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Lenoha letters
Guy
Sloane

5 pages

(5) forwarded 5/91 Sent. Add to list 3504

Mathematisch Instituut
Universiteit van Amsterdam
Roetersstraat 15, Amsterdam-1004
Netherlands
Tel. (020) 522.9111 / 522.2299

Amsterdam, April 24, 1975.

Dear Mr. Sloane:

As an expert on integer sequences you might be interested in the following question which took me several days to decide: let the sequence (a_n) be defined by

$$a_1 = 1, \qquad a_{n+1} = \frac{1 + a_1^2 + a_2^2 + \ldots + a_n^2}{n}$$

new

(so the sequence starts 1, 2, 3, 5, 10, 28, ...); does this sequence obey the first four rules of section 1.5 of your Handbook?

Another question: which rule governs your numbering of the Rules in section 1.5? I couldn't find the answer, but it must be somewhere between mock theta numbers and binary codes, I guess.

Perhaps the solution is contained in the Supplements which I understand you are issuing. I would be most interested in obtaining a copy of these, if possible. Thank you very much.

Sincerely yours,

HWLenstra

Dr (H.W. Lenstra, Jr.)

12) a(12)
466_1345794146_0641338430_9896491930_5264116096

(1 , 2 , 3 , 5 , 10 , 28 , 154 , 3520 , 1551880 , 26_7593772160 , 71_6064269012_2633501504 , 466_1345794146_0641338430_9896491930_52641160_96) a(·)

(2 , 6 , 15 , 40 , 140 , 924 , 24640 , 12415040 , 240_8343949440 , 716_0642690122_6335015040 , 5127_4803735606_7054722740_8861411235_790527_7056 , 217281_4461260320_1307908899_5633000490_1297838505_0783094682_272_1842693_6926713623_0071558272)

New Sequence, please enter

A3504

The University of Calgary
2920 24 AVE. N.W.
CALGARY, CANADA
T2N 1N4

FACULTY OF SCIENCE
Department of Mathematics & Statistics
Area Code 403, Telephone 284-5202

78:11:16

613 547, 5511 (Univ)
2728 (O)

N.J.A. Sloane,
Bell Telephone Laboratories,
600 Mountain Avenue,
Murray Hill,
New Jersey 07974. USA.

613 389 7340 (H)

Dear Neil,

Another sequence which I didn't find in the good book. It was shown to me by Alfred J. van der Poorten; perhaps he can supply a reference. He is at Queen's University (Ontario!) this year. I don't know whether it actually qualifies for membership of the Handbook, since I don't know if it's an integer sequence or not! Perhaps Alf does? Even if it's unknown (or known that it's not!) then I think it should go in (as a joke!). It is defined by

$$x_n = \frac{1 + x_0^2 + \ldots + x_{n-1}^2}{n} \quad , \; x_0 = 1,$$

which gives

(1)

$x_1 = 2$, $x_2 = 3$, $x_3 = 5$, $x_4 = 10 = 2.5$, $x_5 = 28 = 2^2 7$, $x_6 = 154 = 2.7.11$, $x_7 = 3520 = 2^6 5.11$, $x_8 = 1551880 = 2^3 5.11.3527$, $x_9 = 267593772160 = 2^7 5.11.13.829.3527, \ldots$.

From here on it's easier to note that

$$nx_n = 1 + x_0^2 + \ldots + x_{n-1}^2$$

$$(n+1)x_{n+1} = 1 + x_0^2 + \ldots + x_{n-1}^2 + x_n^2$$

$$(n+1)x_{n+1} - nx_n = x_n^2$$

$$(n+1)x_{n+1} = x_n(x_n + n) \; .$$

809.

$x_9 + 9 = 809.330771041$, so $x_{10} = 7160642690122633501504 = 2^6 11.13.829.3527.330771041.$

$x_{10} + 10 = 2.199.769.23396052728279347$, so $x_{11} =$
46613457941460641338430989649193052641160 96
$= 2^7 13.199.769.809.829.3527.330771041.\ 23396052728279347.$

Name: $A(N) = (1 + A(0)**2 + \ldots A(N-1)**2)/N$ ..../2.

— NOT ALWAYS AN INTEGER!

Ref: none

By use of suitable small moduli it can be checked that x_{12}, x_{13}, x_{14}, x_{15}, x_{16} and x_{17} are all integers (of 85,168,334,667,1332 and 2661 decimal digits)*. I don't know the status of $(x_{11} + 11)/3$.

Best wishes,

Yours sincerely,

Richard.

RKG:jw

Richard K. Guy.

cc. A.J. van der Poorten.

P.S have checked up to x_{23}

Bell Laboratories

600 Mountain Avenue
Murray Hill, New Jersey 07974
Phone (201) 582-3000

November 30, 1978

Professor R. K. Guy
Mathematics Department
University of Calgary
Calgary, Alberta
CANADA T2N 1N4

Dear Richard:

Thanks for your letter of November 16 about the sequence $\{x_n\}$. In a letter to me dated May 13, 1975 H. W. Lenstra, Jr. said that he had proved that x_n is an integer if and only if $n \leq 43$.

very best regards,

MH-1216-NJAS-mv N. J. A. Sloane

Copy to
Prof. A. J. van der Poorten

Mathematisch Instituut
Universiteit van Amsterdam
Roetersstraat 15, Amsterdam-1004

Tel. (020) 522.9111 / 522.2299

Amsterdam, May 13, 1975.

HWL/vw/230.75

Dr. N.J.A. Sloane
Mathematics Research Center
Bell Laboratories
600 Mountain Avenue
MURRAY HILL, New Jersey.
07974. U. S. A.

Dear Dr. Sloane,

Thank you very much for sending me the reprint + the first supplement.

The sequence from my letter of April 14:

$a_0 = 1$

$a_1 = 1$

$$a_{n+1} = \frac{1 + a_1^2 + a_2^2 + \ldots + a_n^2}{n}$$

1 2 3 4 5 6 7

(1, 2, 3, 5, 10, 28, 154, 3520, 1551880, 267593772160, ...) was mentioned to me by F. Göbel, when he saw my copy of your book. I was able to explain its absence by proving

$$a_n \in \mathbb{Z} \Leftrightarrow n \leq 43 .$$

(In particular your formula does not hold.) So you could include it by way of exception to Rule 1, just as Rule 2 has its exceptions.

With kindest regards,

H.W. Lenstra, Jr.