file next to Frankel

→217

…ks through conver…
…extensively, e.g. in a…
…RINGER. There it is sh…
…e GAUSS-SEIDEL ite…
…uations can, theoret…
…estimates by COLLA…
…ethod (which the au…
…for checking as well…
…ses. This device doe…
…replacing the n unkn…
$= x_i - x_0$, $i = 1, 2, \cdots$
($i = 1, 2, \cdots, n$) while…
…ation is added: it is form…
the combination of G…
…ards is the most effi…
…e author in the case…
process for the determ…
…ussed, particularly for…
with a complex domin…
…-linear elementary divis…
…ELANDT and for both ca…
…llatz's "inclusion" theor…
…d p…ve (not necessar…
generalization of the ite…
Several methods are giv…
roots. Some of them w…
have not been publish…
…od (by KOCH) to apply…
…nt root by starting wi…
to the dominant root. N…
of one dimension less…
…om the dominant root. I…
…re reported here. Wiela…
…ledge of the dominant r…
…ximation to the next…
to the determination o…
the matrix to an arbi…
to the characteristic p…
…wer degree.

O. TAUSSKY

…den zur Lösung eines…
…erl. Akad. Wetensch.…
…. 53–64, 211–219 = *Ind…*
…. 82–90.

…1 I, L].—G. BLANCH & R. SIEGEL, "Table of modified Bernoulli polynomials," NBS, *Jn. of Research*, v. 44, 1950, p. 103–107.

The polynomials to which the title refers may be defined by their Fourier …s as follows

$$b_{k+1}(x) = -\sum_{n=1}^{\infty} n^{-k} \cos(nx + \tfrac{1}{2}\pi k)$$

…are related to the Bernoulli polynomials

$$B_k(x) = (B + x)^k$$

the relation

$$2k!\,b_k(2\pi x) = (-2\pi)^k B_k(x)$$

…that $b_1(x) = (\pi - x)/2$, $b_2(x) = \dfrac{x^2}{4} - \dfrac{\pi x}{2} + \dfrac{\pi^2}{6}$, etc. The polynomials are …n explicitly for $k = 1(1)11$ and $\left[x = 0\left(\dfrac{\pi}{36}\right)\pi; 17D\right]$. The values were …puted from differences using the IBM 405 tabulator and checked by …mation.

D. H. L.

…2 I].—E. T. FRANKEL, "A calculus of figurate numbers and finite differences," *Amer. Math. Monthly*, v. 57, 1950, p. 14–25.

Figurate numbers are, effectively, taken as defined by generating …ctions

$$(1 - t)^{-n} = \sum F_r^n t^r$$

…d thus are essentially binomial coefficients with sign convention reversed. …r relation to finite differences and sums depends essentially on the …wing results.

If $V(t) = \sum u_r t^r$ is the generating function of u_r ($r = 0, 1, \cdots$), then $(1 - t)^{-1}V(t)$ is the generating function of $u_0 + u_1 + \cdots + u_r$ and $(1 - t)V(t)$ of $u_r - u_{r-1}$. The author writes $Su_r = u_0 + u_1 + \cdots + u_r$ and $S^{-1}u_r = u_r - u_{r-1}$ and defines their iterates in the usual way, which of course …ives figurate numbers. The function generated by the product of two …rating functions, now commonly called the convolution, he calls the criss-cross product. For n-th degree polynomials, special attention is given to numbers $S^{-(n+1)}u_r$, which the author calls d_r, because $d_r = 0$, $r > n$, and all other sums (or differences) of the given number sequence u_r can be expressed in terms of them. Other than illustrative tables, there are two main tables one of figurate numbers F_r^n for $n = -7(1)7$ and $r = 0(1)7$ and one of $d_r = S^{-(n+1)}r^n$ for $n = 1(1)11$ and $r = 1(1)11$. The last have a long history (back to LAPLACE) and have lately been called cumulative numbers (Dwyer) KUMMER numbers (PIZA), triangular permutation numbers (…ANSKY & RIORDAN).

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