The On-Line Encyclopedia of Integer Sequences

An illustrated guide with many unsolved problems

Neil J. A. Sloane, Math 640 Guest Lecture, Rutgers University, April 28 2022

Outline

- OEIS Basics
- Submitting, Editing, Rejecting, Comments
- Gerrymandering
- Facial Recognition and LES Sequences
- Other Topics

OEIS Basics

The database: oeis.org
The OEIS Foundation Inc.: oeisf.org

- Accurate information about 350000 sequences
- Definition, formulas, references, links, programs
- View as list, table, graph, music!
- 30 new entries, 50 updates every day
- Traffic: 1 million hits/day
- 10000 articles and books cite the OEIS
- Often called one of best math sites on the Web
- Begun in 1964 by NJAS. Maintained by dedicated group of unpaid editors. More editors are badly needed.

A serious scientific database.
This is not Social Media.

Also includes fractions, decimal expansions, triangles, arrays.

Since 2021, Russ Cox is President of The OEIS Foundation, NJAS is Chairman.

A classical example: The EKG Sequence

A064413 EKG sequence (or ECG sequence): a(1) = 1; a(2) = 2; for n > 2, a(n) = smallest number not already used which shares a factor with a(n-1).

1, 2, 4, 6, 3, 9, 12, 8, 10, 5, 15, 18, 14, 7, 21, 24, 16, 20, 22, 11, 33, 27, 30, 25, 35, 28, 26, 13, 39, 36, 32, 34, 17, 51, 42, 38, 19, 57, 45, 40, 44, 46, 23, 69, 48, 50, 52, 54, 56, 49, 63, 60, 55, 65, 70, 58, 29, 87, 66, 62, 31, 93, 72, 64, 68, 74, 37, 111, 75, 78, 76, 80, 82 (list; graph; refs; listen; history; edit; text; internal format)

OFFSET 1,2

LINKS Zak Seidov, Table of n, a(n) for n = 1..10000

David L. Applegate, Hans Havermann, Bob Selcoe, Vladimir Shevelev, N. J. A. Sloane, and Reinhard Zumkeller, The Yellowstone Permutation, arXiv preprint arXiv:1501.01669 [math.NT], 2015 and J. Int. Seq. 18 (2015) 15.6.7.

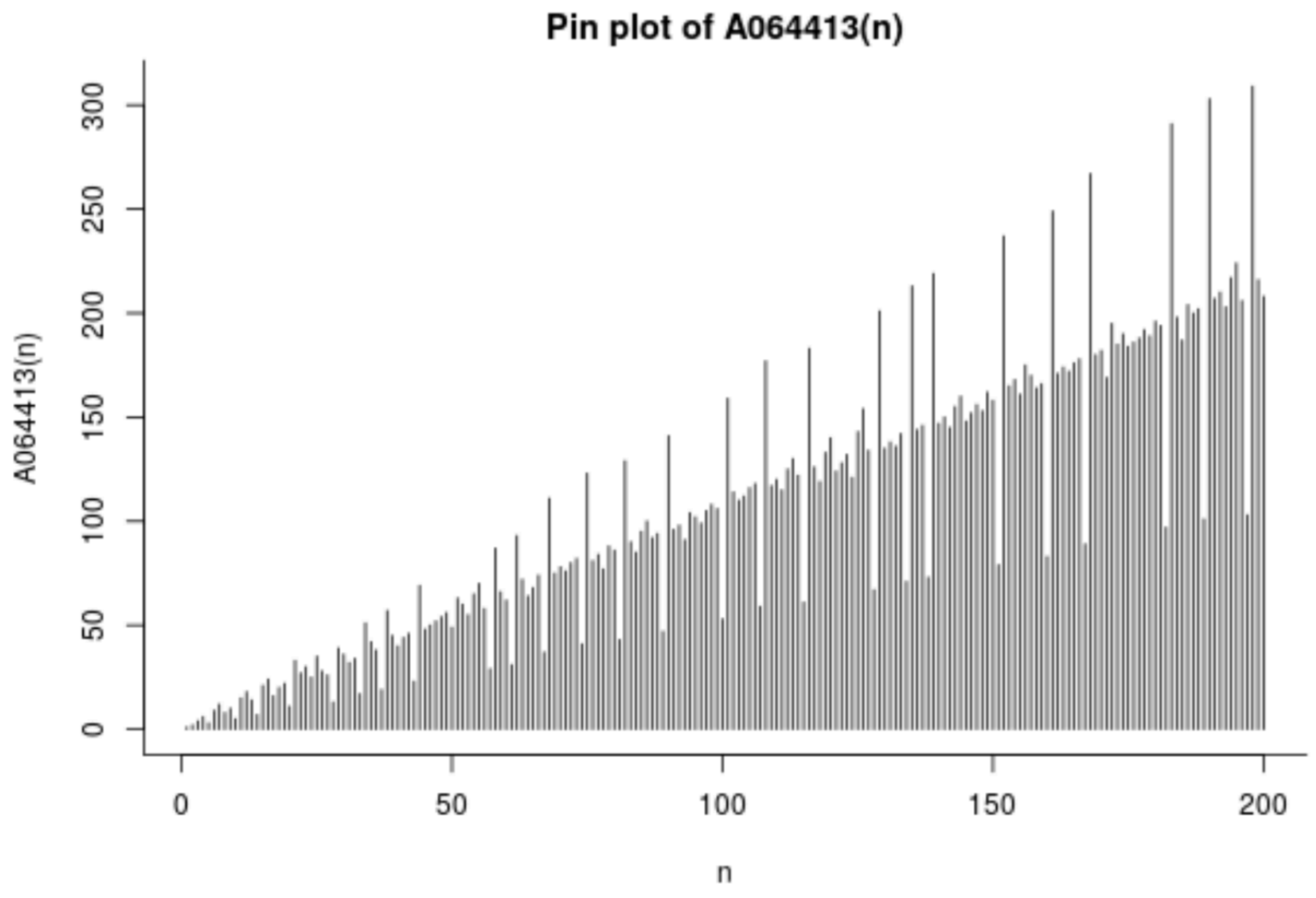
Michael De Vlieger, Annotated plot of a(n) for n=1..120, showing prime p in red, 2p in blue, 3p in green, and other terms in gray.

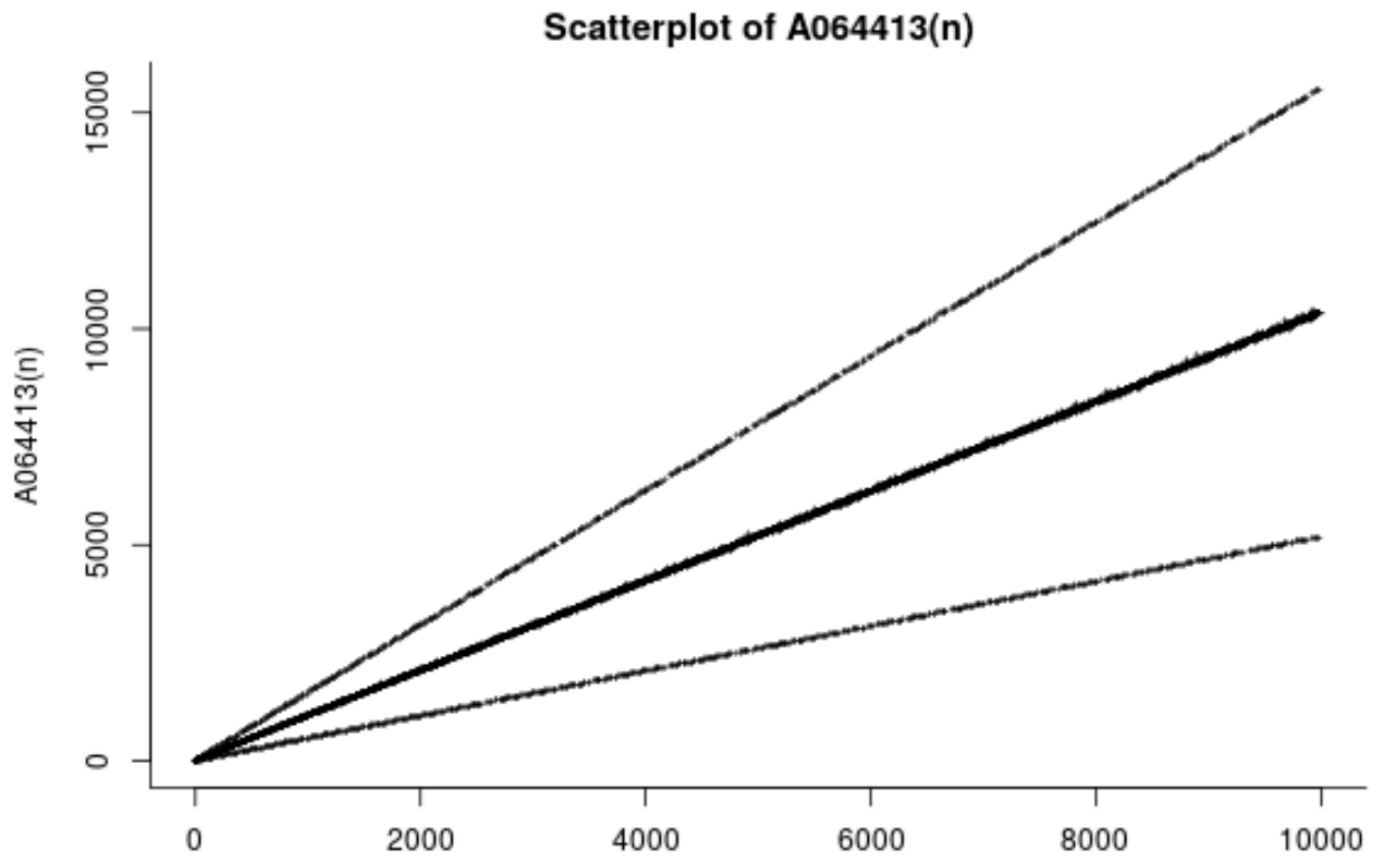
b-file

1	1
2	2
3	4
4	6
5	3
6	9
7	12
8	8
9	10
10	5
11	15
12	18

A064413 as a graph

When you click "graph" you get a pin plot of 200 terms, and a scatterplot of all terms from the b-file





EKG Sequence (A64413)

1, 2, 4, 6, 3, 9, 12, 8, 10, 5, 15, ...

a(1)=1, a(2)=2, a(n) = min k such that

- GCD { a(n-1), k } > 1
- k not already in sequence

Theorem: The EKG sequence is a permutation of the natural numbers 1, 2, ...

"Homework": Give a proof using the "Lean" Proof Assistant.
See Rutgers Colloq. talk 4/20/22 by Heather Macbeth.
Reason: There are similar sequences where the analogous theorems are either very tricky or missing.

LES with GCD(a(n-1),a(n)) > 1 for n>2.

- Jonathan Ayres, 2001

- Analyzed by Lagarias, Rains, NJAS, Exper. Math., 2002

Submitting, Editing, Rejecting, Comments

Lookup | Welcome | Wiki | Register | Music | Plot 2 | Demos | Index | Browse | More | WebCam
Contribute new seq. or comment | Format | Style Sheet | Transforms | Superseeker | Recent
The OEIS Community | Maintained by The OEIS Foundation Inc.

Last modified April 18 11:41 EDT 2022. Contains 352827 sequences. (Running on oeis4.)

Editing is a great way to come across new and interesting problems!

You will often see problems of the "Drop everything and work on this" class.

Editing https://oeis.org/draft

Proposed drafts

(In many browsers, middle-clicking on any of the links below will open it in a new tab.
See also drafts not yet proposed for review.)

Showing entries 1-100 | older changes

Sequence	Status	First Edited	Last Active	Draft by	Reviewed or edited by
A054342	proposed +30 −26	Apr 16 08:02	Apr 16 22:46	Jeppe Stig Nielsen (2/11)	Jon E. Schoenfield
A123123	proposed +8 −4	Apr 12 21:09	Apr 16 22:33	Christoph B. Kassir (3/3)	Jon E. Schoenfield
A000217	proposed +1	Apr 16 21:44	Apr 16 22:19	Ambrosio Valencia-Romero (3/3)	Jon E. Schoenfield
A352949	proposed +56 −2	Apr 10 14:40	Apr 16 22:17	Rémi Guillaume (1/3)	Jon E. Schoenfield
A353019	proposed +17 −2	Apr 16 20:01	Apr 16 21:42	Ambrosio Valencia-Romero (3/3)	
A352991	proposed +19 −2	Apr 16 08:43	Apr 16 20:13	Marco Ripà (3/3)	Kevin Ryde
A129805	proposed +3 −1	Apr 16 19:33	Apr 16 19:35	M. F. Hasler (15/19)	
A146566	proposed +3 −1	Apr 16 19:11	Apr 16 19:30	M. F. Hasler (15/19)	
A086443	proposed +8 −3	Apr 15 19:34	Apr 16 18:52	Enrique Navarrete (2/3)	Kevin Ryde
A352040	proposed +14 −2	Apr 16 16:38	Apr 16 17:25	Chai Wah Wu (1/19)	Amiram Eldar

https://oeis.org/draft

Proposed drafts

(In many browsers, middle-clicking on any of the links below will open it in a new tab.
See also drafts not yet proposed for review.)

newer changes | Showing entries 301-309

Sequence	Status	First Edited	Last Active	Draft by	Reviewed or edited by
A351212	proposed +11 −2	Mar 09 14:10	Mar 12 02:16	Mark Andreas (3/3)	Michel Marcus
A059010	proposed +4 −1	Mar 09 17:24	Mar 11 21:02	John Keith (2/3)	Kevin Ryde
A057030	proposed +6 −1	Mar 10 05:17	Mar 10 05:30	Mikhail Kurkov (3/3)	Michel Marcus
A057032	proposed +5 −1	Mar 10 05:24	Mar 10 05:24	Mikhail Kurkov (3/3)	
A351214	proposed +11 −2	Mar 09 14:16	Mar 09 23:56	Mark Andreas (3/3)	Michel Marcus
A003726	proposed +4 −4	Mar 07 15:08	Mar 09 20:10	John Keith (2/3)	Kevin Ryde
A352249	proposed +7 −2	Mar 09 15:33	Mar 09 16:10	Louise Romana Wade (1/3)	
A007445	proposed +15	Mar 09 01:37	Mar 09 04:33	Davide Rotondo (1/1)	
A130102	proposed +9 −4	Mar 08 21:06	Mar 08 23:34	Ambrosio Valencia-Romero (3/3)	Michel Marcus

A Recently Accepted Sequence (After Editing)

A351922, J. Dushoff (February 2022)
Informally, number of inequivalent expressions involving n variables and the operations +, -, *, /, ^

After editing:

Consider well-formed strings consisting of n operands, n - 1 binary operators +, -, *, /, ^, and n-1 pairs of parentheses, and call two such strings equivalent if they are algebraically identical; a(n) is the number of equivalence classes.

1 8 146 4294

`a+b, a-b, b-a, a*b, a/b, b/a, a^b, b^a.`

We need more terms and a program.

If exponentiation is excluded, we get a much older sequence, A140606, Zhao Hui Du, 2008, with 300 terms, but no program

1, 6, 68, 1170, 27142, 793002

We need a program and verification of these terms. Also there are references in Chinese which are incomplete (authors' names, titles of articles)

Examples of Rejected Sequences or Comments

Primes of the form k! + 7

k=3 gives 13, k=4 gives 31, k=5 gives 127, k=6 gives 727
and there are no more. Rejected.

But the editors created 3 better versions and gave him credit.

E.g.: Triangle: row n lists primes of the form prime(n) + k!

Primes of the form 50^k + 51.

Depends on an arbitrary and large parameter.

101, 2551, 122070312500000000000051

A proposed comment for the triangular numbers:

Number of unique sub-game partitions of a symmetric n-person zero-sum matching pennies game with anonymous players.

**For core sequences like this we only add the most important comments - otherwise we would have a million comments.
I don't think this comment qualifies.**

https://oeis.org/wiki/Deleted_sequences

April 2022 [edit]

- A353009 (Hemjyoti Nath) Arbitrary parameter, NOGI - Hugo Pfoertner (Talk), 04:18, 16 April 2022 (EDT)
- A352991 (H Nath) NFO - N. J. A. Sloane 05:53, 16 April 2022 (EDT)
- A352081 (P Duckett) too contrived - N. J. A. Sloane 05:43, 16 April 2022 (EDT)
- A352040 (J Rivera Romeu) NOGI - N. J. A. Sloane 05:43, 16 April 2022 (EDT)
- A352082 (B Benfield) rifo A352083 - N. J. A. Sloane 05:43, 16 April 2022 (EDT)
- A353006 (Hemjyoti Nath) duplicate of A057733 - Joerg Arndt (Talk) 11:14, 15 April 2022 (EDT)
- A352908 (O Pol) too close to A006877 - N. J. A. Sloane 00:39, 15 April 2022 (EDT)
- A352978 (R El Haddad) duplicate of A112494 - Alois P. Heinz (Talk) 14:47, 13 April 2022 (EDT)
- A351867 (J Krizek) too artificial, NOGI - N. J. A. Sloane 11:33, 12 April 2022 (EDT)
- A352635 (T Vlasic) not well-defined - N. J. A. Sloane 11:33, 12 April 2022 (EDT)
- A353245 (Shannon) withdrawn - Andrey Zabolotskiy (talk) 04:07, 12 April 2022 (EDT)
- A351865 (J Krizek) NOGI - N. J. A. Sloane 15:45, 10 April 2022 (EDT)
- A352802 (Rodrigo) duplicate of A059022 - R. J. Mathar (talk) 13:48, 9 April 2022 (EDT)
- A351745 (A Ratushnyak) rifo A106747 - N. J. A. Sloane 11:01, 7 April 2022 (EDT)

NOGI = Not Of General Interest
NFO = Not For OEIS (not up to our standards)
rifo = retired in favor of (inferior to an existing sequence)

Comments from Users of the OEIS

CiteA

Quality

"*This note could not have been written without the valuable help of the OEIS.*" [Octavio Alberto Agustín Aquino, 2016]

"*... we acknowledge that the On-line Encyclopedia of Integer Sequences has been of great help in this project.*" [Per Alexandersson et al., 2019]

"*Finally, we would like to thank Prof. N.J.A. Sloane in particular since this work would not be possible without OEIS.*" [Alkan and Aybar, 2020]

"*After compiling the results of many explicit computations, we noticed that many of the numbers d_{n,r,S} appear in the existing literature in contexts far removed from the enumerative geometry of rank conditions; we owe this surprising (to us) observation to perusal of [Slo14].*" [P. Aluffi, 2014]

"*Remarkably, this exhaustive enumeration leads us exactly to the integer sequence A001792 of The On-Line Encyclopedia of Integer Sequences... .*" [Milica Andelic et al., 2016]

"*Very important to the results in this paper were the search sites KnotInfo by Cha and Livingston [CL11] and The On-Line Encyclopedia of Integer Sequences by Sloane [Slo11].*" [Cody Armond and Oliver T. Dasbach, 2011]

"*Using the results of these computer searches in the Online Encyclopedia of Integer Sequences, we discovered that this problem, when played on the square grid, is equivalent to several other known problems.*" [Boris Aronov et al., 2017]

"*Something which is rarely mentioned is the value of OEIS as a very comprehensive source of references to mathematical papers--probably the best there is.*" [Email from a user of the OEIS, Jan 24 2021]

"*The contribution you and the OEIS have made to mathematics as a whole is immeasurable and I cannot begin to thank you enough for creating such a brilliantly useful tool.*" [Email from a user of the OEIS, Mar 19 2021]

From XXX Mar 19 2018, Subject: Reminiscence from a young mathematician

Dear Neil, The other day, I had the occasion to use the OEIS, something I haven't done in nearly 15 years (as an algebraic geometer, I don't seem to get that many opportunities)! I was so happy to see it thriving.

I wanted to relay a bit of nostalgia and my heartfelt thanks. Back in the late 1990s, I was a high school in Oregon. While I was interested in mathematics, I had no significant mathematically creative outlet (working class family and subpar mathematics instruction) until I discovered the OEIS in the course of trying to invent some puzzles for myself. I remember becoming a quite active contributor through the early 2000s, and eventually at one point, an editor. My experience with the OEIS, and the eventual intervention of one of my high school teachers, catalyzed my interest in studying mathematics, which I eventually did at XXX College. I went on to a Ph.D. at the University of XXX, various postdocs, and am currently at XXX.

I wanted to thank you for seriously engaging with an 18 year old kid, even though I likely submitted my fair share of mathematically immature sequences.

I doubt I would have become a mathematician without the OEIS!

From one of the most famous computer scientists in the world

The OEIS will last at least as long as the Internet

Three other comments:

```
I am happy to welcome Russ Cox as President and want to take this
opportunity to thank every single one of you for all of the amazing work,
collaboration, criticism, and wisdom that you bring to the OEIS.  The OEIS is
no doubt, a world treasure.

Neil, thanks for everything you've done since you first published the Handbook
in the 1970's! Some of my best work wouldn't have been possible without the
incredible resource you created for all of us.

I hope the world will support the OEIS and ensure its survival for a long time.
```

Gerrymandering (1)

1. A341578 (Sean Chorney), A341721 (Don Reble), Feb 2021: Minimum number of votes needed to win with n voters if all districts must have same size

Rules

Two candidates, a and B, and n voters.
The voters are divided into d equal districts of size n/d.
The districts are winner-takes-all.
Tied districts go to neither candidate.

If there are an even number of districts, it is enough to win half the districts and tie in one further district.

Example: n = 36 voters: optimal strategy is three districts of 12 voters each, and then you can win with only 14 total votes (7+7+0).

EMPTY SQUARES ARE 22 “a” VOTES

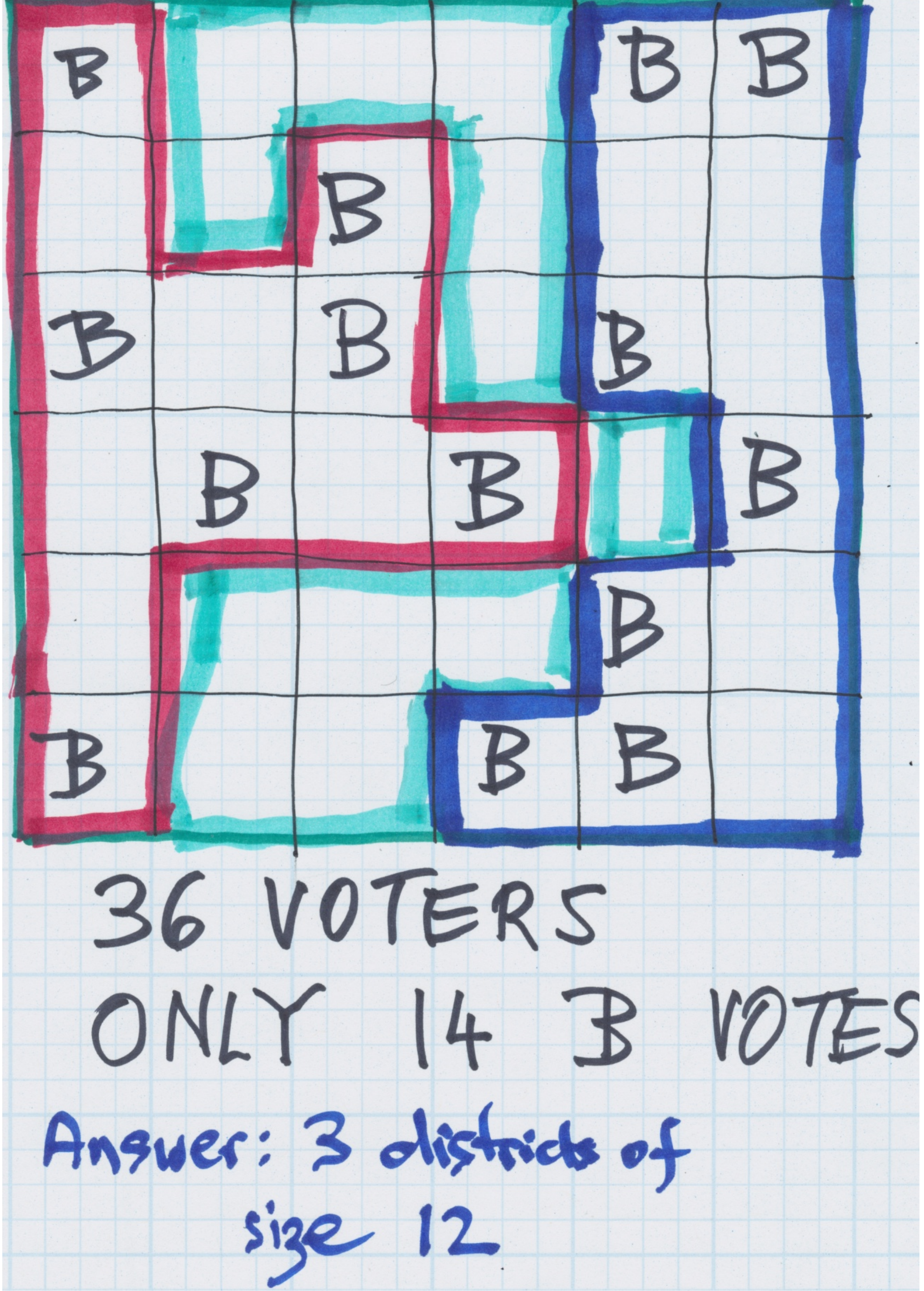
B	a	a	a	B	B
a	a	B	a	a	a
B	a	B	a	B	a
a	B	a	B	a	B
a	a	a	a	B	a
B	a	a	B	B	a

36 VOTERS

ONLY 14 B VOTES

EMPTY SQUARES ARE 22 A VOTES

Open Q.
What if the regions must be edge-connected?



A341578
Sean Chorney
Min. no. to win if n^2 voters

a(6)=14

A341721
Same, if V votes
b(36)=14

Gerrymandering (cont.)

Suggestion for a research project

Moon Duchin heads a study group at Tufts University (the Metric Geometry and Gerrymandering Group) which has produced many papers.

For instance, how can you detect, or prove, that Gerrymandering has taken place?

How to measure Gerrymandering?

There should be new sequences (as a function of the number of voters, or number of districts) that arise from this work!

Another resource:
Princeton Gerrymandering Project
"We bridge the gap between mathematics and the law to achieve fair representation through redistricting reform."
https://gerrymander.princeton.edu/

Do a Google Scholar search for Moon Duchin, Redistricting, to see many articles.

An old paper: Moon Duchin, Gerrymandering metrics: How to measure? What's the baseline? arXiv:1801.02004, Jan 06 2018

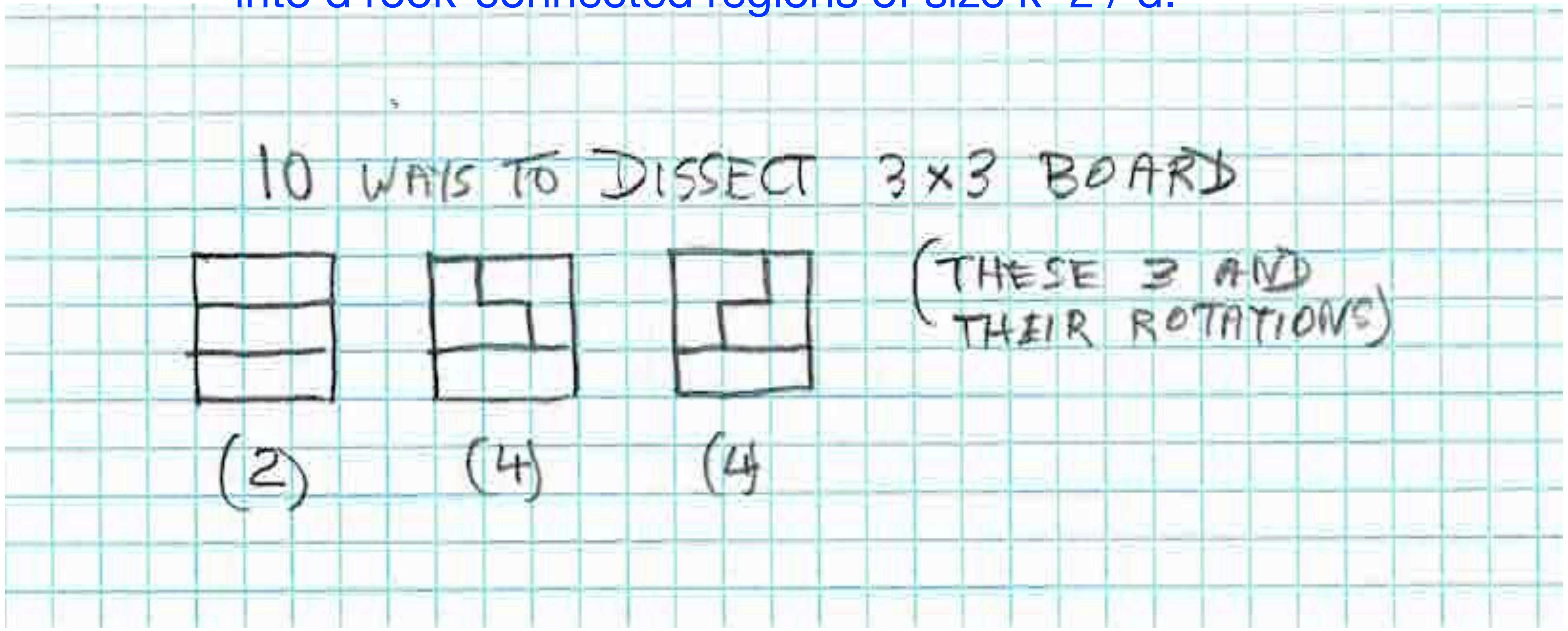
Gerrymandering (2)

What if the districts must be connected?

Lovely problem, much harder

T(k, d) = no. of ways to dissect a k X k square board
into d rook-connected regions of size k^2 / d.

T(3,3) = 10:



Gerrymandering (2), cont.

(A348452, A348456, A172477, A004003)

T(k, d) = no. of ways to dissect a k X k square board into d rook-connected regions of size k^2 / d.

$k\backslash d$	1	2	3	4	5	6	7	8	9	...	
1	1										
2	1	2	0	1							
3	1	0	10^a	0	0	0	0	0	1		
4	1	70^b	0	117	0	0	0	36^c	0	...	$1_{@16}$
5	1	0	0	0	4096	0	0	0	0	...	$1_{@25}$
6	1	80518	264500	442791	0	451206	0	0	✓	...	$1_{@36}$
7	1	0	0	0	0	0	✓	0	0	...	$1_{@49}$
8	1	?	0	?	0	0	0	✓	0	...	$1_{@16}$

Most wanted: T(8,2) = no. of ways to cut chessboard into 2 rook-connected regions of area 32

Ignore colors of chessboard squares; rotations, reflections count as different;   regions need not have same shape.

How large will T(8,2) be, roughly?   How would you program it?   How would you parallelize it?

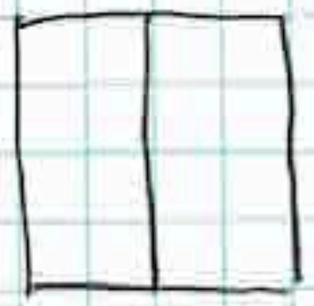
Paul Zimmermann et al. in 2020 solved one of the RSA Challenge Problem,
It took them 2700 core years. How does T(8,2) compare?

Gerrymandering (2), cont.

T(4,2) = 70:

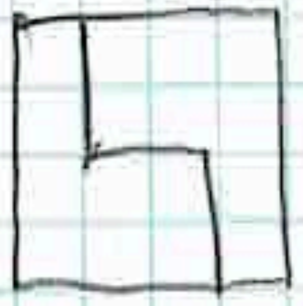
70 WAYS TO DISSECT 4 × 4 BOARD

CODE — BOUNDARY — COUNT



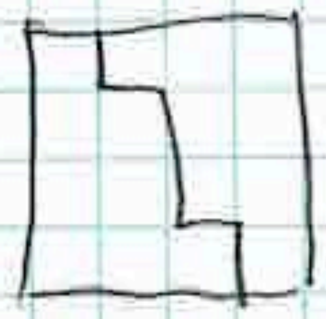
2222 B = 4 # = 2
C = 0 G = 4
CORNERS GROUP ORDER

1



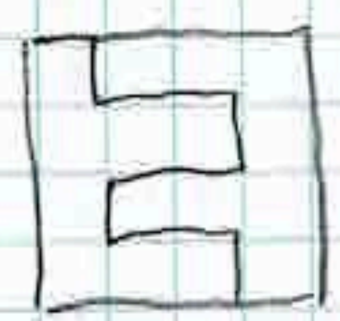
1133 B = 6 # = 4
C = 2 G = 2

2



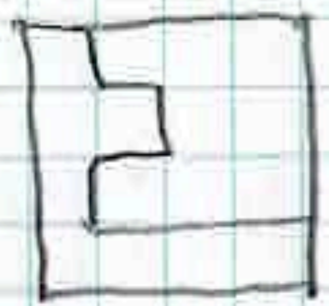
1223 B = 6 # = 4
C = 4 G = 2

3



1313 B = 10 # = 4
C = 6 G = 2

4

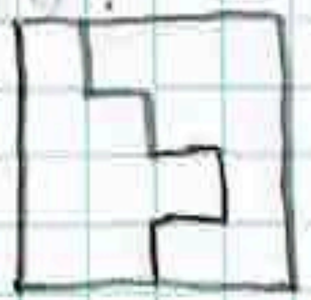


1214 B = 8 # = 8
C = 5 G = 1

5

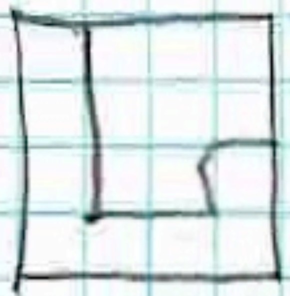
1322 B = 7 # = 8
C = 4 G = 1

6



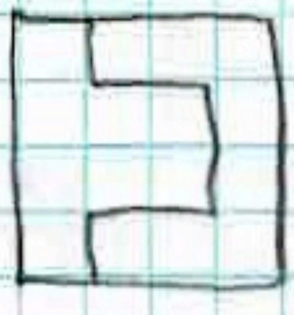
1232 B = 7 # = 8
C = 6 G = 1

7



1114 B = 7 # = 8
C = 3 G = 1

8

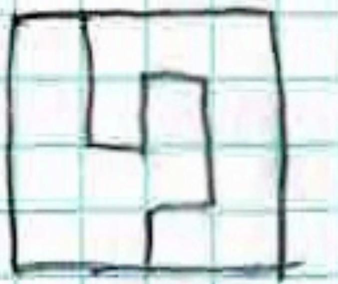


1331 B = 8 # = 4
C = 4 G = 2

9

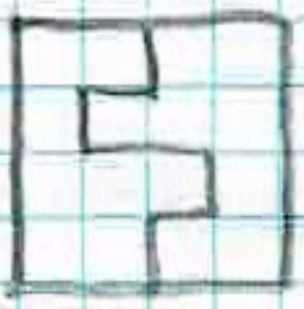
1124 B = 6 # = 4
C = 3 G = 2

10



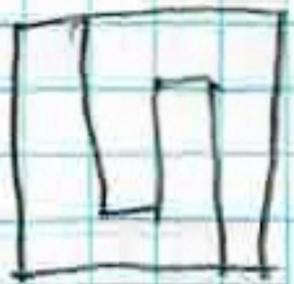
1132 B = 9 # = 8
C = 6 G = 1

11



2132 B = 8 # = 4
C = 6 G = 2

12



1113 B = 10 # = 4
C = 4 G = 2

13

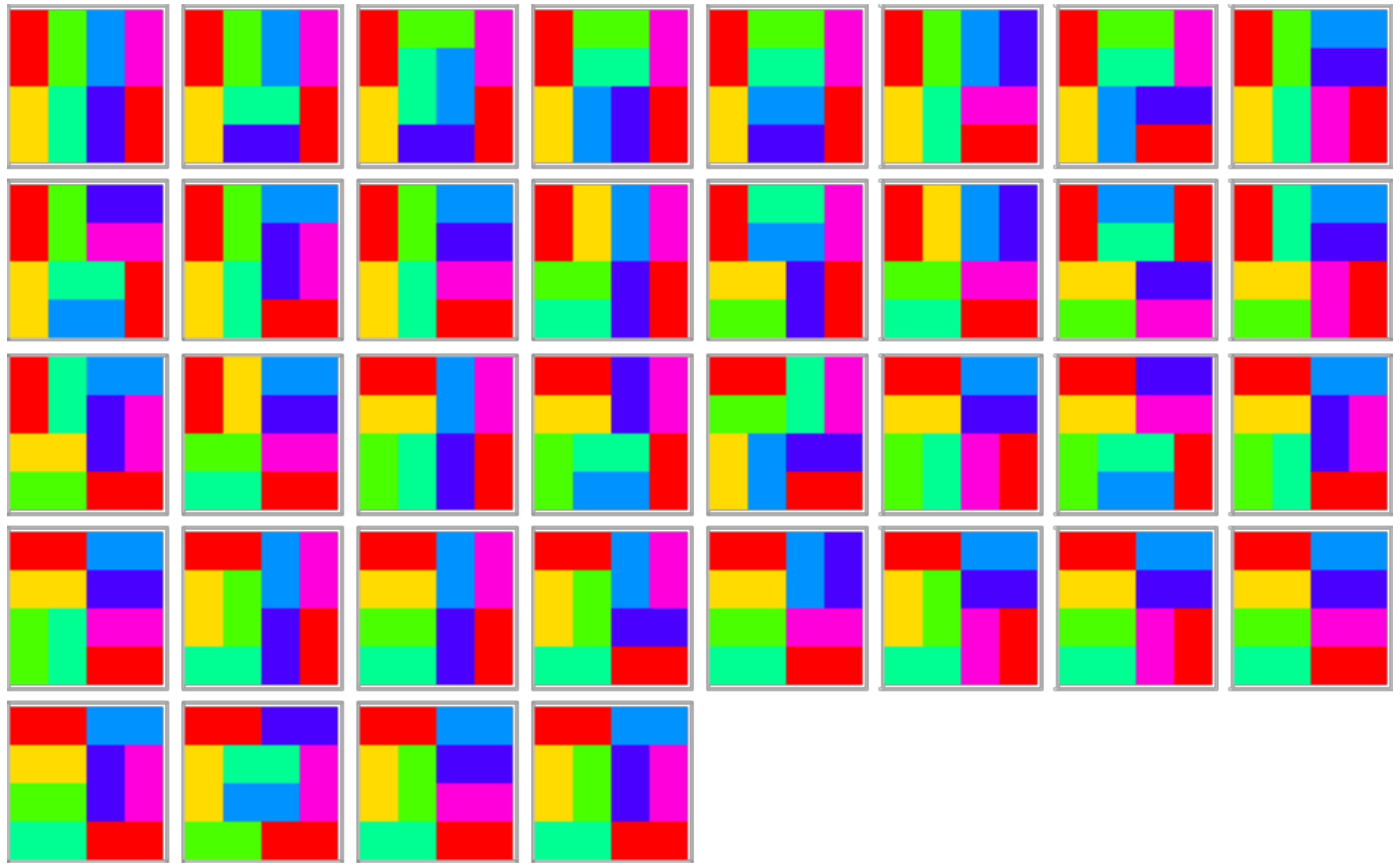
TOTAL = 70

Tiling a Square with Dominoes

T(4,8) = 36:

36 ways to tile a 4X4 square

a(2)=36



1, 2, 36, 6728, 12988816, 258584046368, 53060477521960000, ...

(A4003)

$$a(n) = \prod_{j=1}^{n} \prod_{k=1}^{n} \left(4\cos^2 \frac{j\pi}{2n+1} + 4\cos^2 \frac{k\pi}{2n+1} \right)$$

(Kastelyn, 1961)

"Facial Recognition" and LES Sequences

Lexicographic Order on Sequences of Nonnegative Integers

`(blank) < 0 < 1 < 2 < 3 < 4 < ...`

1, 2, 4, 6, ... comes before 1, 2, 5, 6, ...

1, 2 (blank) comes before 1, 2, 0, 0, 5, ...

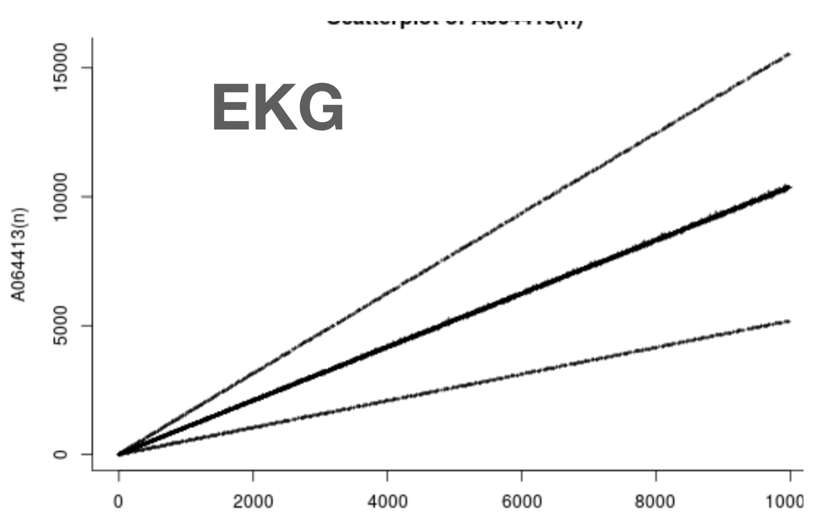
"LES" class of sequences: Lexicographically Earliest Infinite Sequence of Distinct Positive Numbers Such That ...

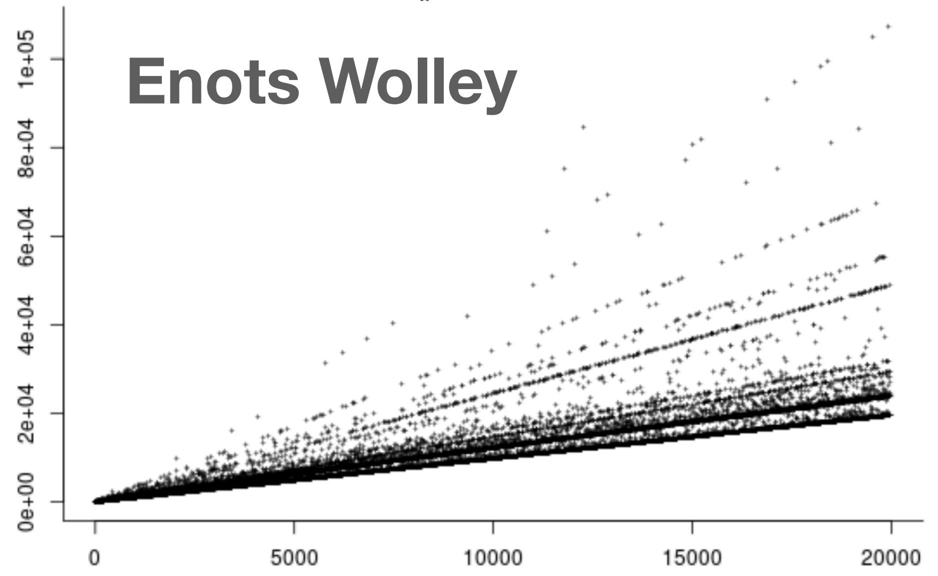
EKG sequence is a classical example: LES such that gcd(a(n-1), a(n)) > 1 for n>2

(A064413)

Enots Wolley: LES such that gcd(a(n-1), a(n)) > 1 and gcd(a(n-2),a(n)) = 1 for n>2

(A336957)





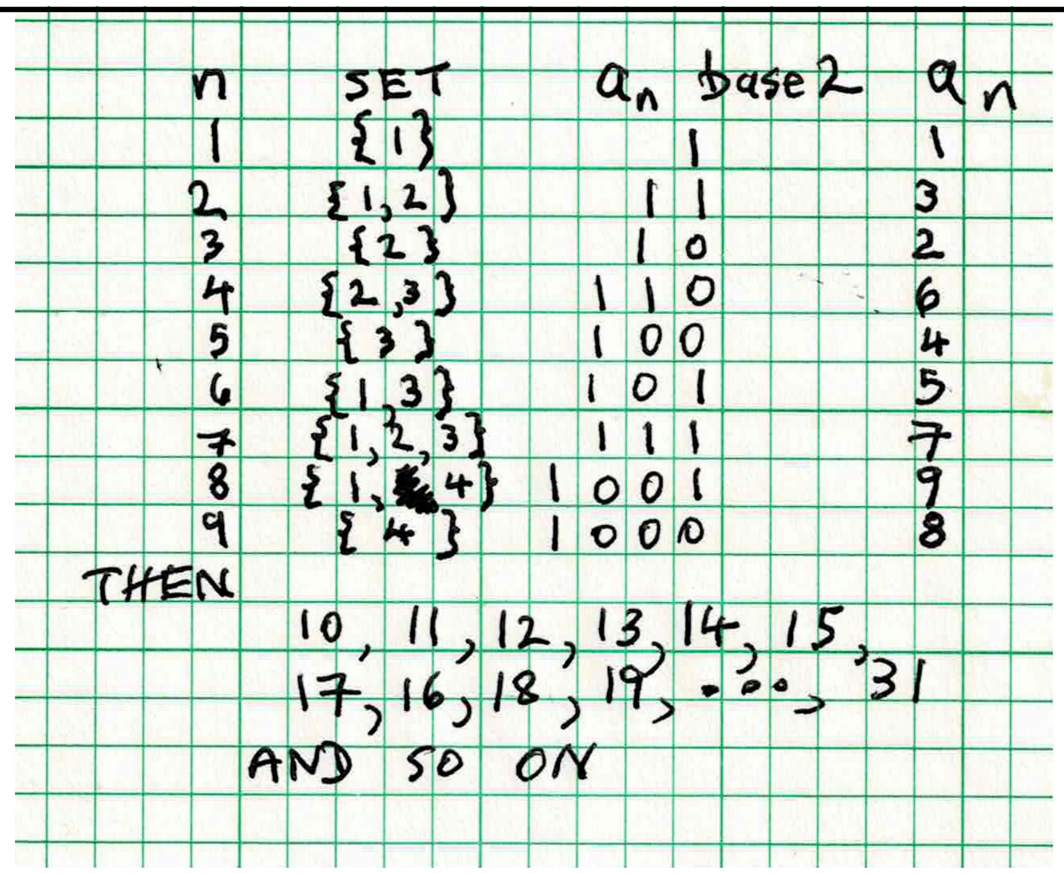
"Facial Recognition" (cont.)

Set theory analogs of EKG etc.

Replace gcd(x, y) = 1 with $x \cap y = \varnothing$

Replace gcd(x, y) > 1 with $x \cap y \neq \varnothing$

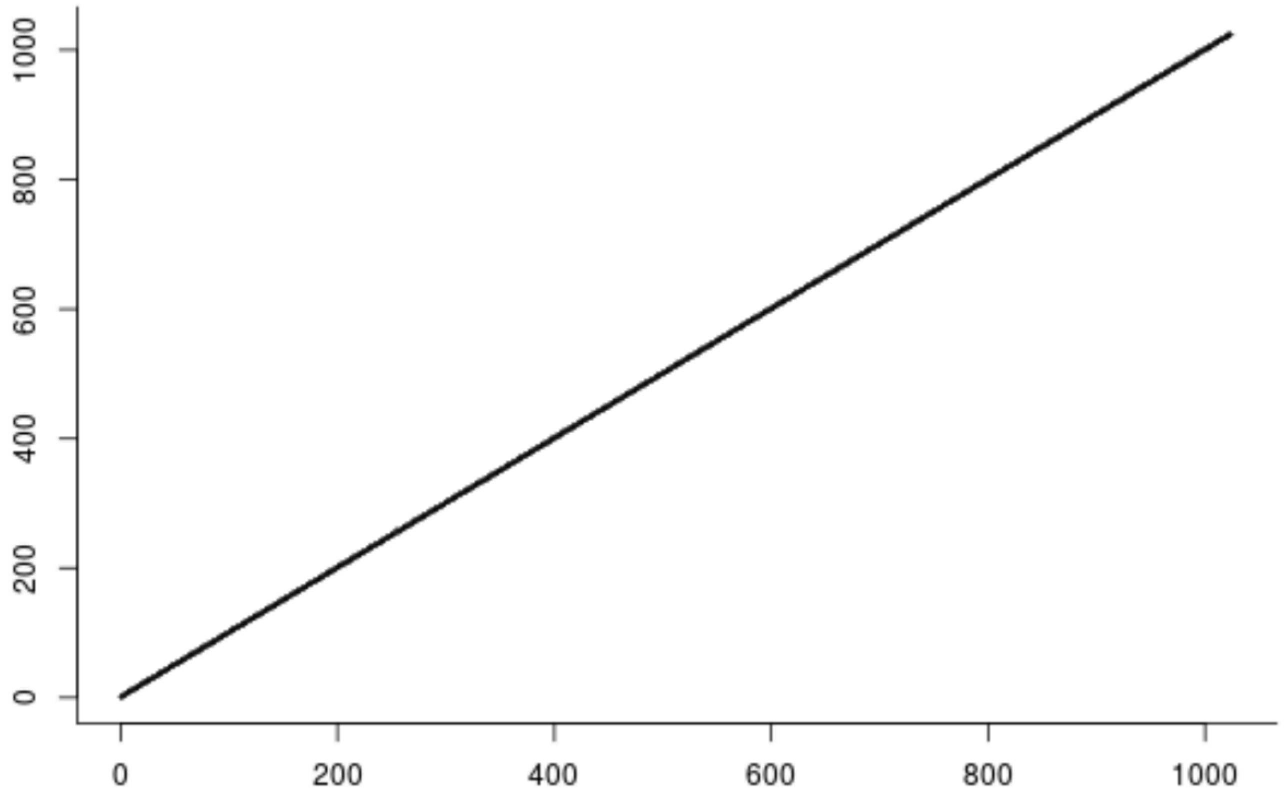
Set theory analog of EKG: $a_{n-1} \cap a_n \neq \varnothing$ as subsets of N



n	SET	a_n base 2	a_n
1	{1}	1	1
2	{1,2}	11	3
3	{2}	10	2
4	{2,3}	110	6
5	{3}	100	4
6	{1,3}	101	5
7	{1,2,3}	111	7
8	{1,4}	1001	9
9	{4}	1000	8

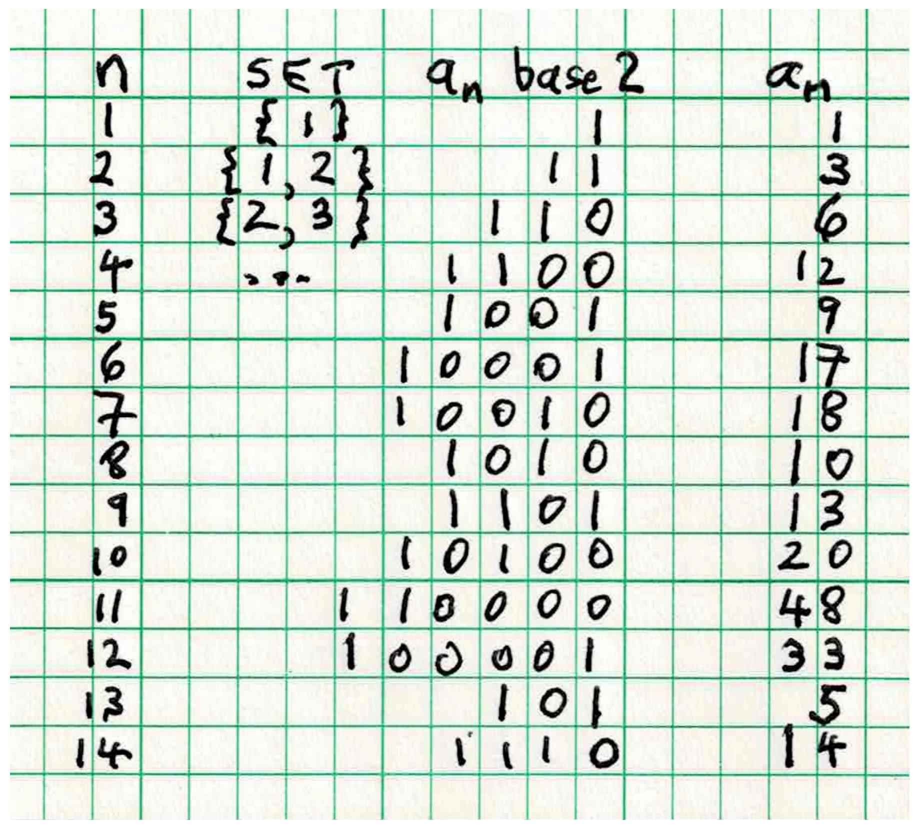
THEN 10, 11, 12, 13, 14, 15, 17, 16, 18, 19, ..., 31 AND SO ON

A115510



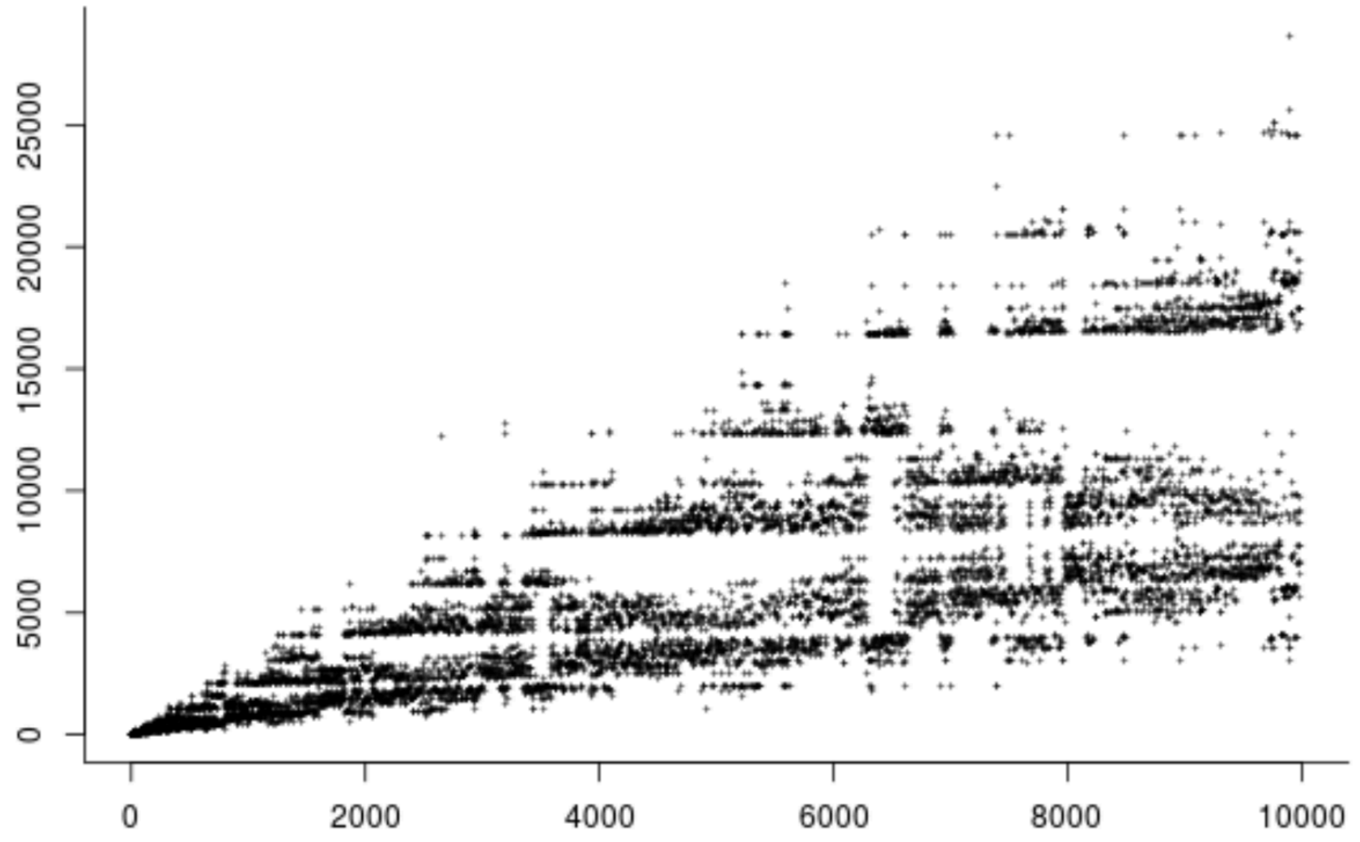
Easy Theorem: Every number appears

Set theory analog of Enots Wolley: add $a_{n-2} \cap a_n = \varnothing$ and $a_n \backslash a_{n-1} \neq \varnothing$



n	SET	a_n base 2	a_n
1	{1}	1	1
2	{1,2}	11	3
3	{2,3}	110	6
4	...	1100	12
5		1001	9
6		10001	17
7		10010	18
8		1010	10
9		1101	13
10		10100	20
11		110000	48
12		100001	33
13		101	5
14		1110	14

A338833



What ???

Theorem (Nathan Nichols): This is a permutation of { N excluding 2,4,8,16,... }

"Homework" 1. Check the proof. [I really hope someone - or some collaboration - will do this. I have not checked it myself. It needs to be done.]
2. Can the proof be adapted to show that Enots Wolley is a permutation of N ?

"Facial Recognition" (cont.)

In fact there is a simple and basic sequence with that same mysterious graph!

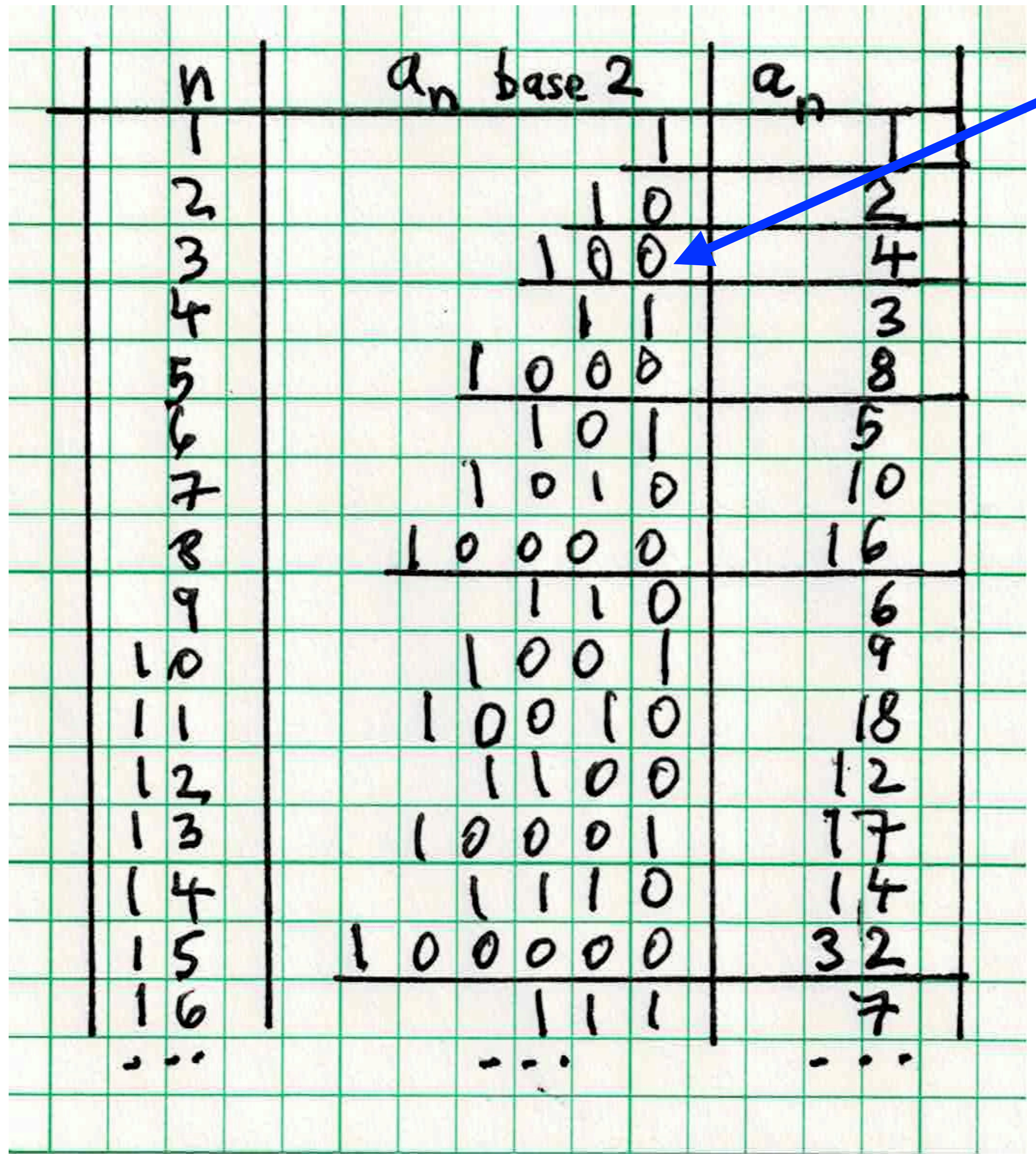
Number theory version	Set theory version
LES, gcd(a(n-1), a(n)) = 1	LES, $a_{n-1} \cap a_n = \varnothing$

1, 2, 3, 4, 5, 6, ...

(N, A000027)

The Natural Numbers

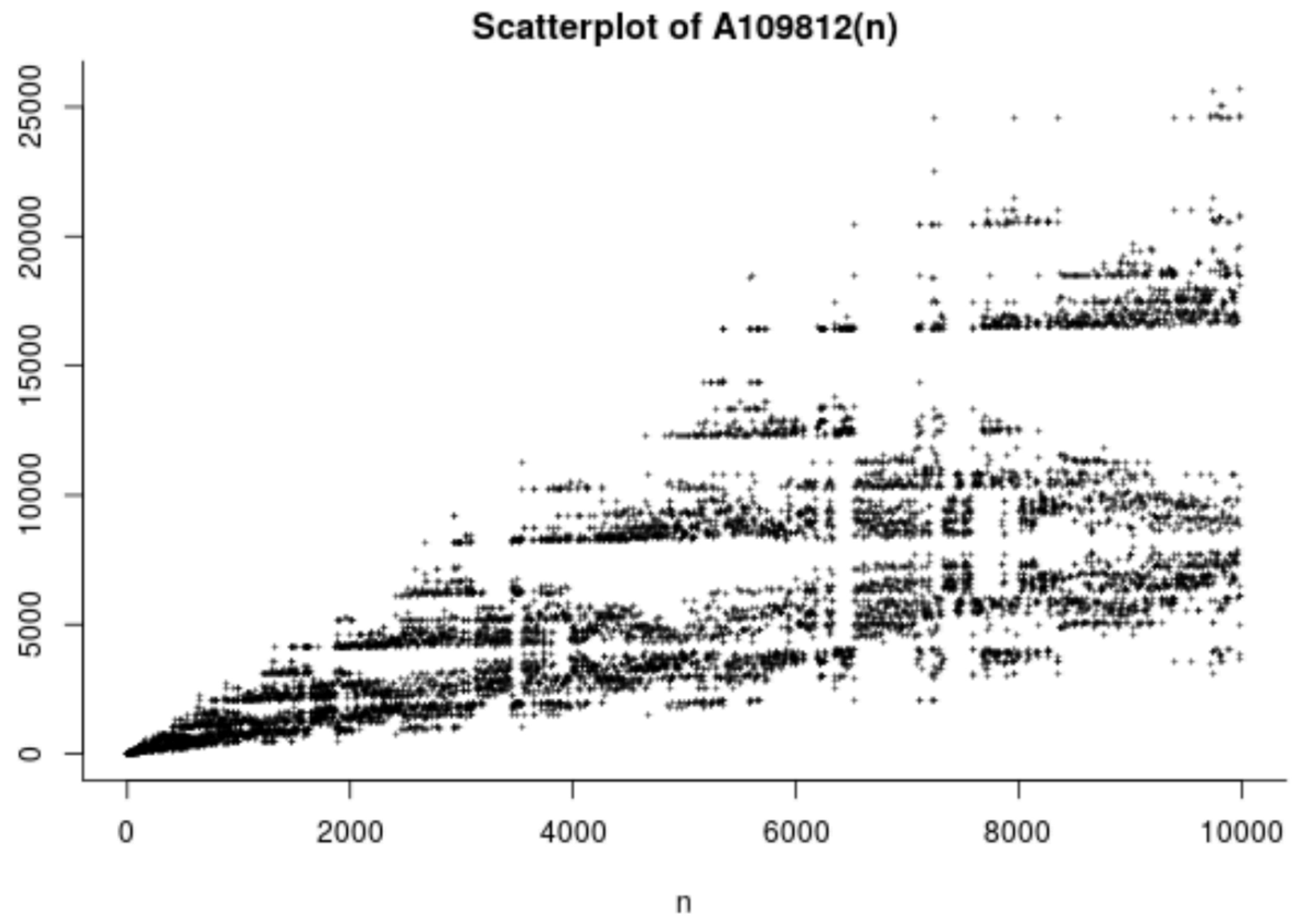
Binary expansions of successive terms must be disjoint



n	a_n base 2	a_n
1	1	1
2	10	2
3	100	4
4	11	3
5	1000	8
6	101	5
7	1010	10
8	10000	16
9	110	6
10	1001	9
11	10010	18
12	1100	12
13	10001	17
14	1110	14
15	100000	32
16	111	7
...	...	...

The "Tetris" Sequence

(A109812)



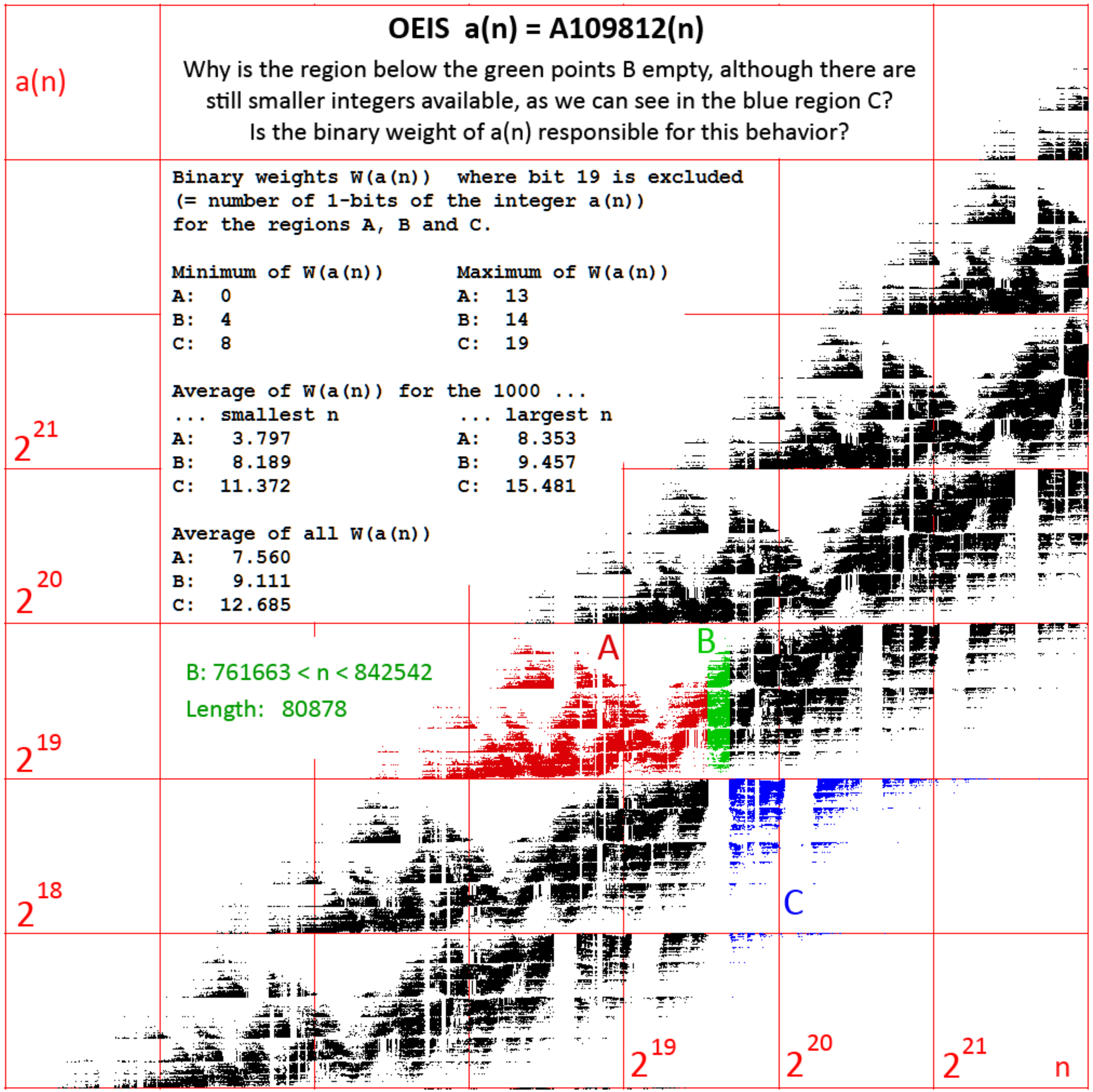
The same mysterious graph!

"Facial Recognition" (cont.)

The Tetris Sequence A109812

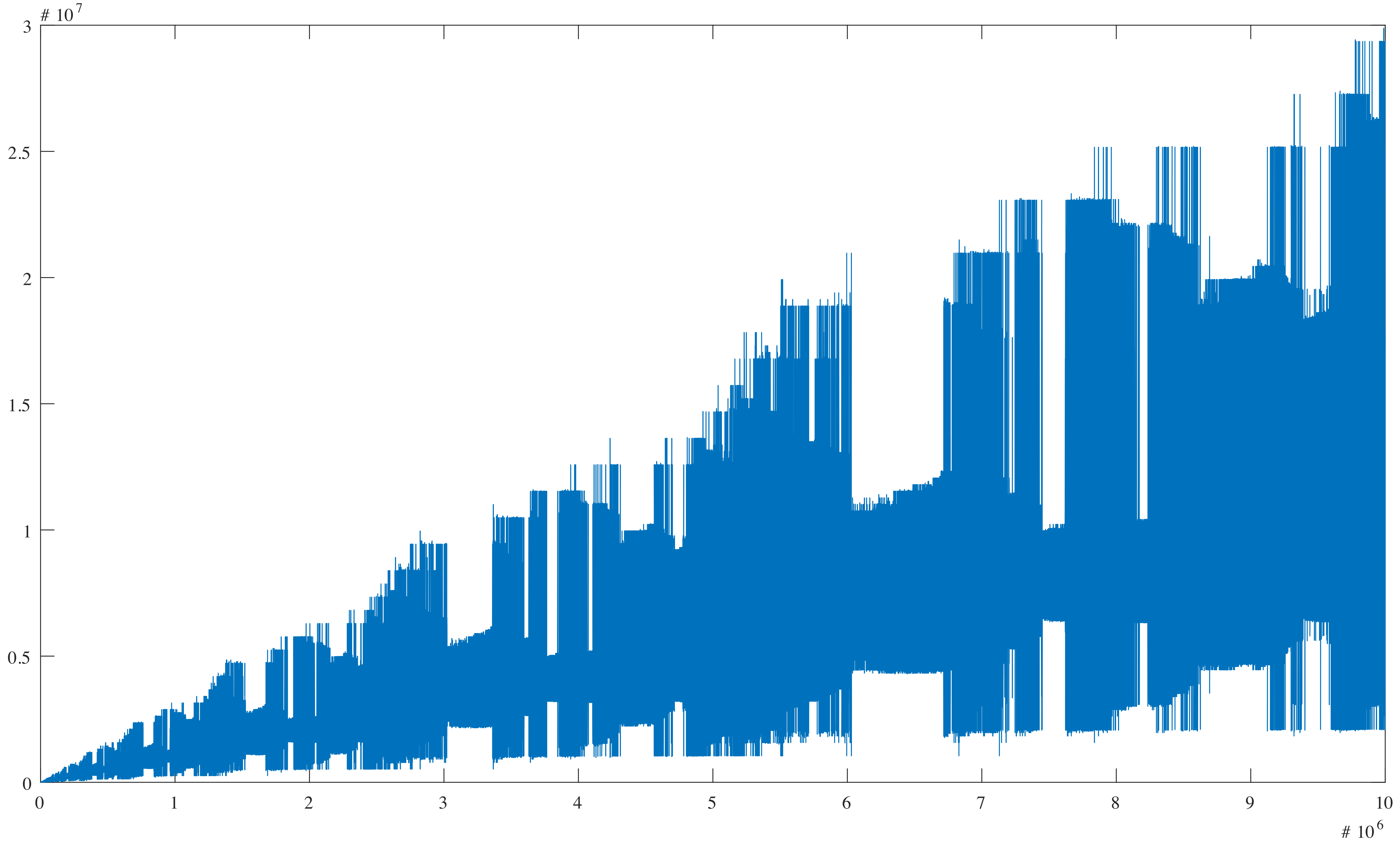
The subject of intensive study in April 2022 by me and many friends

An analysis by Walter Trump
April 14 2022



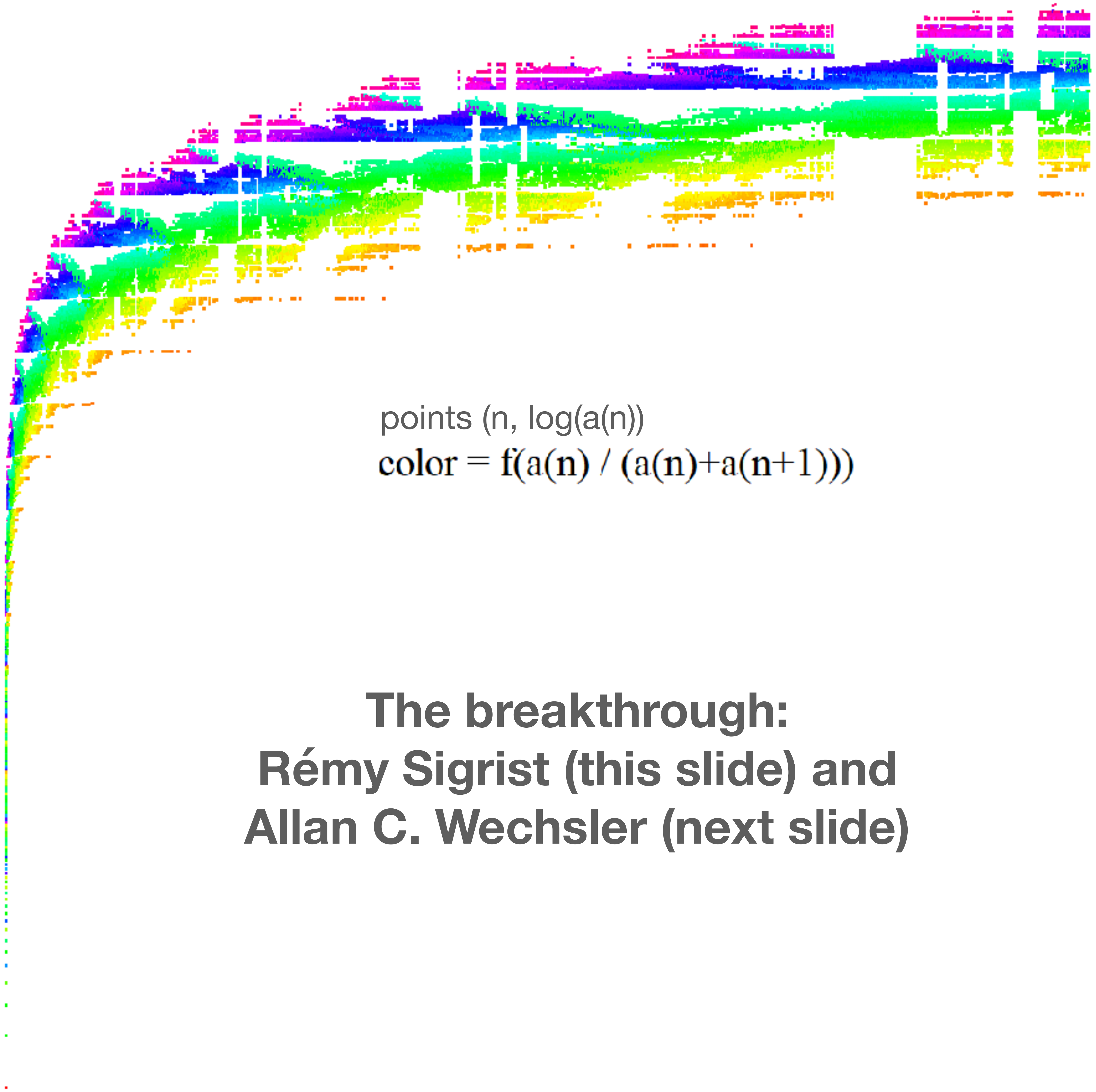
The Tetris Sequence A109812

Graph of first 10^7 points from Thomas Scheuerle



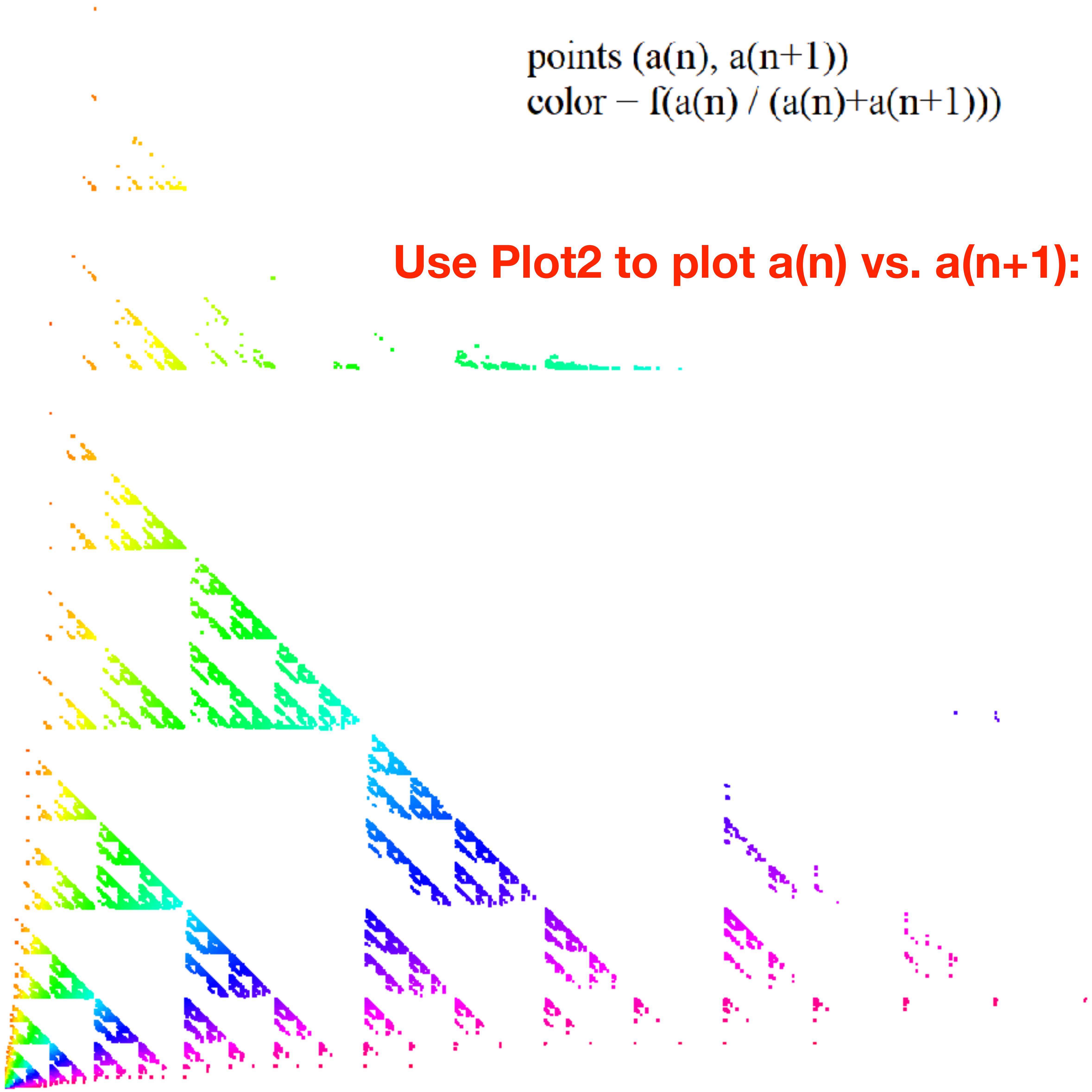
(Successive points are joined by a line)

The Tetris Sequence A109812 (cont.)



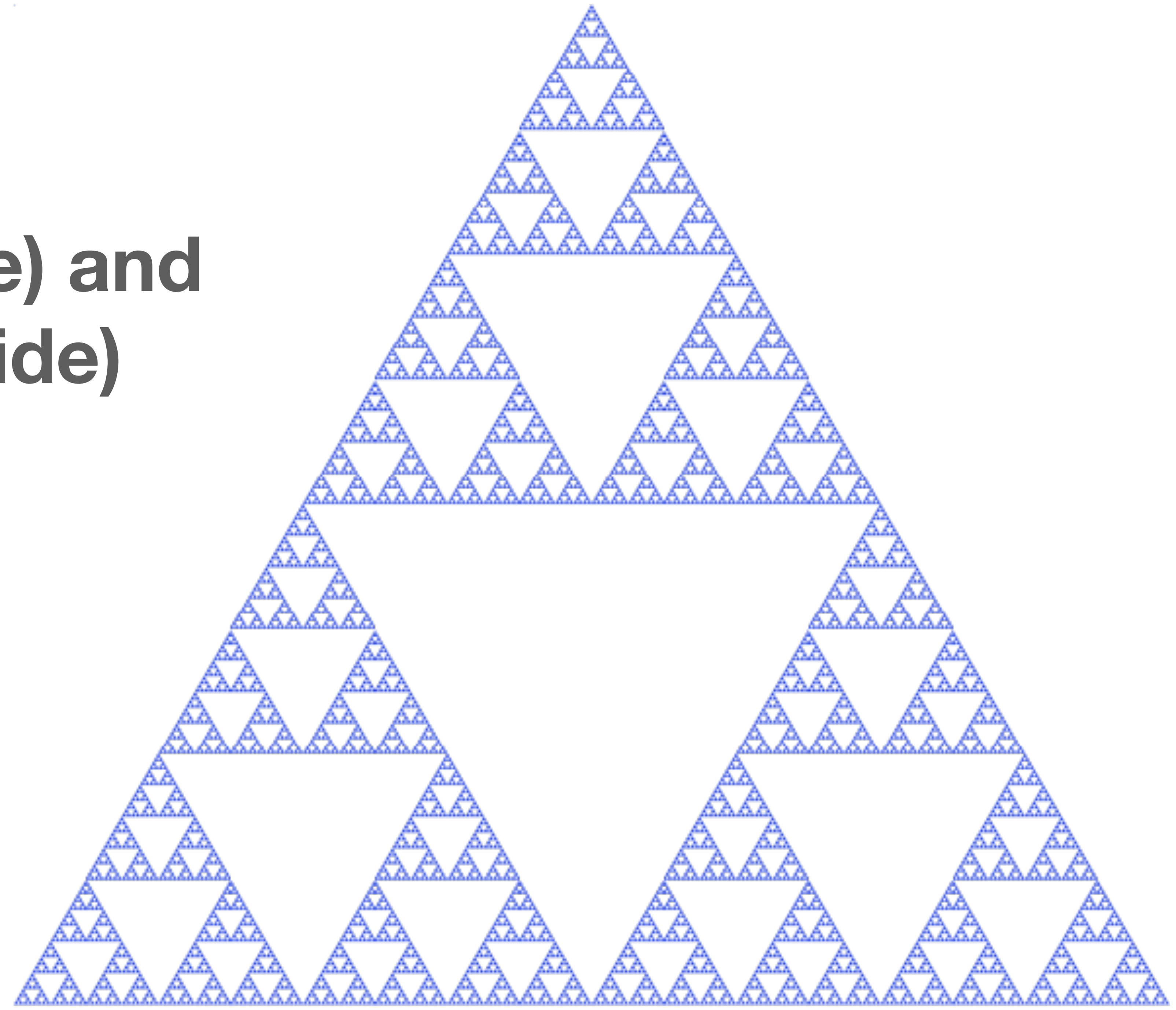
The breakthrough:
Rémy Sigrist (this slide) and
Allan C. Wechsler (next slide)

Use Plot2 to plot a(n) vs. a(n+1):



The Tetris Sequence A109812 (cont.)

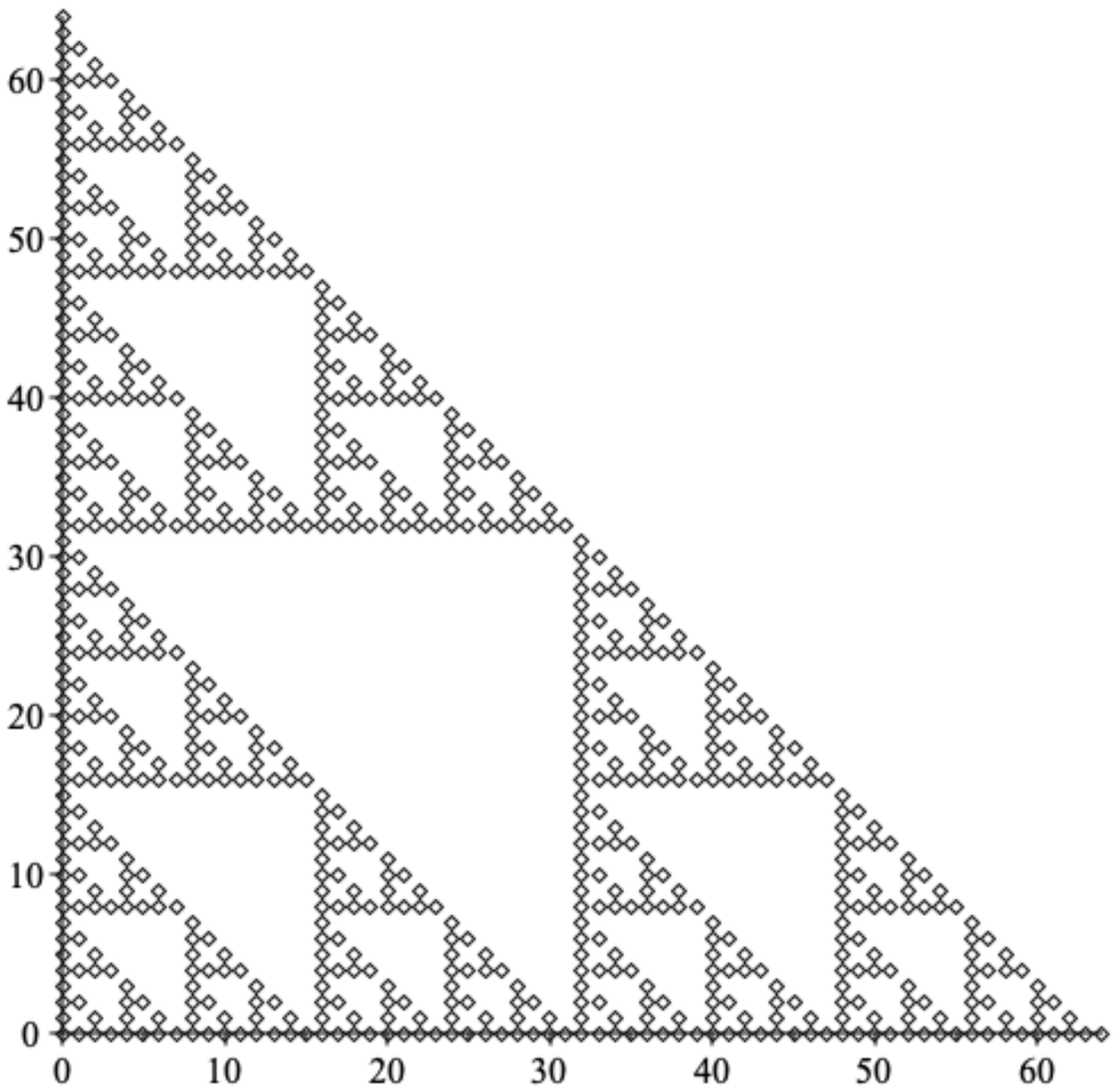
The breakthrough:
Rémy Sigrist (previous slide) and
Allan C. Wechsler (this slide)



Sierpinski Gasket
Pascal's Triangle mod 2

$$T(n,k) = \binom{n}{k} \pmod 2 = 1 \ \textit{iff} \ k \subset n$$

(Lucas's Theorem)

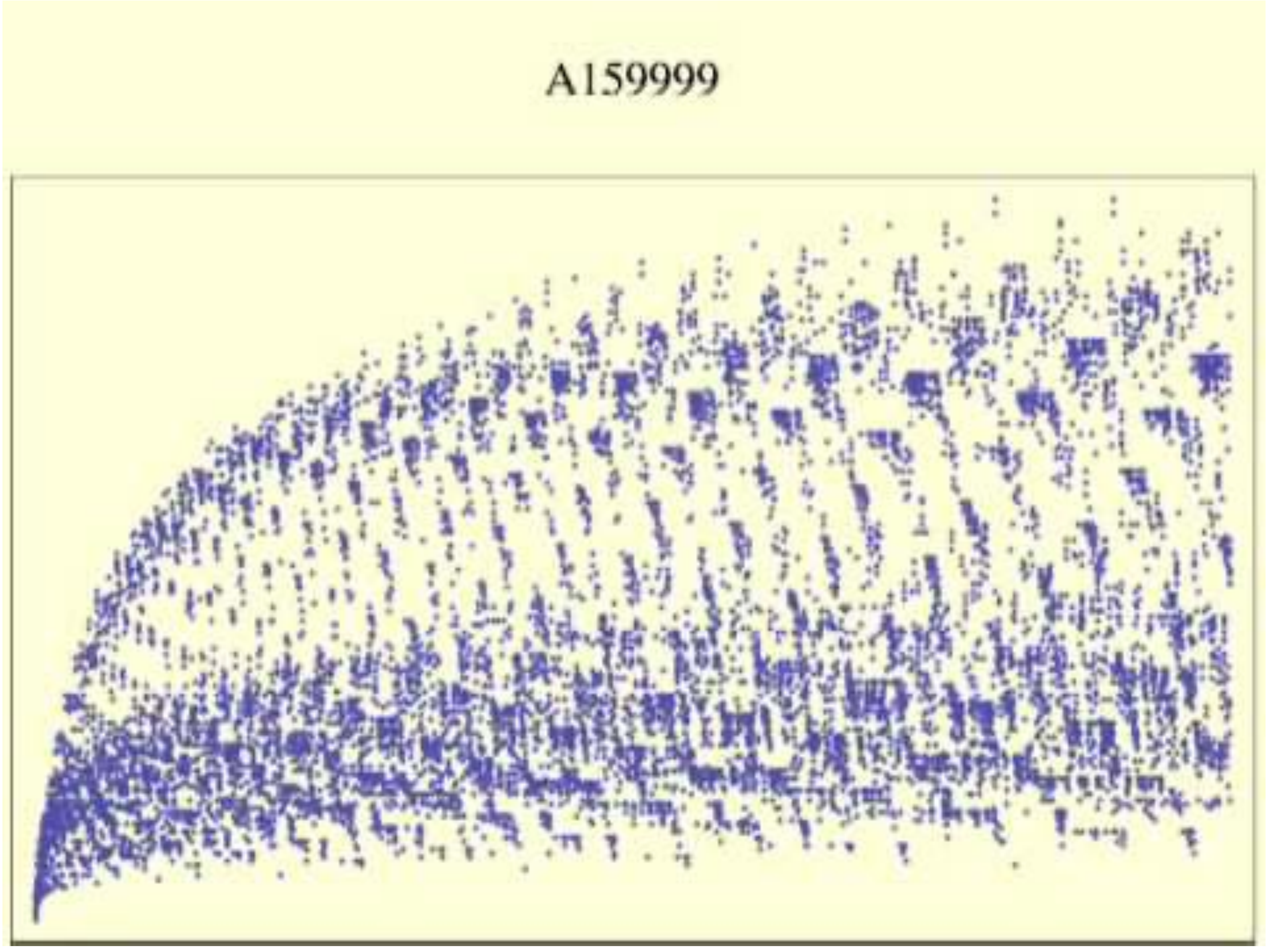


Sierpinski Gasket

G(x,y) = 1 iff x and y disjoint in binary

Idea: Build a virtual database or movie of graphs of 10000 terms of 100K sequences. Use all sequences that have b-files with 10K or more terms. The INPUT is a sequence we are studying. Use facial recognition techniques to find sequences that are the best match.

In March 2010, to celebrate the launching of The OEIS Foundation Inc., Tony Noe made a real movie using Mathematica that showed 1000 terms of 1000 sequences: YouTube, watch?v=LCWglXljevY. Soundtrack is Recaman's sequence A005132,

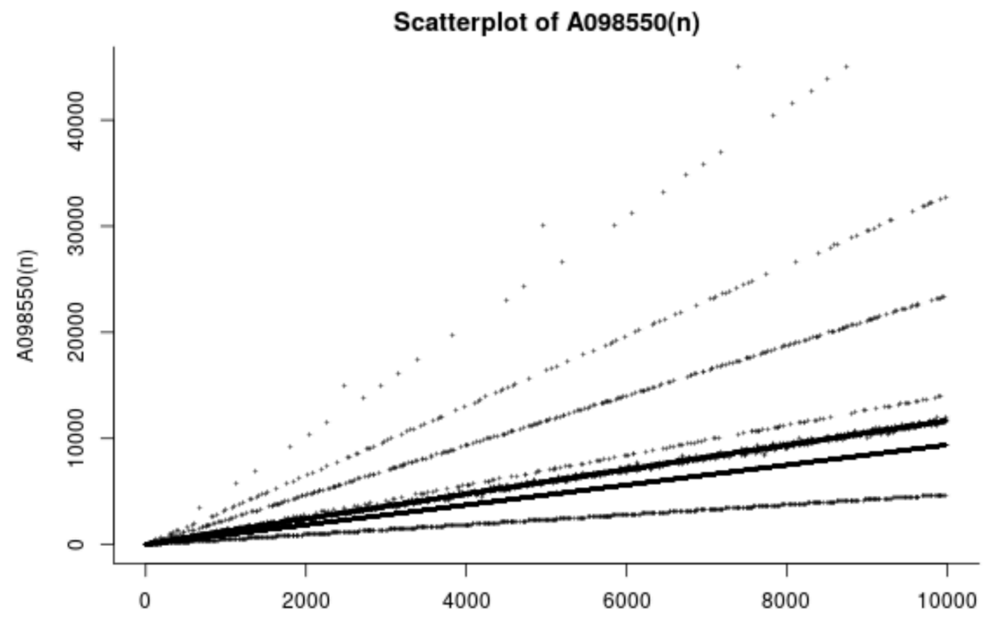


Questions: If INPUT sequence has only M terms, should we truncate the sequences in the database when looking for a match? Humans have two eyes and a nose and a mouth. There is greater variation in sequences! What metrics to use? (See next slide.)

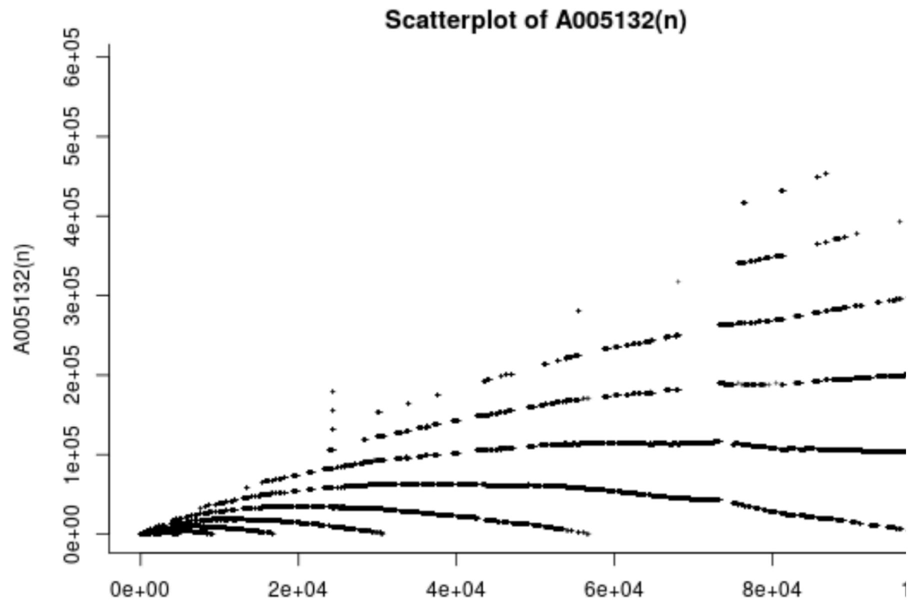
"Facial Recognition" (cont.)

Human

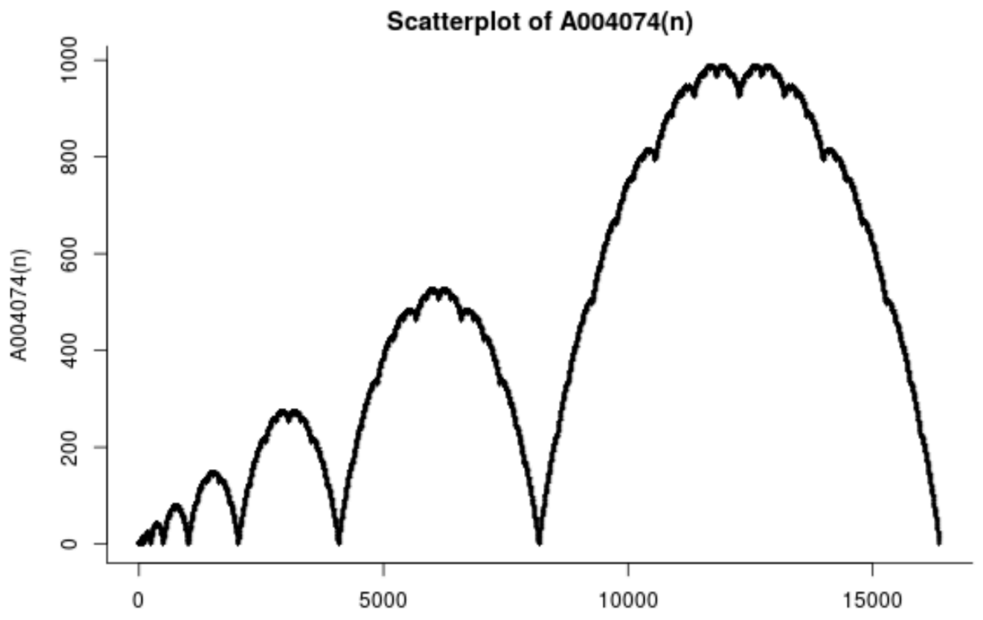
Sequences



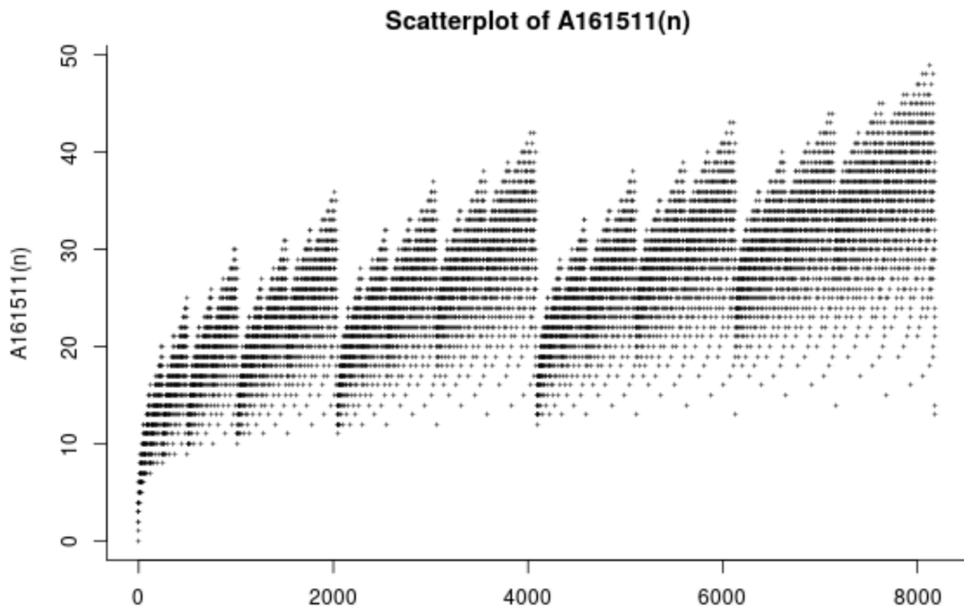
Yellowstone (A098550)



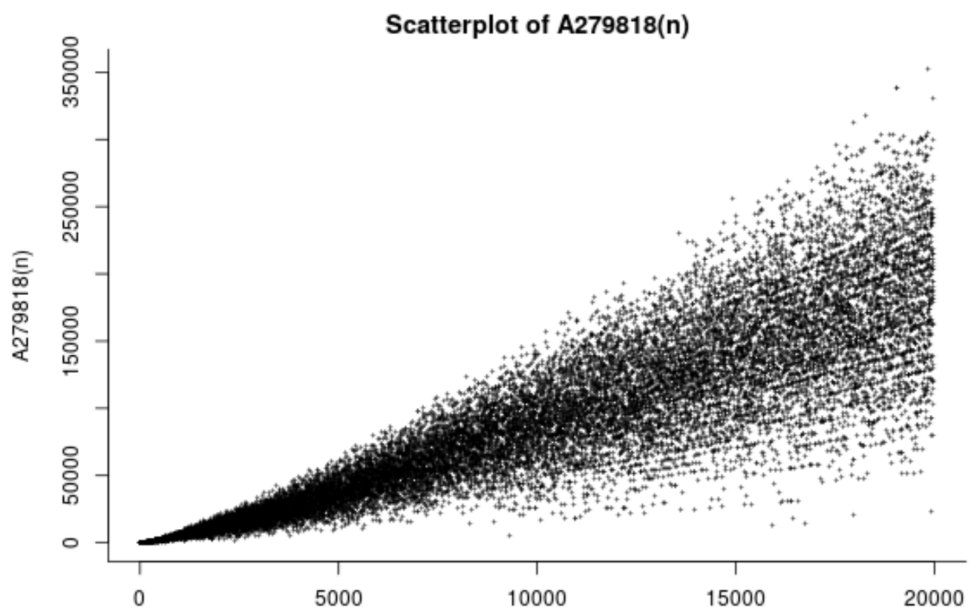
Recaman (A005132)



Blancmange (A004074)



Flames (A161511)



Paint sprayer (A279818)

Others: Wisteria, A063543; Squares: A000290; Sqrt(n): A000196; log(n): A000195; Integer log: A000144; EKG: A064413; ribbon: A055748; forest fire: A131813.

There are probably thousands of types. For a very partial list see Index to OEIS, Section Gra, Subsection "graphs (or plots) , sequences with interesting"

Questions: How does facial recognition work, and how would you modify it to handle "faces" that are sequences of the "Yellowstone" class, say?

Other Topics

- **Stepping Stones,** A337663: Youtube watch?v=m4Uth-EaTZ8, Numberphile video, January 2022. See the Code Golf link in A337663.
- **How many squares can you make with n points in the plane? A051602**
- **Stan Wagon's Problem of the Week 1321: A352178**
- **Find formulas for the "Most Wanted Sequences" A067151, A292104, A349784, A350606.**

- **Stan Wagon's Problem of the Week 1321**

An Impossible-Sounding Problem

Stan Wagon's
Problem of the Week 1321

S = set of n different integers.
f(S) = number of pairs s < t in S such that s+t = a power of 2.
W(n) = max of f(S) over all choices for S.

(The sums s+t, the powers of 2, do not need to be distinct.)

W(2) = 1 from {0,1}
W(3) = 3 from {-1, 3, 5}

Until last month, all W(4), W(5), ...were unkown.

W(4) >= 4 from { -3, -1, 3, 5 }

W(5) >= 6 from { -3, -1, 3, 5, 11 }

W(10) >= 15 from { -5, -3, -1, 1, 3, 5, 7, 9, 11, 13 }

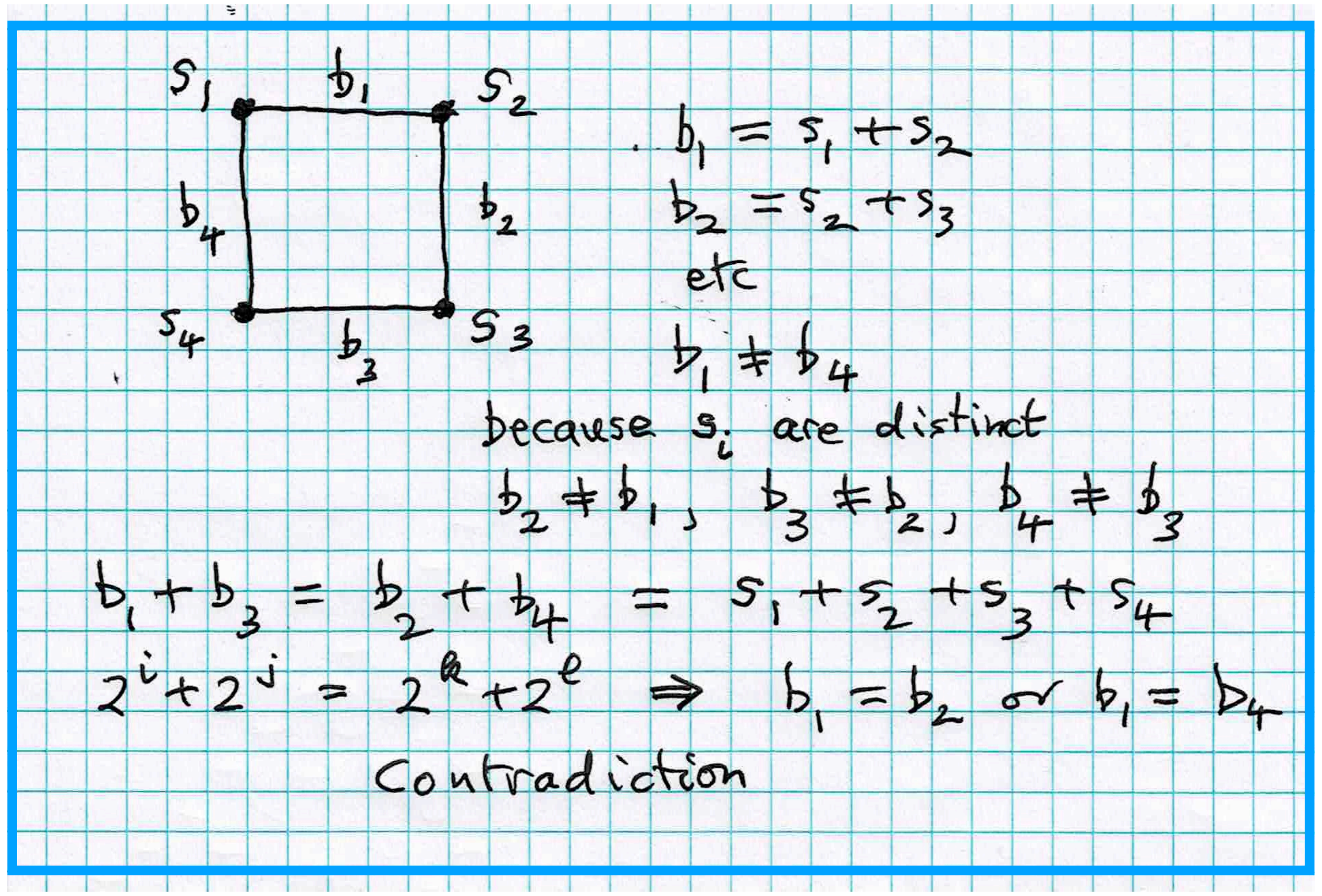
A347301, A352178

Problem of the Week 1321 (cont.)

Email from M. S. Smith, March 6 2022, almost solved the whole problem!

Form a graph G with a node for each s in S, join s and t iff s+t = power of 2.

Theorem (M. S. Smith): G contains no 4-cycle. Proof:



Which implies:

W(n) <= A006855(n) = max no. of edges in C_4 -free graph

We now know W(1) , ..., W(9)

W(10) is 15 or 16

W(n) is bounded by n sqrt(n) / 2.

We went from knowing almost nothing to almost a full solution!