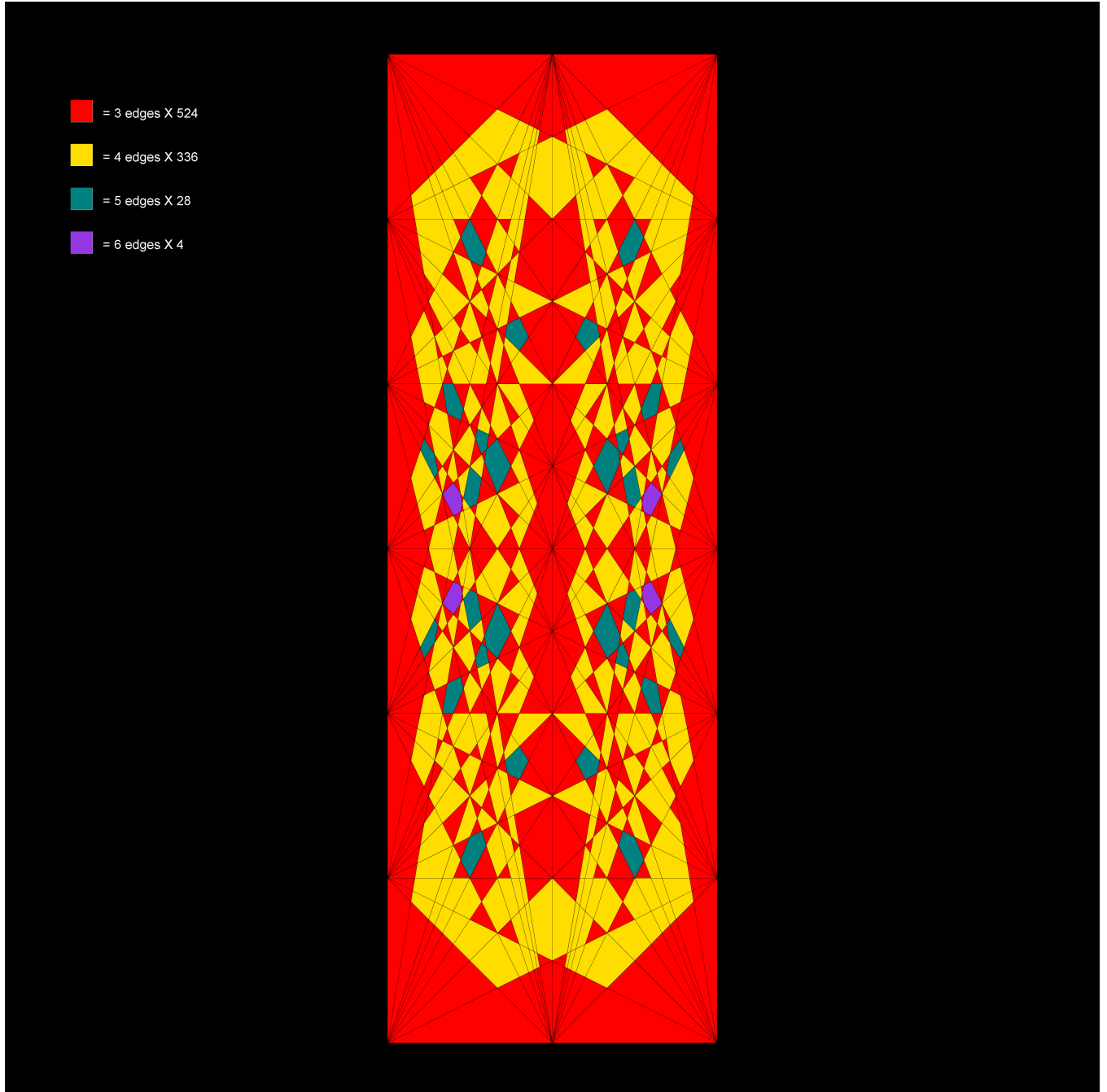


Figure 8: The graph $BC(6,2)$: 524 triangles (red), 336 quadrilaterals (yellow), 28 pentagons (green), 4 hexagons (purple).

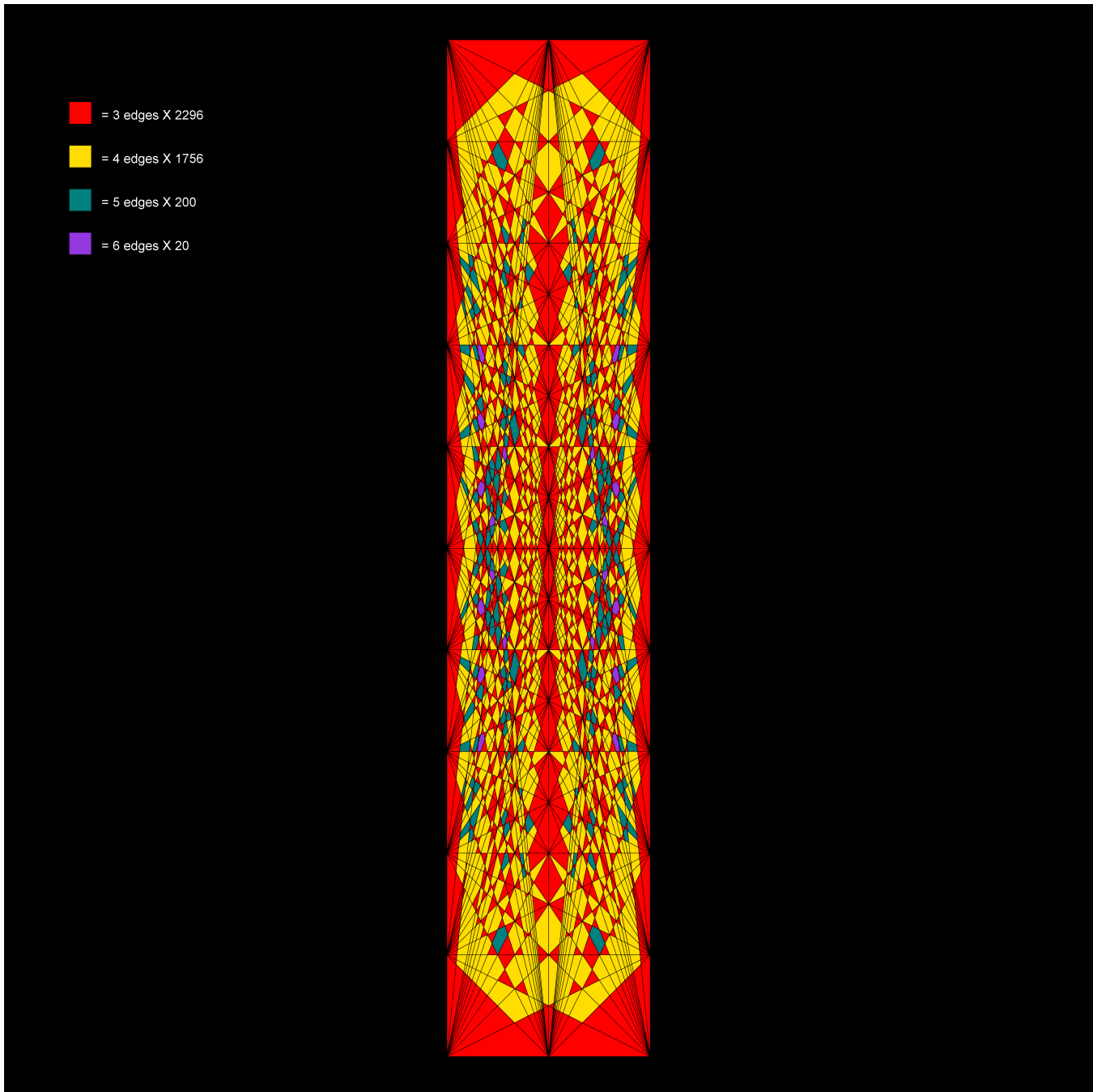


Figure 9: The graph $BC(10, 2)$: 2296 triangles (red), 1756 quadrilaterals (yellow), 200 pentagons (green), 20 hexagons (purple).

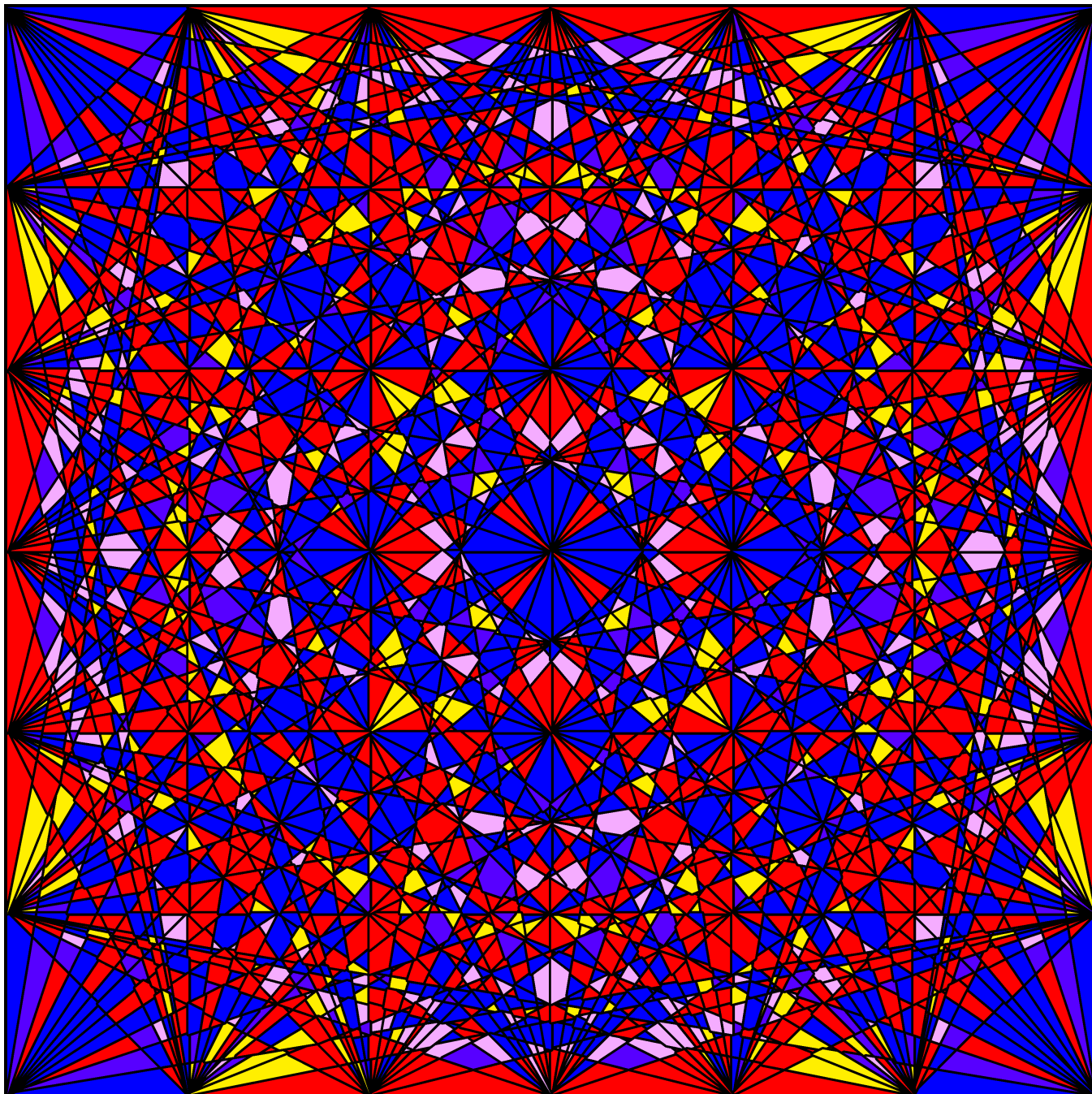


Figure 10: The graph $BC(6,6)$.

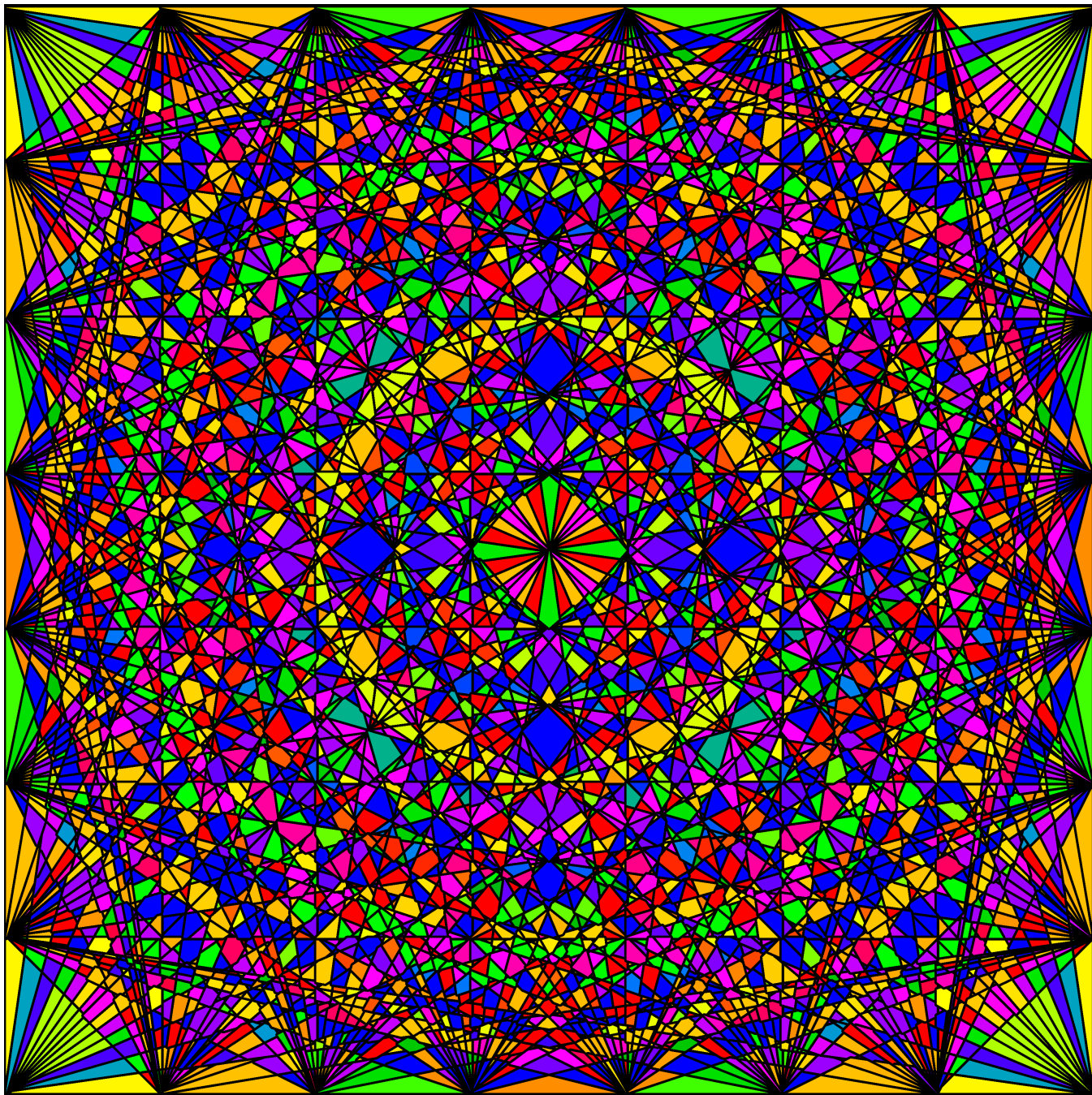


Figure 11: The graph $BC(7,7)$.

8 The graphs $AC(m, n)$.

In contrast to the previous illustrations, which use "hot" colors, appropriate for a cold cathedral, the next three figures use a cooler palette, which would be more suitable for a mosaic floor or carpet. Sequence [A288187](#) has further figures.

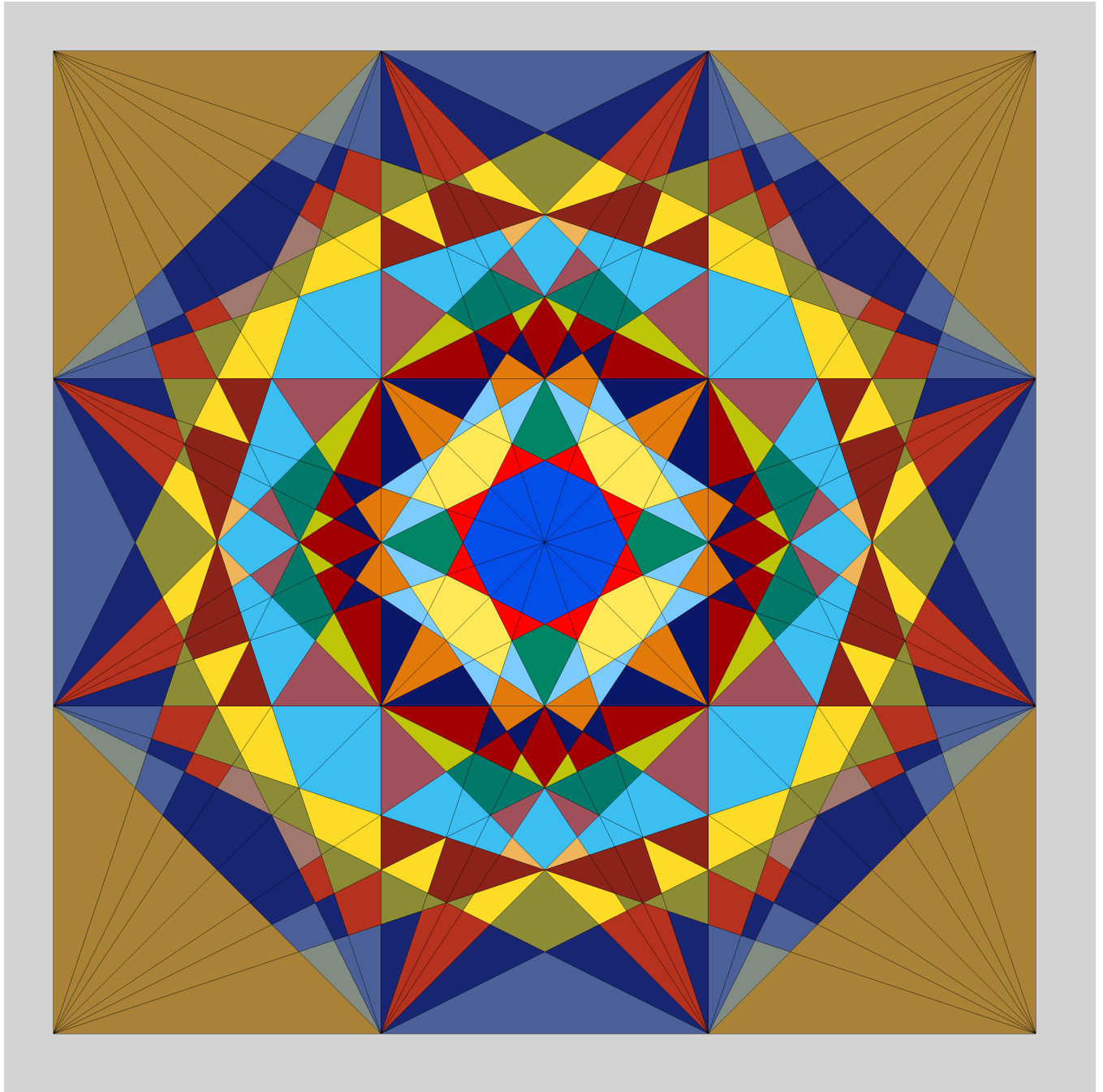


Figure 12: The graph $AC(3,3)$ (this is a colored version of Fig. 18 in Part 1).

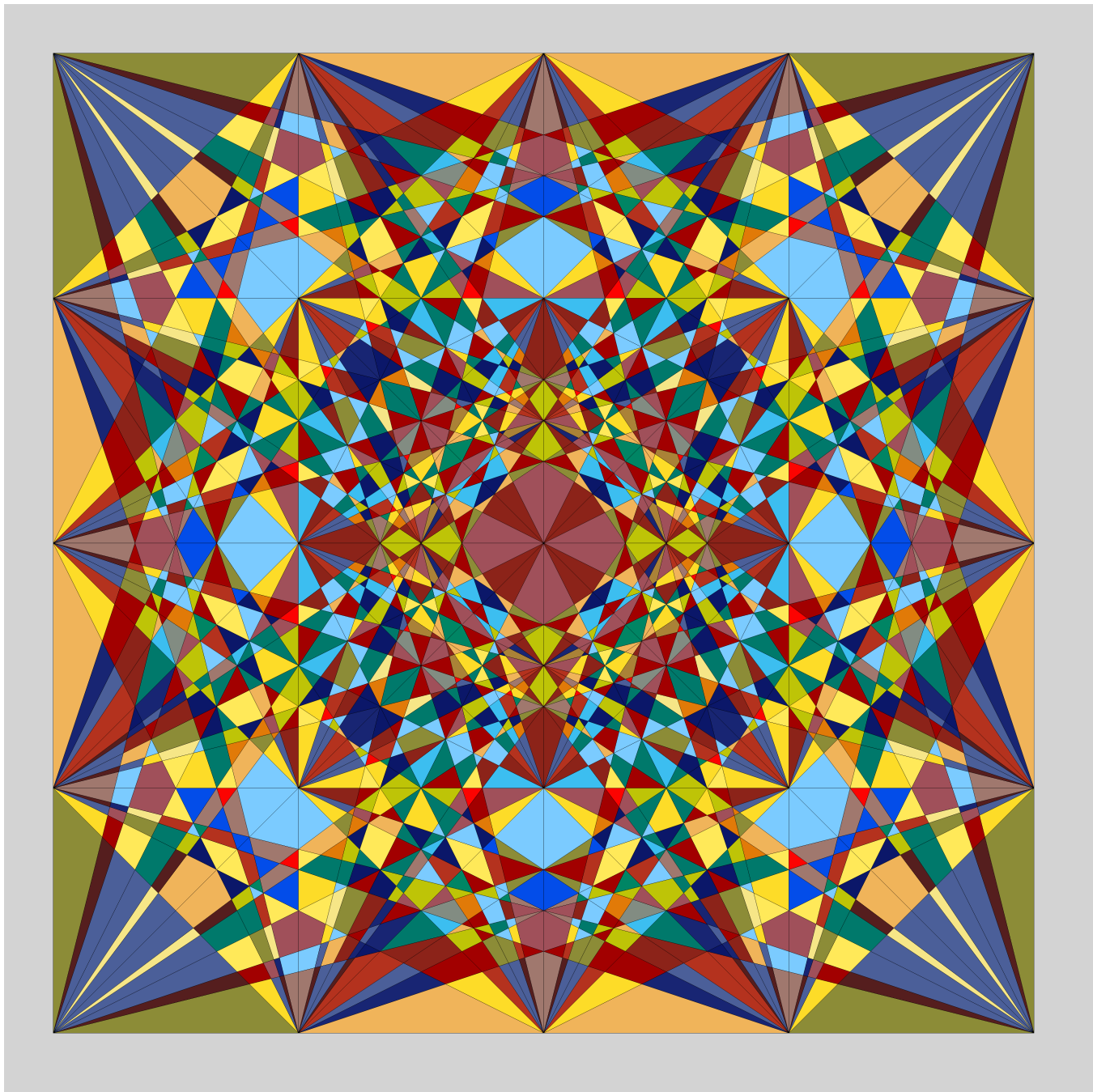


Figure 13: The graph $AC(4,4)$.

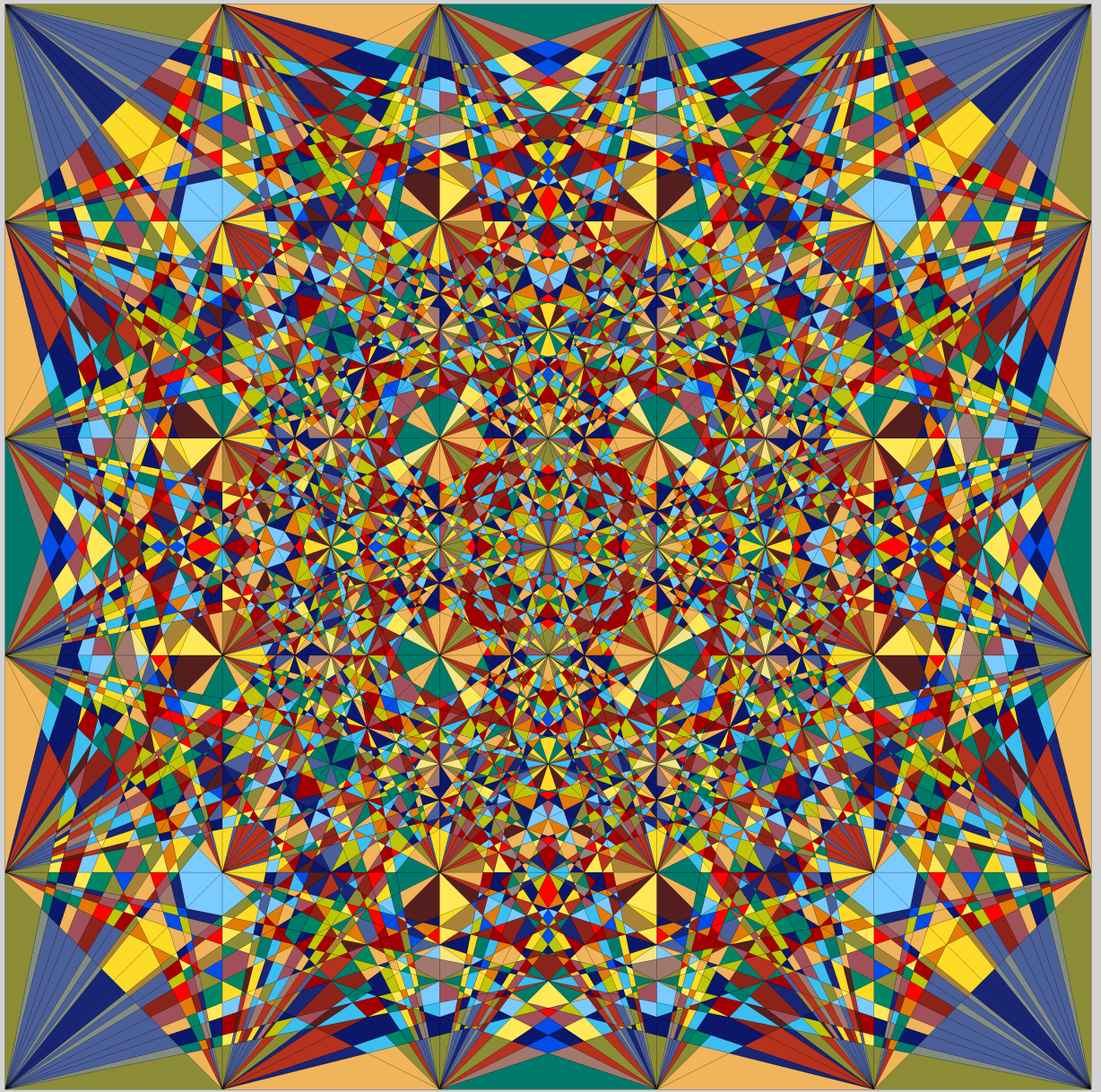


Figure 14: The graph $AC(5,5)$.

9 The graphs $LC(m, n)$.

$LC(3, 3)$ is shown in Fig. 19 of Part 1. Sequence [A333282](#) has further drawings.

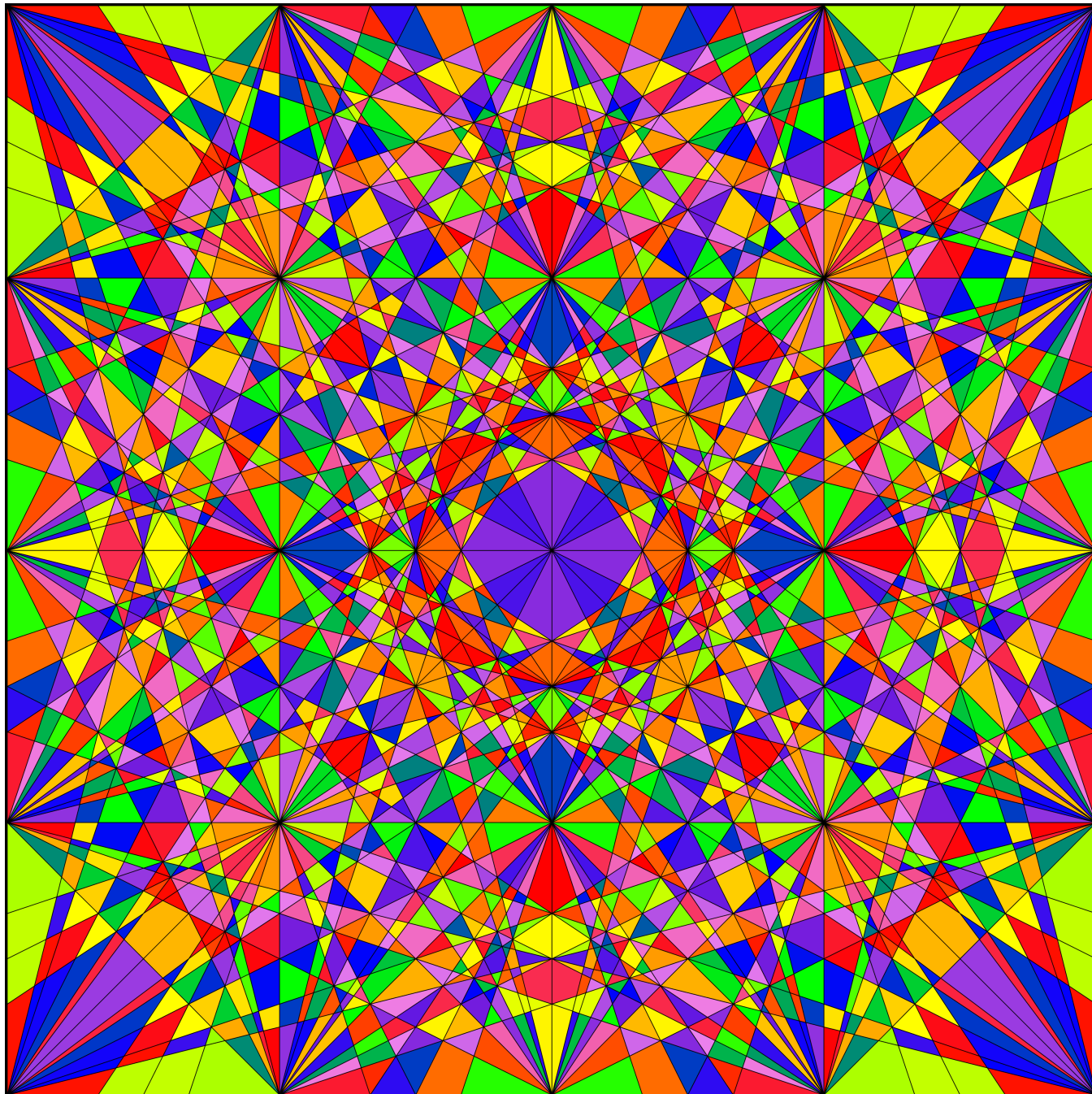


Figure 15: The graph $LC(4, 4)$ (random coloring).

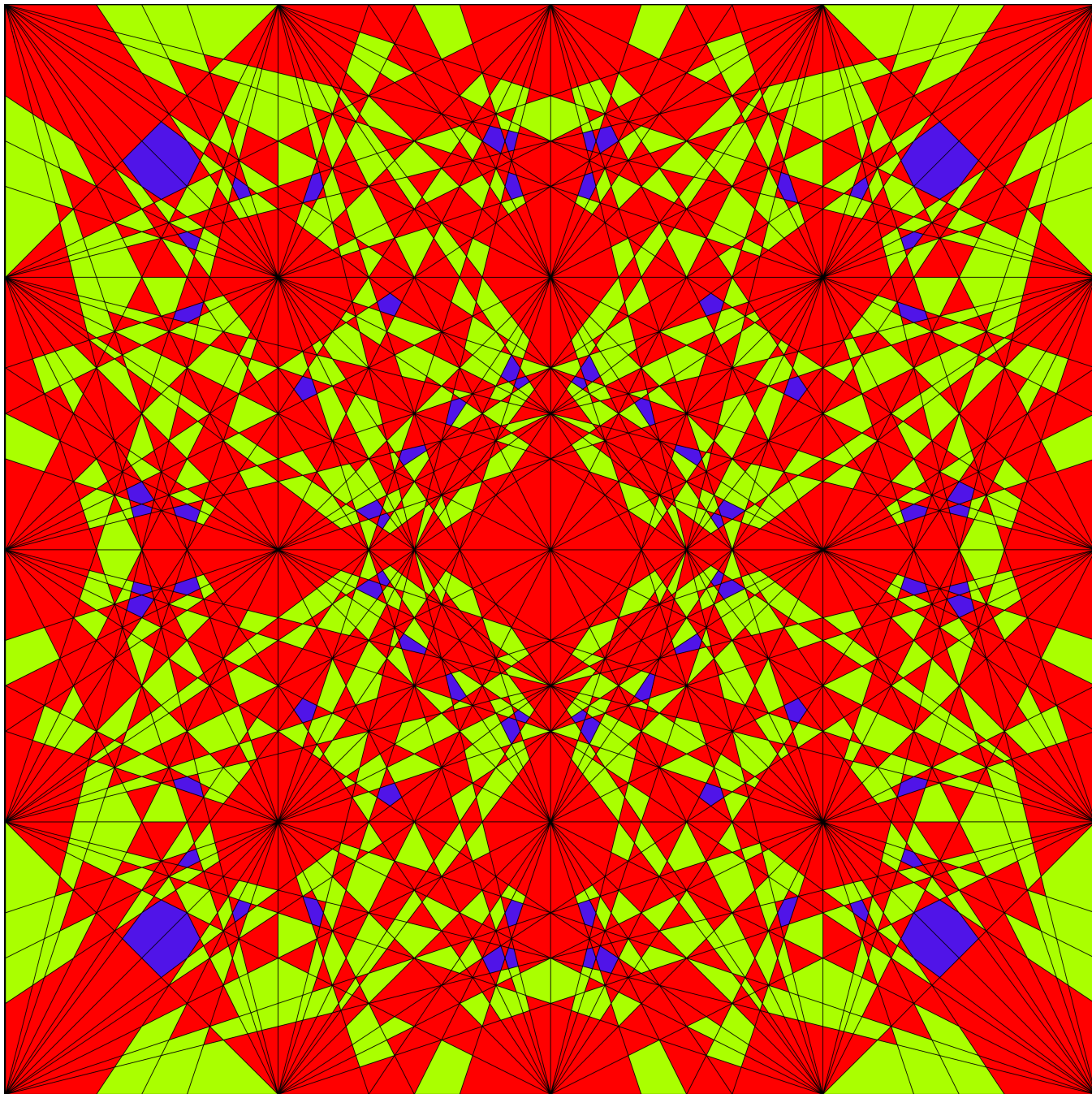


Figure 16: The graph $LC(4,4)$ (number-of-sides coloring): 2272 triangles (red), 936 quadrilaterals (yellow), 80 pentagons (purple).



Figure 17: The graph $LC(5,5)$ (random coloring).

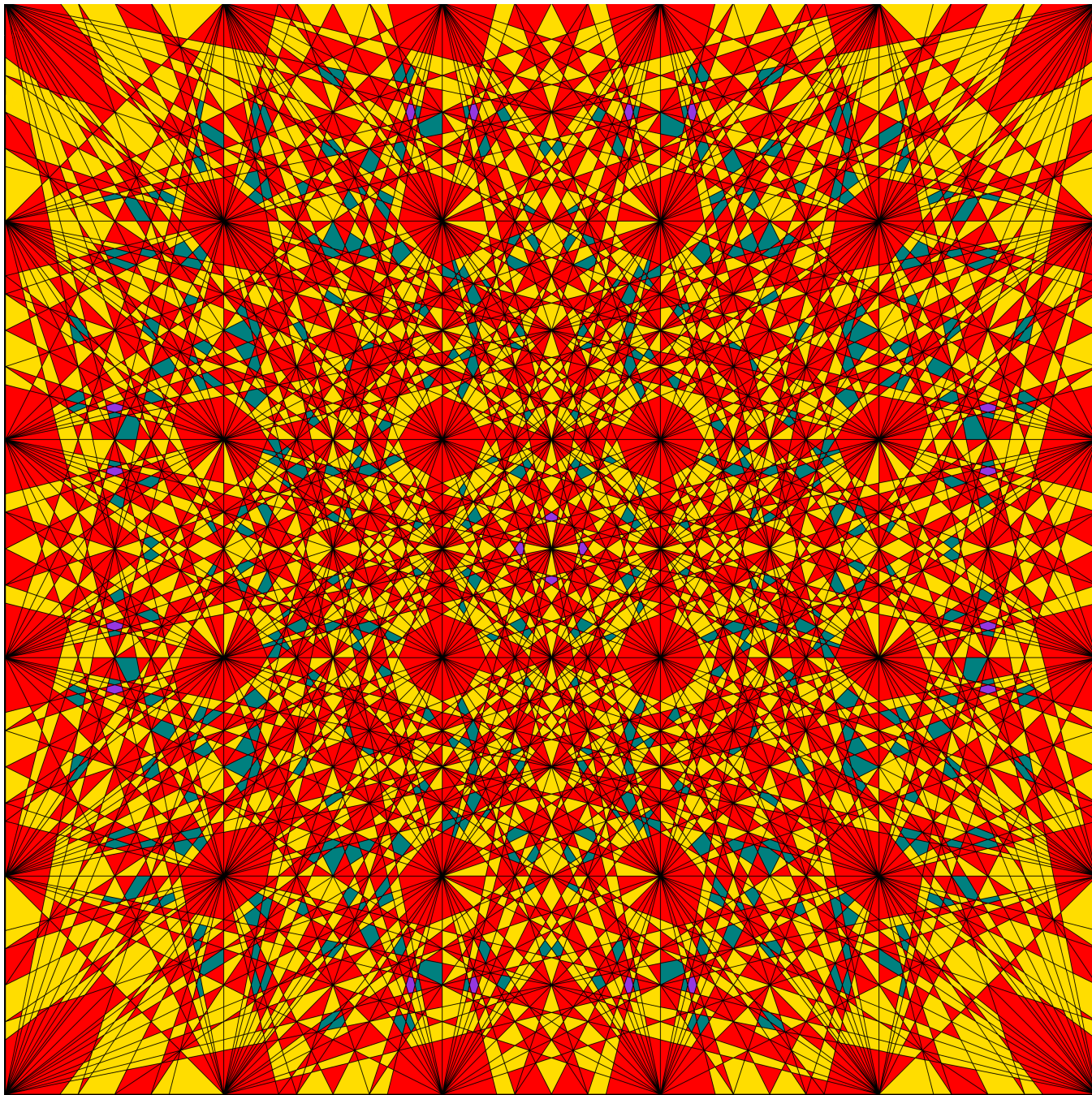


Figure 18: The graph $LC(5, 5)$ (number-of-sides coloring): 11200 triangles (red), 5196 quadrilaterals (yellow), 496 pentagons (green), 20 hexagons (purple).

References

- [1] L. Blomberg, S. R. Shannon, and N. J. A. Sloane, Graphical enumeration and stained glass windows, 1: rectangular grids, *INTEGERS*, (2021) to appear.
- [2] The OEIS Foundation Inc., *The On-Line Encyclopedia of Integer Sequences*, 2021; <https://oeis.org>.