Problem 1321: Powers of Two

Let A denote a finite set of distinct integers (no restriction on size or sign). Let $f(A)$ be the number of pairs (x, y) from A such that $x + y$ is a power of 2. For example, if $A = \{-1, 3, 5\}$, then $f(A) = 3$. Let $g(n)$ be the maximum of $f(A)$ over all sets A of size n. Show that $g(10) \geq 14$.

Can you improve on 14 (which is our best)?

Any values of $g(n)$ are welcome. The main question here (open problem): Is there $r > 1$ and positive constant C such that $g(n) \geq C\,n^r$ for sufficiently large n?

Source: Dan Ullman and Stan Wagon.

Solution. The problem as stated was solved by Jim Henle, Mark Rickert, and Freddy Barrera. The following solvers improved my 14 to 15, and Pratt proved that 15 is the maximum when the domain is retricted to $[-n-3, n+3]$: Barry Cox, Chris Hanusa, Joseph DeVincentis, Al Zimmerman, Piotr Zielinski, Jim Tilley, Nagendra Gd, Rob Pratt, and Witold Jarnicki. To get 15 pairs, use $\{-5, -3, -1, 1, 3, 5, 7, 9, 11, 13\}$.

Further investigations led to the following general formula, which leads to $O(n \log n)$ pairs. It seems that these examples are optimal, but a proof is lacking. Piotr Zielinski proved an upper bound $O\left(n (\log n)^2\right)$. Given n, the best examples (apparently) arise from the set $\{M-n, M-n+2, M-n+4, \ldots, M+n-2\}$, the question being: What is the best choice of M? Computation shows this to be the following. It is easiest to restrict to $n = 4^p$. It appears that the best choice of M is given by the EXACT formula $M = 1 + 2^p = 1 + \sqrt{n}$. And the number of pairs attained is EXACTLY $n\,p - \frac{n}{2} + \frac{3}{2}\sqrt{n} - 1$. This is $O(n \log n)$. I have not proved this, but got the formula by studying the clear pattern that arises for the number of times 1 is attained, 2 is attained, etc.