How Much Does Formula vs. Chaining Matter for a Cost-of-Living Index? The CPI-U vs. the C-CPI-U

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Abstract

A large economics literature has debated the best formula to estimate a cost-of-living index (COLI). This study shows that formula does not matter for many purposes for an index chained at a monthly frequency once chain drift has been removed. Spurious chain drift is removed with a new method revealing the large majority of the difference between the CPI-U and the C-CPI-U (a COLI) is due to the CPI-U weights effectively chaining at the biennial frequency, rather than the difference in formula assumptions. This sufficiently justifies the C-CPI-U and similar chained indexes while also showing their assumptions are not critical. (C43, C82, E31)

Keywords: Consumer Price Index, CPI, Divisia, Index Number, Inflation, Price Index, Cost of Living

Abreviations: chained urban CPI (C-CPI-U), Consumer Expenditure Survey (CE), cost-of-living index (COLI), consumer price index (CPI), urban CPI (CPI-U), cost-of-goods index (COGI), Constant Quantity Törnqvist (CQTQ), constant quantity (CQ), long term (LT), Geometric Lowe index (LTGL), Törnqvist (TQ)

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1 Introduction

A large economics literature has debated the formula to estimate a cost-of-living index (COLI) (see Reinsdorf and Triplett (2009), and also including Diewert (1976, 1978); Balk and Diewert (2003); Pollak (1983); Divisia (1925); Hill (2006); Konüs (1939); CPI Manual ILO 2004). Without knowing what consumer preferences are, a formula for a COLI over a long span of time must assume them. However, most official price indexes are chained indexes, and it is well established that these indexes all approximate the continuous Divisia index (Balk 2005; Diewert 1976, 1978, 1980; Divisia 1926; Reinsdorf 1998b; CPI Manual ILO 2004). As the time interval of each chain link shrinks, these approximations all converge to the Divisia index, the change in which for each point in time measures the current COLI.[1] The only question is how short this interval needs to be so that the formula is no longer relevant. Diewert (1978) showed that at least the annual frequency is generally sufficient for the superlative class of formulas, such as the Törnqvist, which track each other closely at that frequency, while the Laspeyres and Paasche formulas track much less closely to

[1] The instantaneous COLI at a point in time should not be confused with a direct COLI over a discrete time interval. As discussed by Hulten (1973), Samuelson and Swamy (1974), and Diewert (2004), the Divisia index (which is an integral over time) will only equal a direct COLI over that time period if consumer preferences are homothetic. Thus, a chained (or integrated over continuous time) index such as the Laspeyres or Törnqvist, even as the interval shrinks, will only be the true COLI over that period in the case that preferences are homothetic. However, Reinsdorf (1998b) shows that the Divisia index is relevant in a more general sense even if preferences are not homothetic.

the superlatives and each other.[2] I answer this question more generally. The monthly frequency, which is what is actually used for most official indexes, is sufficient, so that even the Laspeyres, an upper bound to a COLI and a type of cost-of-goods index (COGI), is close to the superlative Törnqvist formula. To uncover this, the obstacle of spurious chain drift in a monthly chained Laspeyres index must be overcome. I develop a new method, suitable in this context, to remove this 'bad' chain drift, dubbed 'resonance drift'.[3] Thus, if resonance drift is controlled for, all relevant formulas should give similar results which are thus not dependent on their particular assumptions.

I demonstrate this by explaining the difference between the CPI-U (urban CPI) compared to the C-CPI-U (chained urban CPI), which are produced by the U.S. Bureau of Labor Statistics. The CPI-U is the current U.S. headline CPI, but the C-CPI-U is intended as a better measure of a COLI, and in typical months and cumulatively over months is lower than the CPI-U. As shown below, the difference is about 0.3% a year. This is a very significant difference because it compounds over time. For example, over 16 years real income measures would be about 5% higher if deflated using the C-CPI-U vs. the CPI-U. It has replaced the CPI-U for indexing tax brackets and other tax related limits[4], and is being considered to replace the CPI-U for indexing social security.

However, the C-CPI-U has been is restricted by a specific functional form. While both indexes use the same elementary item-area indexes as inputs, they use different ag-

[2] Hill (2006) shows that the formula is quite important and even superlative indexes are not necessarily close for unchained indexes at frequencies of one to 17 years or across countries, for which the results of this paper do not apply.

[3] There is an alternative definition of chain drift from what is used in this paper which is the deviation of an index from circularity, or the difference between 1 and the final index value if all prices return to their original values. Under this definition, all drift is "bad".

[4] https://www.congress.gov/bill/115th-congress/house-bill/1/text

gregation formulas and different weights to estimate aggregate price change. I show that the effects of chaining frequency can be seen by the importance of the weighting differences between the two formulas. While both indexes are at the monthly frequency, the CPI-U only updates its weights every two years, and so is similar to a biennial index. Therefore, the effects of weights and chaining are often discussed interchangeably. Weights in the CPI-U implicitly constrain quantities to be constant over long periods of time, while weights in the C-CPI-U allow quantities to change every month. I show that differences over only a one month period at a time, whether these differences are holding a quantity or share weight constant, or a geometric vs. arithmetic mean formula, matter very little since elementary price indexes change very little over one month, even when aggregated over time. But the CPI-U weights have implicit quantities lagged for 36 months on average, so the total effect of this dominates since it is roughly on the order of 36 times of a one month difference.

Nearly the same results as the C-CPI-U can be obtained with a resonance-drift-controlled Laspeyres index, without needing the formula assumptions of the C-CPI-U. More generally, this means provides evidence that a sufficiently frequently updated Laspeyres index can be a COLI. Thus on the scale of the difference between the CPI-U and the C-CPI-U, this both justifies COLIs such as the Törnqvist and shows their preference assumptions aren't needed.

Section 2 describes the CPI-U and C-CPI-U, and the differences between them due to formula differences and chaining with current weights. Section 3 gives a brief overview of the methods used to breakdown the differences by making intermediate indexes which change by one difference at a time, an overview of results, and describes the problem of resonance chain drift. Section 4 describes in detail the methods of changing the weights in intermediate indexes before the formula, and section 5 describes the methods of changing the formula first before changing the weights, in two different ways. This is to show that the results, which have the same conclusions as changing the weights first, do not depend on the order of the breakdown or the removal of resonance chain drift. Section 6 concludes.

2 The CPI-U and C-CPI-U: COGI versus COLI

Upper level of aggregation the CPI-U uses a Lowe, or 'modified Laspeyres' formula, which is a Laspeyres index with lagged weights. The Lowe formula is a COGI: as described in "At What Price?: Conceptualizing and Measuring Cost-of-Living and Price Indexes", pp. 2-3. A COGI is the ratio of the expenditure needed in the current period to the expenditure needed in the past period to buy a fixed basket of goods. The CPI-U uses an implicit basket of goods defined over a multi-year period base period, which is updated every several years with a lag for processing. The CPI-U index relative, the ratio of index levels, between months t-1 and t is

$$CPIU_{t-1,t} = \frac{\Sigma_{ia} Aggweight_{iaB} I_{iat}}{\Sigma_{ia} Aggweight_{iaB} I_{i,a,t-1}} \tag{1}$$

where I_{iat} denotes the item-area cell index level for item i in area a for month t, and $Aggweight_{iaB}$ denotes the aggregation weight for item i in area a for base period B. The aggregation weight is total expenditure over the base period, measured by the Consumer Expenditure Survey (CE), divided by the average index level for that item-area over the base period.

The Lowe formula can be rewritten as an arithmetic mean of item-area index relatives, denoted R_{iat} for the index relative for item i in area a between months t and $t-1$, weighted by expenditure shares that hold the implicit quantity constant,

$$CPIU_{t-1,t} = \Sigma_{ia} \frac{Aggweight_{iaB} I_{i,a,t-1}}{\Sigma_{ia} Aggweight_{iaB} I_{i,a,t-1}} R_{iat} \tag{2}$$

where $\frac{Aggweight_{iaB} I_{i,a,t-1}}{\Sigma_{ia} Aggweight_{iaB} I_{i,a,t-1}}$ is an expenditure share updated from the base period B by the index levels $I_{i,a,t-1}$ holding the quantities constant. Therefore, the Lowe weights are updated shares, but those updates do not use new expenditure information – they only use the index relatives. The shares are only updated in such a way that assumed constant quantities.

Conversely, at the upper level of aggregation, the C-CPI-U uses the Törnqvist formula. The Törnqvist formula is meant to be an approximation to a cost of living index, or COLI.[5] As defined in Konüs (1939) and Pollak (1983), a COLI is the ratio of the expenditure needed with current prices to the expenditure needed in the past period to purchase a base standard of living, meaning that a consumer would be indifferent to choosing between these two expenditure-price combinations.[6] As described in Diewert (1976), the Törnqvist approximates an arbitrary COLI by approximating the function defining the consumer's necessary expenditure (or cost) given prices and a standard of living.[7]

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[5] The C-CPI-U is actually a Conditional-Cost-of-Living index, because many things that affect the standard of living, including weather, crime, various government services, etc., as described in "At What Price?: Conceptualizing and Measuring Cost-of-Living and Price Indexes" pp. 94-105, are out of the scope of the index. The CPI-U and all other indexes constructed herein are also conditional indexes, and have the same scope.

[6] Balk and Diewert (2003) show that a Lowe index can be considered an approximation of a COLI, but only if there are no significant relative price trends in the data. Below it will be shown that these different trends in the data do make a difference.

[7] From Diewert (1976), Theorem (2.16), p. 122, this approximation represents the expenditure as a second order translog function,

$$\ln C(u;p) \equiv \alpha_0^* + \Sigma_{i=1}^N \alpha_i^* \ln p_i + \frac{1}{2} \Sigma_{j=1}^N \Sigma_{k=1}^N \gamma_{jk}^* \ln p_j \ln p_k + \beta^* \ln u + \delta^* (\ln u)^2 + \Sigma_{i=1}^N \varepsilon_i^* \ln u \ln p_i$$

Where i, j, and k denote items, with N total items, u denotes utility, p_i denotes the price of item i, and the $\alpha, \beta, \gamma, \varepsilon$ terms are parameters. This implies the Törnqvist formula, evaluated at $u = \sqrt{u_{t-1} u_t}$.

The Törnqvist formula, denoting $T_{t-1,t}$ as the Törnqvist relative from period t-1 to t, is

$$T_{t-1,t} = \Pi_i \left(\frac{p_{it}}{p_{i,t-1}} \right)^{\frac{s_{it}+s_{i,t-1}}{2}} \tag{3}$$

, where p_{it} denotes the price of item i in period t, and s_{it} denotes the expenditure share of item i in period t.

Using item-area cells in place of items, item-area index relatives, R_{iat}, in place of price relatives (the ratios of prices), and the CE measured expenditure share for item i in area a for month t as s_{iat}, this implies the formula for the C-CPI-U index relative[8] of

$$T_{t-1,t} = \Pi_{ia} R_{iat}^{\frac{s_{iat}+s_{i,a,t-1}}{2}} \tag{4}$$

.

The CPI-U and C-CPI-U use the same item-area index relatives, and both use CE survey measured expenditures to construct the weights.[9] Thus, the two indexes differ in two ways: weights and formula. The Lowe index is an arithmetic mean formula of index relatives with weights updated by holding the base period quantity fixed, while the Törnqvist is a geometric mean formula of index relatives with the average of current and previous period observed share weights.

There are three types of differences in the weights used in the CPI-U versus the

There are other approximations to COLIs, but the indexes in the class of the Tornqvist, superlatives, are generally found to be close to each other. See Diewert (1978).

[8]The C-CPI-U actually uses interpolated relatives described below, which will be denoted as $Rint_{iat}$ but for simplicity here are denoted the same as other relatives as R_{iat}.

[9]Due to the time needed to process the CE, the final C-CPI-U is made with a two year lag. Only initial and interim estimates of what the final index will be are published earlier.

C-CPI-U: time span of data, lag, and frequency of updating. The CPI-U weights are derived from a multiple year base period. The total expenditure for each item-area over that period is used to derive an aggregation weight that is used in the index. The quantities in those shares are fixed by the base period expenditure, updated by the inflation of the item-area index relatives. For 1998 through 2001, there was a three year base period, and since 2002 it has used a series of two-year base periods. Conversely, the C-CPI-U weights are derived from only two months of data, the current and previous months.[10] Using current shares instead of the share updating of the Lowe index allows the implicit quantity weights to change, even if the new shares don't change. For example, if the current period shares don't differ from the older ones, but the index relative has risen by 2%, then the implicit quantity weight has fallen by 2%. The second is that the CPI-U weights are updated with a long lag, due chiefly to the processing time needed. For 1998, the lag was two years after the end of the base period, and since 2002, it was one year since the end of the base period. Therefore, the total lag for a given month has been two to five years from 1999 through 2001 and then one to three years since 2002. The C-CPI-U weights, however, have no lag, since they use information from the current (and previous) month, which is why the C-CPI-U cannot be computed in real time. Third, before 1998, the CPI-U weights were updated around once every 10 years. The weights were updated once in 1998, and then every two years starting in 2002, while the C-CPI-U weights are updated every month.

To the extent that the Törnqvist formula approximates a COLI, those differences

[10] It has been argued that the C-CPI-U is not a true Törnqvist index due to the fact that the CE survey-measured weights (which have a high variance because the sample size is low) are smoothed. In a process called ratio allocation, the total share for an item in a given month is allocated across areas by the average share for the item over the previous 11 months. However, it makes almost no difference in the index when the raw weights are actually used instead, as is shown in Appendix D.

that make a COGI different from a COLI would explain the differences between the CPI-U and the C-CPI-U. A COLI index with a base at the previous period, for preferences that are constant over that period, will be as low or lower than a COGI over the same period which has a base basket in the previous period, due to consumer substitution. This is because if there are relative price changes, consumers can substitute relatively cheaper goods for relatively more expensive ones, and be better off than if they purchased a fixed basket. Therefore, the cost of maintaining the same standard of living is as low or lower than the cost of buying the same basket of goods.

This was formalized by Konüs (1939) who showed that for preferences that don't change over the period, a Laspeyres index with a base in the previous period is an upper bound for a COLI with the same base period, since a Laspeyres index is a COGI with the basket fixed in the previous period. He also showed that a Paasche index, which is a COGI with the base basket in the current period, is a lower bound for a COLI with a base in the current period, also because of consumer substitution. The Laspeyres and Paasche both bound a COLI with a base in-between the two periods.

The Törnqvist formula has two ways to incorporate changes in consumer purchases to approximate a COLI: (i) the current share weights are direct information on changes in consumer purchases that do not hold quantities constant over long periods of time, while the quantity weights in the Lowe formula are only information on past purchases; (ii) the use of a geometric mean to aggregate the item-area indexes instead of the Lowe/Laspeyres arithmetic mean assumes a certain amount of consumer substitution. Because different weights could be used in either an arithmetic or geometric mean, for clarity and simplicity (i) will be referred to as the effects of weights, while in contrast to how 'formula' was used before this point, (ii), use of an arithmetic vs. geometric mean, will refer to the effects of formula for the rest of the paper.

Updating quantity weights more frequently will generally make a Lowe index fall, as described in Greenlees & Williams (2010). Since consumers substitute away from items with rising prices, those items with higher inflation will have relatively falling quantities in the long run, all else equal. Therefore updated weights in a Lowe index will give lower weight to higher inflation goods, and lower the long run index because of consumer substitution.

This fall in a Lowe index towards a COLI is not a coincidence. The CPI-U is effectively chained biennially, and the C-CPI-U is chained at the monthly frequency. However, a Lowe index with more frequently updated weights will approach a monthly chained Laspeyres.[11] As shown by Diewert (1976, 1978, 1980), Reinsdorf (1998b), and Balk (2005), both the Laspeyres and the Törnqvist formulas (as well as the Paasche and others), approximate a Divisia index, introduced by Divisia (1925),[12] which is a price index for continuous time,

$$P_{t',t}^{Div} = \exp\left(\int_{t'}^{t} \Sigma_{i=1}^{N} s_i\left(\tau\right) \frac{d\ln p_i\left(\tau\right)}{d\tau} d\tau\right) \tag{5}$$

where $s_i\left(\tau\right)$ denotes the share of item i at point in time τ, and $p_i\left(\tau\right)$ denotes the price of item i at τ. Because all of the discrete time indexes approximate a Divisia index around a point where all the prices have no change, as the time interval for a Laspeyres or Törnqvist index shrinks, they both approach the Divisia index and thus approach each other. The above studies point out that in that sense, as the interval shrinks, the formula doesn't matter.

At a given point in time, the change in the Divisia index is in fact the "instantaneous"

[11] Balk and Diewert (2003) discuss the relation ship between the Lowe and Laspeyres indexes.
[12] Divisia (1925) also showed that the Laspeyres index was a first order approximation to his index.

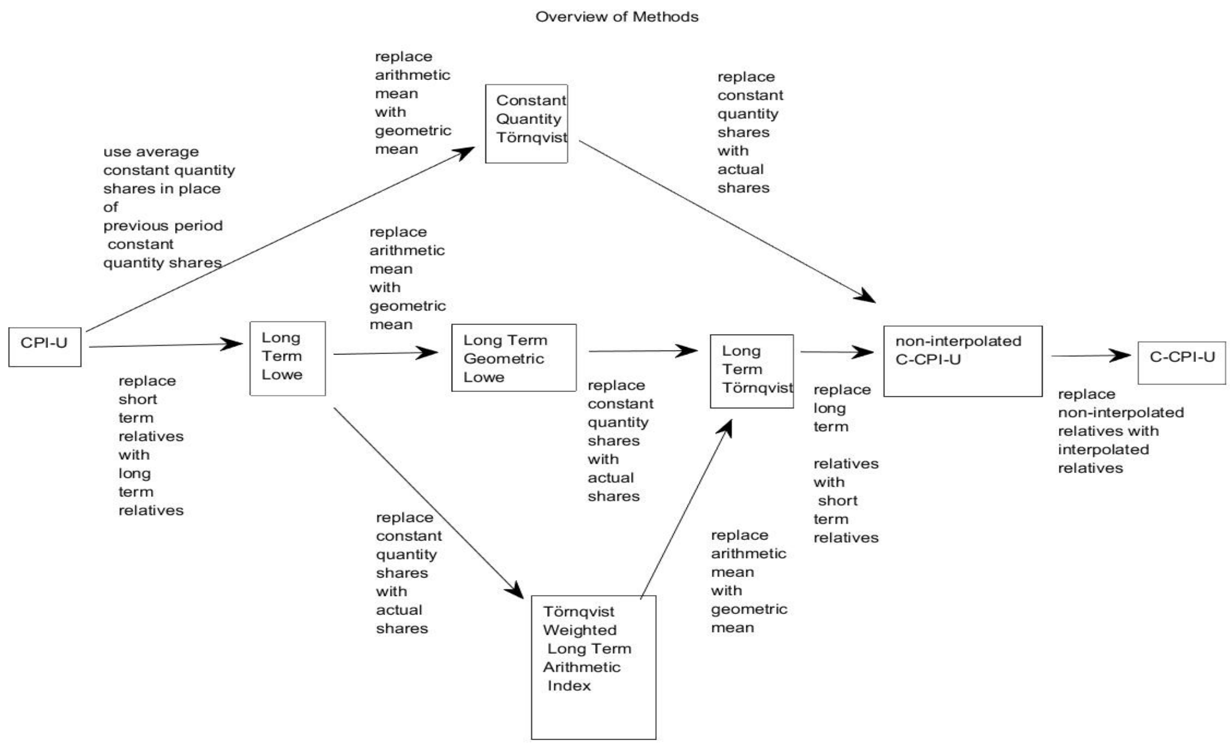
cost-of-living index at that point. Denoting the expenditure needed at a price vector $p(\tau)$ at time τ to obtain the standard of living or utility level $u(\tau)$ as $e(p(\tau), u(\tau))$, the expenditure function at $p(\tau)$ and $u(\tau)$, note that the integrand of (5),

$$\Sigma_{i=1}^{N} s_i(\tau) \frac{d \ln p_i(\tau)}{d\tau} = \frac{\partial \ln e(p(\tau), u(\tau))}{\partial p(\tau)} \tag{6}$$

, where $\frac{\partial \ln e(p(\tau),u(\tau))}{\partial p(\tau)}$ can be considered as an "instantaneous" analogy to a COLI $\frac{e(p(\tau'),u(\tau))}{e(p(\tau),u(\tau))}$ between time periods τ and τ'. As described in Diewert (1983), since the COLI with a base reference period is bounded above by the Laspeyres, and a COLI with a current reference is bounded below by the Paasche, as the interval shrinks, both indexes approach the COLI at that point. Since the Törnqvist approximates a COLI at the point where prices are unchanged, and Balk and Diewert (2003) also show that both the Laspeyres (as a specific case of the Lowe index) approximates a COLI at that same point, both indexes approach the instantaneous COLI as the interval shrinks. If official price indexes are supposed to report the current inflation rate, then as the interval shrinks they approach the appropriate target. The only question is how close to the target does monthly chaining get.

The goal of this project is to determine how important quantity weight updating and the geometric vs. arithmetic mean formula are in moving the CPI-U to the C-CPI-U. This analysis implies that the formula plays a minor role in causing the divergence between the CPI-U and the C-CPI-U. Instead, it is using expenditure share weights that don't hold quantities constant over long periods that are responsible for the majority of the divergence. The qualitative nature of this result holds across three different approaches to breaking down the divergence.

Figure 1:



3 Breakdown Overview

To explain how much the weight and formula effects matter for the different levels of the indexes, I construct a number of intermediate indexes which each incorporate a different change in the CPI-U that either made no significant difference or made it more like the C-CPI-U. The total difference between the intermediate indexes relative to the total difference between the CPI-U and C-CPI-U is the effect of that change.

There is more than one way to move from the CPI-U to the C-CPI-U. For comparison and robustness, I use three methods, each giving qualitatively similar results. The methods are described in turn below. Figure 1 gives an overview of the methods.

The first updates the weights first, and the second and third change the formula first.

The intermediate steps must be meaningful in themselves so that the results can be interpreted in a meaningful way. If not, the differences would be irrelevant to explaining the difference between the CPI-U and C-CPI-U. If the C-CPI-U is a relevant approximation of a COLI, this means each change should be designed to move a COGI to a COLI.

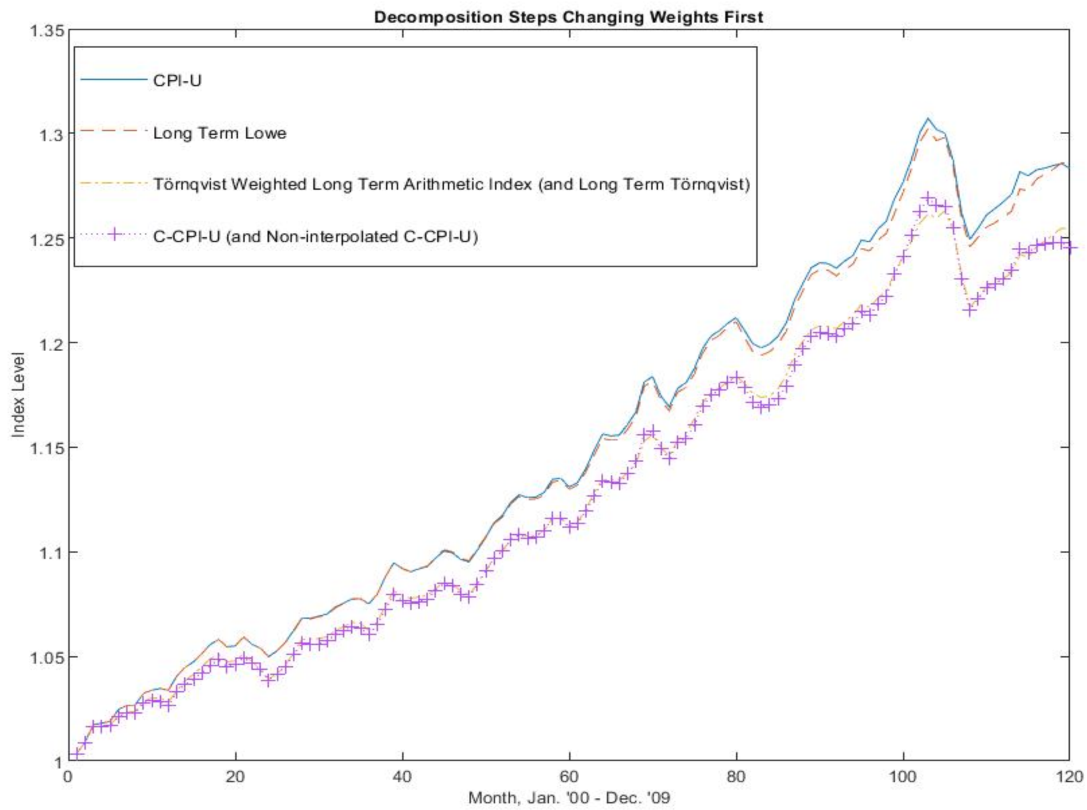
While any change to an index is likely to change the month-to-month movements, the differences in month-to-month changes are small enough that it is difficult to see the significance of any particular change just looking at those movements. Only when the monthly changes are cumulated over time to make the index levels do obvious differences become visible. Therefore, instead of graphing monthly changes, this study focuses on the differences in the index levels.

The period studied is the ten years covering December 1999 through December 2009. The C-CPI-U began to be published in January 2002, and because the final values are published with a two year lag, the first month of the index was January 2000, which used the index relative from December 1999 to January 2000. Thus the first month of data used is December 1999, month 0, for which the indexes are normalized to 1, so the reported indexes begin in January 2000, which is month 1 out of 120. CPI data is used, which has the expenditures, index levels, and index relatives needed to construct the indexes.

The results are summarized in Table 1, which is described in detail in the following sections below. To give an overview, Figure 2 graphs the index levels for the CPI-U, C-CPI-U, and the intermediate indexes of changing the weights first.[13] The main feature to note is that the indexes are in two clumps, around the CPI-U and the C-CPI-U, and the gap

[13] Because the differences between these indexes cannot be seen on the scale of the figure, the Törnqvist Weighted Long Term Arithmetic Mean Index lines and the Long Term Törnqvist lines were combined, as were the C-CPI-U and the Non-interpolated C-CPI-U lines.

Figure 2:



in the middle is the effect of using the Törnqvist weights. None of the other changes are very significant on this scale. This gap illustrates how updating the CPI-U weights more frequently move it closer to a COLI.

Table 1: Effects of each Step by Method of Breakdown as % of Total CPI-U vs. C-CPI-U Difference[14]

	Method		
	Weights First	Formula First - Geomeans	Formula First - CQTQ
Replace Short Term Relatives with Long Term Relatives[15]	8.38%	Same	NA
Weight Effect: Replace Constant Quantity Shares with Actual Shares[16]	85.98%	86.25%	96.22%
Formula Effect: Replace Arithmetic Mean with Geometric Mean[17]	2.05%	1.79%	1.45%
Replace Long Term Relatives with Short Term Relatives[18]	1.25%	Same	NA
Replace Non-interpolated Relatives with Interpolated Relatives[19]	2.34%	Same	Same

[14] The total is given by equation (27).

[15] This refers to equation (26), the difference between the solid and dashed lines in Figure 2.

[16] These refer to equations (31), (42), and (44) for the three columns respectively, and to the difference between the dashed and dash-dot lines in Figure 2, the dot-square and dash-dot lines in Figure 8, and between the dot-period and dot lines in Figure 8 respectively.

[17] These refer to equations (33), (41), and (43) for the three columns respectively, and the second two columns refer to difference between the dashed and dot-square lines in Figure 8, and between the solid and dot-period lines in Figure 8 respectively. The first column is not shown in Figure 2 because the difference between indexes is too small to be seen.

[18] This is equation (34), the difference between the dash-dot and dot-plus lines in Figure 2.

[19] This is equation (36), and not shown in Figure 2 because the difference is too small to be discernable.

3.1 Chain Drift

The convergence of index formulas to a Divisia index from quantity weight updating, and thus from more frequent chaining, is not necessarily direct or monotonic. In other words, a smaller interval can make the difference with a Divisia index even larger, even if a still further reduction in the interval would have a smaller difference. Changing the weights every month can cause a spurious correlation between the weights and index relatives, referred to here as 'resonance' chain drift. This makes the index diverge from a Divisia index. This barrier is why the convergence of these index formulas at the monthly frequency has not been discovered before - when a Laspeyres index in such a context is chained monthly, it flies upward away from a Törnqvist.

As noted above, both the CPI-U and the C-CPI-U are chained indexes, not direct indexes from a base period to the present.[20] The difference between a direct index and a chained index of the same formula over the same time span is called chain drift. Chained indexes are a better measure of the current inflation rate as opposed to the total inflation over a span of time. This is because for longer time spans, the base period of a direct index becomes less relevant to the present. Thus the chain drift due to consumer substitution is 'good' drift. But due to the discrete chaining interval, the weights could be correlated with the price changes at certain chaining frequencies. In fact, overall inflation may be higher for a chained index than even the inflation rate of the highest inflation item. Denoting this as 'resonance' drift, it could cause a chained index to diverge from a Divisia index, and is thus 'bad' chain drift.[21]

[20] See CPI Manual (ILO 2004), chapters 9, 15, and 19.

[21] Certain chained indexes that had constant weights would, however, be circular, or the same as a direct

Resonance drift in a Laspeyres index is caused by price oscillations, or 'bouncing', and was described by Szulc (1983)[22]. It is also described in Hill (1988) and is similar to the problem of formula bias described in Reinsdorf (1998a). As an example, consider the following index relative formula, a Young index, given by

$$Y_{t-1,t} = \Sigma_i s_i^b R_{it} \tag{7}$$

over items i between period $t-1$ and t. This is Laspeyres-like index which implicitly changes the quantity weights each period such that the expenditure shares remain constant over time at a base period of b. Consider a chained Young index using the same base period b. Suppose there are two goods with equal expenditure shares when the weights are set, so that $s_1 = s_2$. Also suppose each good's price bounces between 1 and 2 every period, so that each price relative is either 2 or $\frac{1}{2}$. There would be no long run inflation for either good. Yet this index relative would give an inflation rate of $\frac{1}{2}(2 + \frac{1}{2}) = 1.25$, or 25% inflation every period.

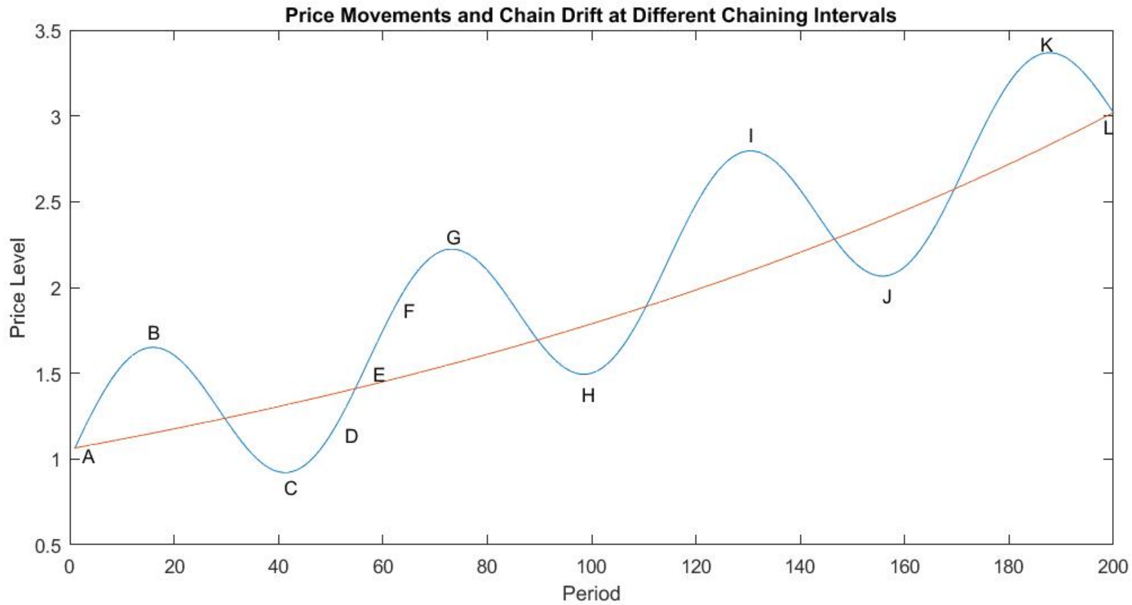
As another, more general example than the Young index, Figure 3 graphs a possible common pattern of continuous price change for an item-area in the CPI.

In this example, the price level tends to rise in the long run, due to overall inflation: but the price level also goes up and down in the short run. If the index is chained at a long interval, so that one index period is from B to L, the short run variation matters little.

index, since in that case there would be no correlation between weights and price movements. For example, if the Lowe index never updated its weights at all, multiplying successive periods' inflation rates together would mean the numerator for each period would equal and divide out the denominator from the past period, and the result would simply be the direct index between the first and last periods.

[22] Hill (2006) provides a formal definition of the conditions for chain drift in the spread between the Laspeyres and Paasche indexes.

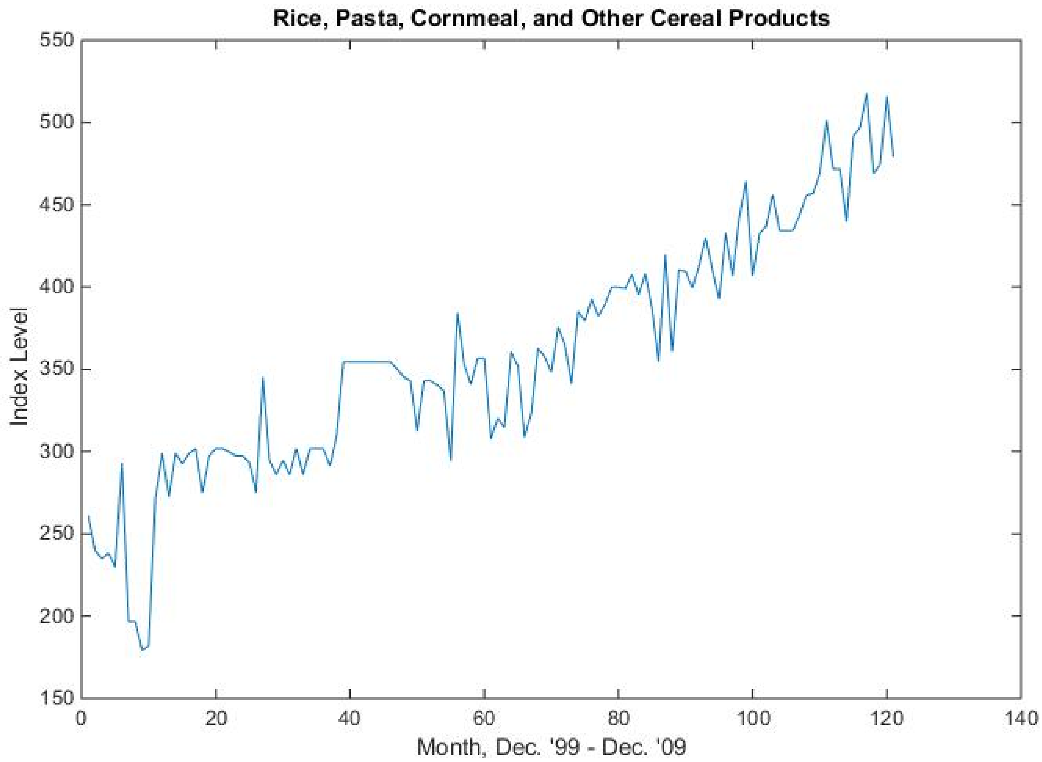
Figure 3:



Likewise, if the index is chained at a frequency such that the chaining points are C-H-J, or B-G-I-K, then the short run variation will not influence the long run movement of the index. But consider if the index was chained at the frequency such that the chaining points were B-C-G-H-I-J-K, which are the peaks and troughs over the price movements. At B, the price is relatively high, so consumers will buy relatively less of it, and the quantity will be relatively low. At C, the reverse is true, and the quantity will be relatively high. A Laspeyres index chained at this frequency will put a low weight on the price decline from B to C, but a high weight on the price rise from C to G. Because of overweighting the price increases relative to declines, it could give a higher overall inflation rate attributed to that item than its long run inflation, shown by the curve from A to L. This curve is in fact the index series defined in (16) of the long term relatives from A to L, with the period of point A as period 0 and the period of L being t. Since it's a smooth curve, it has no correlation with the weights.

One example of an item-area cell that could have this pattern is Rice, Pasta, Corn-

Figure 4:



meal, and Other Cereal Products in Philadelphia, in Figure 4, where point is a month. If the monthly frequency corresponded to the B-C-G-H-I-J-K frequency in Figure 3, the resulting chaining points could look like this.

However, if the frequency of chaining increased further so that the chaining points are points like C-D-E-F-G-etc., the oscillations will once again not matter as much. If it was continuously chained, the oscillations wouldn't cause any resonance drift, and it would be a Divisia index.

Thus, only if the index was chained close enough to the B-C-G-H-I-J-K frequency do the oscillations matter. Szulc (1983) used the Tacoma Narrows Bridge, which collapsed in 1940, as an analogy to this resonance effect. Only at certain chaining frequencies would a chained Laspeyres diverge from the continuous Divisia index.

A chained Laspeyres index is, in a way, a contradiction. A Laspeyres index measures the change in the cost of buying last period's fixed basket, but a chained Laspeyres changes the basket every period. As a measure of the change in the cost of living, it assumes quantities are fixed but then changes quantities. It opens up the possibility of a garbage-in-garbage-out result that doesn't necessarily mean anything. This could outweigh any gain of a chained index in estimating a COLI over using a direct index.

Indeed, garbage-in-garbage-out is what actually happens if the Laspeyres arithmetic mean of index relatives is used with the monthly updated or Törnqvist share weights in place of the Lowe constant quantity share weights. The Chained Laspeyres index relative is

$$L^C_{t-1,t} = \Sigma_{ia} s_{i,a,t-1} R_{iat} \tag{8}$$

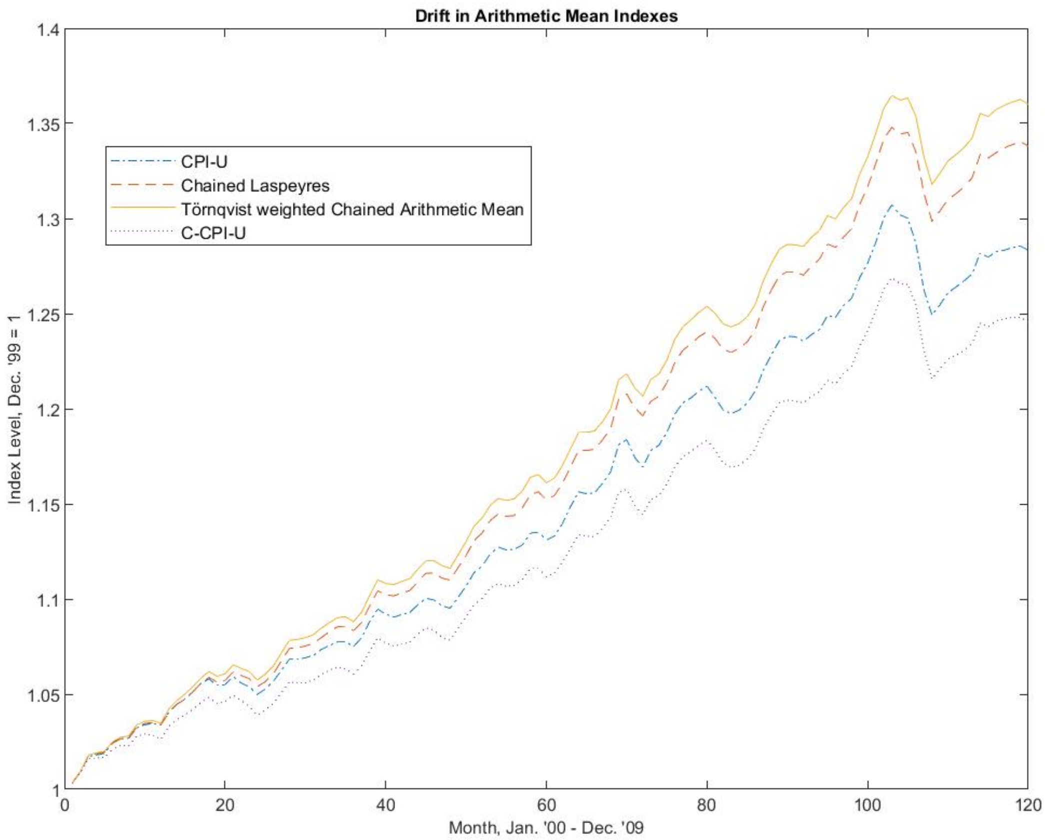
. A Törnqvist weighted chained arithmetic mean index relative, which uses the TQ mean expenditure shares in a chained Laspeyres index in place of a single month's previous period share, is

$$L^{TQ}_{t-1,t} = \Sigma_{ia} s^{TQ}_{iat} R_{iat} \tag{9}$$

where $s^{TQ}_{iat} = \frac{s_{iat}+s_{i,a,t-1}}{2}$. As seen in Figure 5, the Chained Laspeyres is higher than the CPI-U by 102% of the distance between the CPI-U and C-CPI-U, while the Törnqvist weighted chained arithmetic mean index is higher than the CPI-U by 148% of the distance.

The reason for this pattern is that holding the actual shares constant matters little for index construction, at least on the scale of comparing the CPI-U to the C-CPI-U. This means that they can be considered as effectively constant for this purpose. Cage, Greenlees,

Figure 5:


Drift in Arithmetic Mean Indexes
CPI-U
Chained Laspeyres
Törnqvist weighted Chained Arithmetic Mean
C-CPI-U
Index Level, Dec. '99 = 1
Month, Jan. '00 - Dec. '09
1.4
1.35
1.3
1.25
1.2
1.15
1.1
1.05
1
0
20
40
60
80
100
120

and Jackman (2002) in fact report that the shares do not have large changes over time. Therefore, using updated share weights is not very different from using the index relative in (7). This is described in Appendix C.

4 Changing Weights First

4.1 The Long Term Relatives

To see how much weight updating moves the CPI-U to the C-CPI-U, the resonance drift must be taken out first. One suggested method for a similar kind of chain drift is used by Ivancic, Diewert, and Fox (2011). They use scanner data to construct various price indexes at quarterly, monthly, and weekly levels, and find drift. They suggest using a method of combining longer spanning indexes to create drift-free indexes, called the GEKS method.

I use a different method that is more direct and intuitive for upper level CPI aggregation. The problem is solved by simply making a smooth line from the first to last index levels for each item-area. This is illustrated in the smooth curve in Figure 3. This modified index therefore has no short run correlation between the index relatives and implicit quantity weights, but has the same total inflation and total consumer substitution effects, removing the 'bad' resonance drift but not the 'good' substitution drift.

The modified index series must have the same first and last levels as the actual levels, and have a constant rate of change. Denoting the modified index level in month τ used to make a smooth index level series from month 0 to month t as $I_{ia\tau}^{LT,t}$, for long term (LT) index, and the modified constant relative for that series as d_{iat} for item i in area a for a series from

0 to t,

$$I_{ia0}^{LT,t} = I_{ia0} \tag{10}$$

$$I_{iat}^{LT,t} = I_{iat} \tag{11}$$

$$\frac{I_{ial}^{LT,t}}{I_{i,a,l-1}^{LT,t}} = \frac{I_{iak}^{LT,t}}{I_{i,a,k-1}^{LT,t}} = d_{iat} \tag{12}$$

for any two months $l, k < t$.

Since each item-area index level is the first month's level, denoted by $I_{ia0}^{LT,t}$, multiplied by all the intervening index relatives which are constant,

$$I_{iat}^{LT,t} = I_{ia0}^{LT,t} d_{iat}^{t} \tag{13}$$

, the only d_{iat} that would satisfy this is

$$d_{iat} = \left(\frac{I_{iat}^{LT,t}}{I_{ia0}^{LT,t}} \right)^{\frac{1}{t}} = \left(\frac{I_{iat}}{I_{ia0}} \right)^{\frac{1}{t}} \tag{14}$$

. It therefore requires only the price information that a direct index between month 0 and t would use. Equivalently,

$$d_{iat} = \Pi_{\tau=1}^{t} R_{ia\tau}^{\frac{1}{t}} \tag{15}$$

.

The long term relatives d_{iat} are different for each last month t, given the initial month 0. Each d_{iat} therefore defines a different series of index levels for each item i in area a tracing a smooth inflation path from 0 to t. From (13) and (10), for a given month t, the long term index levels in each series are given by

$$I_{ia\tau}^{LT,t} = I_{ia0} d_{iat}^{\tau} \tag{16}$$

for the long term index level at month τ leading from 0 to t.

These series were aggregated by a given formula over all items and areas to create a different all-items national index series from 0 to t. The intervening levels from 0 to t-1 of the modified series are discarded. Only the final month of each series is used as the level for the long term overall index for month t. The Lowe index level made with the long term relatives, or long term Lowe index for month t, denoted by $Lowe_t^{LT}$ is then

$$Lowe_t^{LT} = \Pi_{\tau=1}^{t} \frac{\Sigma_{ia} Aggweight_{iaB} I_{ia\tau}^{LT,t}}{\Sigma_{ia} Aggweight_{iaB} I_{i,a,\tau-1}^{LT,t}} = \Pi_{\tau=1}^{t} \Sigma_{ia} \frac{Aggweight_{iaB} I_{i,a,\tau-1}^{LT,t}}{\Sigma_{ia} Aggweight_{iaB} I_{i,a,\tau-1}^{LT,t}} d_{iat} \tag{17}$$

. This is illustrated in Figure 6, which graphs six examples of the $\Sigma_{ia} \frac{Aggweight_{iaB} I_{i,a,\tau-1}^{LT,t}}{\Sigma_{ia} Aggweight_{iaB} I_{i,a,\tau-1}^{LT,t}} d_{iat}$ series over $\tau = 0$ to t, for six different values of t, and also graphs the entire set of endpoints, $Lowe_t^{LT}$. Each of the six series for $\tau = 0$ to t is a nearly smooth curve itself from 1 to $Lowe_t^{LT}$.

The long term Lowe index is in fact a direct index over the two year period that the weights don't change. After that, it is as direct as the Lowe index: it is the same as chaining every two years. In fact, they are exactly the same for the first two years before the first CPI-U weight update in the data, as seen in Figure 2. This is not surprising since the Lowe index is circular (the same as a direct Lowe index from the first to last months) over each

Figure 6:

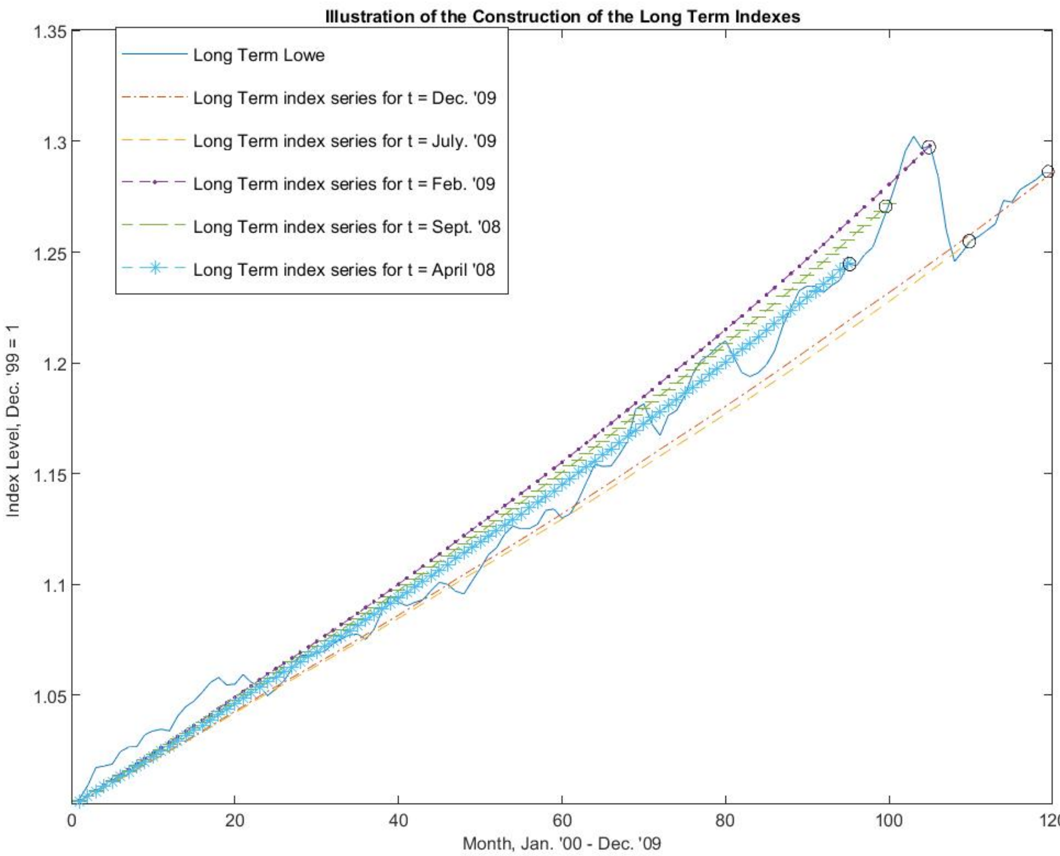


Illustration of the Construction of the Long Term Indexes
Long Term Lowe
Long Term index series for t = Dec. '09
Long Term index series for t = July. '09
Long Term index series for t = Feb. '09
Long Term index series for t = Sept. '08
Long Term index series for t = April '08
Index Level, Dec. '99 = 1
Month, Jan. '00 - Dec. '09

two year period in which the quantity weights do not change.[23] The only difference between the long term Lowe index and the CPI-U is due to the correlations between the long run relatives and the biennial weight updates which is slightly different than the correlations between the short term relatives.

As shown empirically, the LT Lowe index made with the LT relatives is very close to the CPI-U. Since the Törnqvist is generally known to have little chain drift, the use of the LT relatives is not driving the convergence between these formulas. Theorem 1 formalizes this by proving that the chained long term Laspeyres index approaches the chained Laspeyres as the interval shrinks. Therefore it only bypasses the resonance drift barrier.

Theorem 1 *Let $I_{iat}(t)$ denote the index level for item i in area a as a function of continuous*

[23] The chained Lowe is

$$\Pi_{\tau=1}^{t} \frac{\Sigma_{ia} Aggweight_{iaB} I_{iat}}{\Sigma_{ia} Aggweight_{iaB} I_{i,a,t-1}} =$$

$$\frac{\Sigma_{ia} Aggweight_{iaB} I_{ia1}}{\Sigma_{ia} Aggweight_{iaB} I_{ia0}} \cdot \frac{\Sigma_{ia} Aggweight_{iaB} I_{ia2}}{\Sigma_{ia} Aggweight_{iaB} I_{ia1}} \tag{18}$$

$$\cdot \frac{\Sigma_{ia} Aggweight_{iaB} I_{ia3}}{\Sigma_{ia} Aggweight_{iaB} I_{ia2}} \cdots \frac{\Sigma_{ia} Aggweight_{iaB} I_{iat}}{\Sigma_{ia} Aggweight_{iaB} I_{i,a,t-1}} \tag{19}$$

$$= \frac{\Sigma_{ia} Aggweight_{iaB} I_{iat}}{\Sigma_{ia} Aggweight_{iaB} I_{ia0}} \tag{20}$$

over the period that the $Aggweight_{iaB}$ doesn't change. Using (16), the chained long term Lowe is

$$\Pi_{\tau=1}^{t} \frac{\Sigma_{ia} Aggweight_{iaB} I_{ia0} d_{iat}^{\tau}}{\Sigma_{ia} Aggweight_{iaB} I_{ia0} d_{iat}^{\tau-1}} =$$

$$\frac{\Sigma_{ia} Aggweight_{iaB} I_{ia0} d_{iat}^{1}}{\Sigma_{ia} Aggweight_{iaB} I_{ia0} d_{iat}^{0}} \cdot \frac{\Sigma_{ia} Aggweight_{iaB} I_{ia0} d_{iat}^{2}}{\Sigma_{ia} Aggweight_{iaB} I_{ia0} d_{iat}^{1}} \tag{21}$$

$$\cdot \frac{\Sigma_{ia} Aggweight_{iaB} I_{ia0} d_{iat}^{3}}{\Sigma_{ia} Aggweight_{iaB} I_{ia0} d_{iat}^{2}} \cdots \frac{\Sigma_{ia} Aggweight_{iaB} I_{ia0} d_{iat}^{t}}{\Sigma_{ia} Aggweight_{iaB} I_{ia0} d_{iat}^{t-1}} \tag{22}$$

$$= \frac{\Sigma_{ia} Aggweight_{iaB} I_{ia0} d_{iat}^{t}}{\Sigma_{ia} Aggweight_{iaB} I_{ia0} d_{iat}^{0}} \tag{23}$$

$$= \frac{\Sigma_{ia} Aggweight_{iaB} I_{ia0} \left(\frac{I_{iat}}{I_{ia0}}\right)}{\Sigma_{ia} Aggweight_{iaB} I_{ia0}} = \frac{\Sigma_{ia} Aggweight_{iaB} I_{iat}}{\Sigma_{ia} Aggweight_{iaB} I_{ia0}} \tag{24}$$

which is the same as the chained Lowe over that same period.

time at t. Let $\tilde{R}_{iat} \equiv \frac{\partial I_{iat}(t)}{\partial t}$*. When the expenditure share weights are constant,* $s_{ia\tau} = s_{ia.}$ *for all* τ*, a continuously chained Laspeyres index using item-area index relatives is equal to a continuously chained Laspeyres index using the long term relatives defined in* (14) *and* (15)*,*

$$\exp\left(\int\limits_{\tau=0}^{t} \ln\left(\Sigma_{ia} s_{ia.} \tilde{R}_{ia\tau}\right) d\tau\right) = \exp\left(\int\limits_{\tau=0}^{t} \ln\left(\Sigma_{ia} s_{ia.} d_{ia\tau}\right) d\tau\right) \tag{25}$$

.

Proof. *In Appendix B.* ■

4.2 Breakdown Changing Weights First

The first step in the breakdown by changing weights first is to take the total difference between the CPI-U and the long term Lowe index, from month 1 to T = 120, Jan. '00 to Dec. '09 respectively. Denoting the level of the CPI-U in month t as $CPIU_t$, this is given by

$$\Sigma_{t=1}^{T}\left[CPIU_t - Lowe_t^{LT}\right]$$

$$= \Sigma_{t=1}^{T}\left[\Pi_{\tau=1}^{t}\Sigma_{ia}\frac{Aggweight_{iaB}I_{i,a,\tau-1}}{\Sigma_{ia}Aggweight_{iaB}I_{i,a,\tau-1}}R_{ia\tau} - \Pi_{\tau=1}^{t}\Sigma_{ia}\frac{Aggweight_{iaB}I_{i,a,\tau-1}^{LT,t}}{\Sigma_{ia}Aggweight_{iaB}I_{i,a,\tau-1}^{LT,t}}d_{iat}\right] \tag{26}$$

. This is 8.38% of the total difference between the CPI-U and C-CPI-U, which denoting the level of the C-CPI-U as T_t, is given by

$$\Sigma_{t=1}^{T}\left[CPIU_t - T_t\right] \tag{27}$$

$$= \Sigma_{t=1}^{T}\left[\Pi_{\tau=1}^{t}\Sigma_{ia}\frac{Aggweight_{iaB}I_{i,a,\tau-1}}{\Sigma_{ia}Aggweight_{iaB}I_{i,a,\tau-1}}R_{ia\tau} - \Pi_{\tau=1}^{t}\Pi_{ia}Rint_{ia\tau}^{s_{ia\tau}^{TQ}}\right] \tag{28}$$

, where the Törnqvist (TQ) mean shares are given by

$$s_{ia\tau}^{TQ} = \frac{s_{i,a,\tau-1} + s_{ia\tau}}{2} \tag{29}$$

and $Rint_{ia\tau}$ denotes the interpolated index relatives used in the C-CPI-U as described below. As mentioned, the difference for the first two years is zero.

The second step is to use the Törnqvist weights in place of the Lowe weights, to see the effect of implicit weight updating free of drift, which is the only change from the long term Lowe index.

Because the Törnqvist weights are an average of the current month's expenditure share and the previous month's share, the weights contain implicit quantity information from both months. This is a property the Törnqvist weighted long term arithmetic index shares with superlative indexes such as the Fisher and Walsh indexes, and pseudo-superlative indexes such as the Marshall-Edgeworth index. Because superlative indexes tend to give similar results, it raises the question of whether simply using the Törnqvist weights may have given the index a superlative-like quality and explain why it is so close to the C-CPI-U without using a superlative formula.

However, a chained long term Laspeyres index, which uses monthly updated expenditure share weights from the previous month only and not the current, is very close to the Törnqvist weighted long term arithmetic index, with a total difference of 1.41% of the difference between the CPI-U and C-CPI-U. This is explained by the fact that the share trends make little difference, so using a lagged share vs. an average with the current share would make little difference. Also, as will be shown below with the Constant Quantity Törnqvist, using an average weight instead of the previous month's weight makes little difference when quantities are held constant.

The Törnqvist weighted long term arithmetic index, denoted A_t^{LTTQ}, is

$$A_t^{LTTQ} = \Pi_{\tau=1}^{t} \Sigma_{ia} s_{ia\tau}^{TQ} d_{iat} \quad (30)$$

.

The difference between the long term Lowe index and the Törnqvist weighted long term arithmetic index,

$$\Sigma_{t=1}^{T} \left[Lowe_t^{LT} - A_t^{LTTQ} \right]$$

$$= \Sigma_{t=1}^{T} \left[\Pi_{\tau=1}^{t} \Sigma_{ia} \frac{Aggweight_{iaB} I_{i,a,\tau-1}^{LT,t}}{\Sigma_{ia} Aggweight_{iaB} I_{i,a,\tau-1}^{LT,t}} d_{iat} - \Pi_{\tau=1}^{t} \Sigma_{ia} s_{ia\tau}^{TQ} d_{iat} \right] \quad (31)$$

is 85.98% of the total difference between the CPI-U and C-CPI-U.

The third step is to change the arithmetic mean to a geometric mean, making the long term Törnqvist index level, denoted T_t^{LT},

$$T_t^{LT} = \Pi_{\tau=1}^{t} \Pi_{ia} d_{iat}^{s_{ia\tau}^{TQ}} \quad (32)$$

. The difference between the Törnqvist weighted long term arithmetic index and the long term Törnqvist,

$$\Sigma_{t=1}^{T} \left[A_t^{LTTQ} - T_t^{LT} \right] = \Sigma_{t=1}^{T} \left[\Pi_{\tau=1}^{t} \Sigma_{ia} s_{ia\tau}^{TQ} d_{iat} - \Pi_{\tau=1}^{t} \Pi_{ia} d_{iat}^{s_{ia\tau}^{TQ}} \right] \quad (33)$$

is only 2.05% of the total difference. As seen in Figure 2, this is a very small difference relative to the overall CPI-U to C-CPI-U difference. This is consistent with geometric and arithmetic means being approximately equal as interval the index is chained at shrinks. If one month is a short enough interval, a small difference is what would be expected.

Another difference between the CPI-U and C-CPI-U is that the CPI-U uses the non-interpolated item-area index relatives, while the C-CPI-U uses interpolated relatives. Many item-area cells are only priced bimonthly. For those cells, the index relative is 1 and the index doesn't change for the unpriced months. For use in the C-CPI-U, however, the unpriced months are given the square root of the next priced index relative, for both the priced and unpriced months, smoothing the inflation over the two months. The long term relatives were made with the non-interpolated index relatives.

The fourth step, from the long term Törnqvist to the Törnqvist made with non-interpolated relatives, denoted T_t^{nI}, adds back the short run variation in the item-area index relatives. This yields the difference that using the long term relatives makes for the Törnqvist. The difference is

$$\Sigma_{t=1}^{T}\left[T_t^{LT} - T_t^{nI}\right] = \Sigma_{t=1}^{T}\left[\Pi_{\tau=1}^{t}\Pi_{ia}d_{iat}^{s_{ia\tau}^{TQ}} - \Pi_{\tau=1}^{t}\Pi_{ia}R_{ia\tau}^{s_{ia\tau}^{TQ}}\right] \tag{34}$$

which comes to only 1.25% of the total difference. It is even less of an effect than the difference between the CPI-U and the long term Lowe index. Using the long term relatives, which holds the inflation rate constant, doesn't bias the C-CPI-U. This is because the trends in the shares matter little, so it makes little difference when the inflation in a certain item-area occurred when using expenditure shares in an index – all the relatives would be weighted about the same. This is similar to the effect of a long term Lowe index, since by construction the CPI-U holds the quantity weights constant most of the time. In fact, if the shares are constant, the long term Törnqvist is exactly the same as the normal Törnqvist, as shown by Theorem 2.

Theorem 2 *When the expenditure share weights are constant,* $s_{ia\tau}^{TQ} = \frac{1}{2}\left(s_{i,a,\tau-1} + s_{ia\tau}\right) = s_{ia.}$ *for all* τ*, a Törnqvist index using item-area index relatives is equal to a Törnqvist index using the long term relatives defined in* (14) *and* (15),

$$\exp\left(\Sigma_{t=1}^{T}\Sigma_{ia}s_{ia.}\ln R_{iat}\right) = \exp\left(\Sigma_{t=1}^{T}\Sigma_{ia}s_{ia.}\ln d_{iat}\right) \tag{35}$$

.

Proof. *In Appendix B.* ■

Finally, the fifth step is to move from the interpolated relatives Törnqvist to the actual C-CPI-U. Denoting the interpolated index relative for item i in area a for month τ as $Rint_{ia\tau}$, the difference is

$$\Sigma_{t=1}^{T}\left[T_t^{nI} - T_t\right] = \Sigma_{t=1}^{T}\left[\Pi_{\tau=1}^{t}\Pi_{ia}R_{ia\tau}^{s_{ia\tau}^{TQ}} - \Pi_{\tau=1}^{t}\Pi_{ia}Rint_{ia\tau}^{s_{ia\tau}^{TQ}}\right] \tag{36}$$

which is 2.34% of the total difference.

If the first and fourth steps are considered part of the formula effect, the total formula effect is 8.38%+2.05%+1.25% = about 11.86% of the total, while the weight updating effect, step 2, is 85.98% of the total, which is the large majority. The remainder is the effect of using interpolated index relatives.

5 Changing Formula First

Another set of valid intermediate steps that move from the CPI-U to the C-CPI-U involves changing the formula first, and then the weights. The first step is now to change from an arithmetic mean to a geometric mean, keeping the share weights the same as in equation (2).

However, this change would suffer from a different kind of drift. Consider a Geometric Lowe index, the log of which is

$$\begin{aligned}\ln G^{LoCQ}_{\mathbf{t-1},t} &= \Sigma_{ia} \frac{Aggweight_{iaB} I_{i,a,\mathbf{t}-1}}{\Sigma_{ia} Aggweight_{iaB} I_{i,a,\mathbf{t}-1}} \ln R_{ia\mathbf{t}} \\ &= \Sigma_{ia} \frac{Aggweight_{iaB} I_{i,a,\mathbf{t}-1}}{\Sigma_{ia} Aggweight_{iaB} I_{i,a,\mathbf{t}-1}} \ln \left(\frac{I_{iat}}{I_{i,a,t-1}} \right) \end{aligned} \tag{37}$$

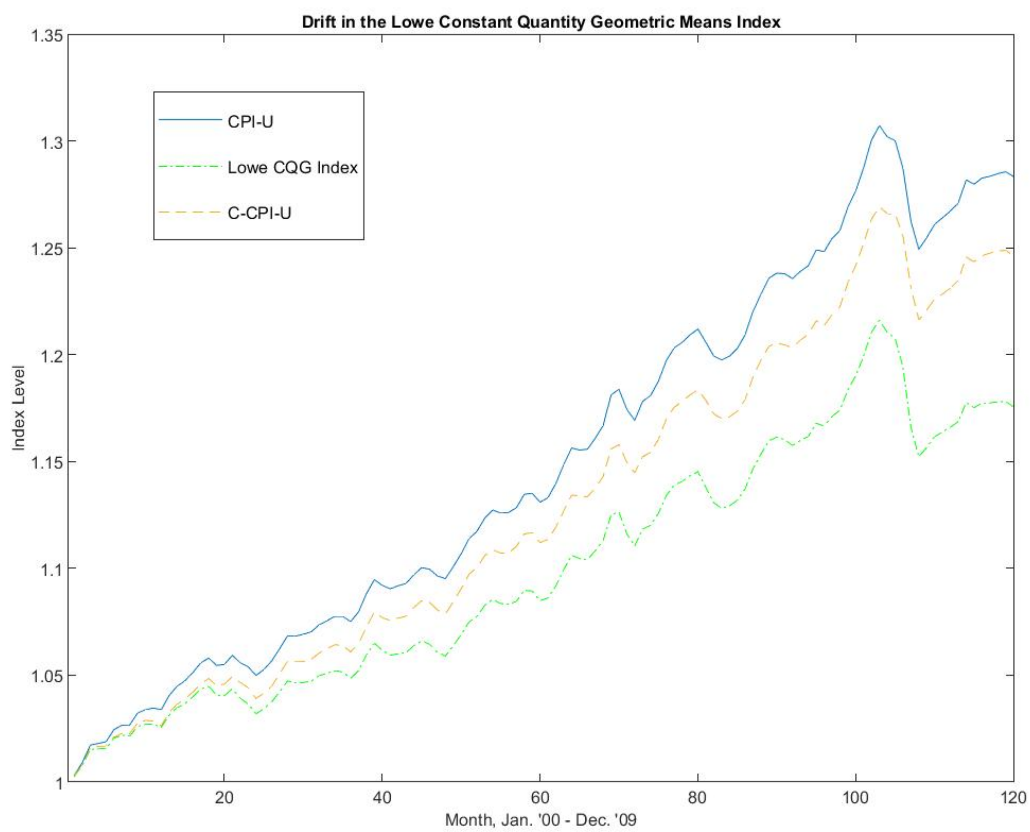
. This uses the Lowe $I_{i,a,t-1}$ term in both the share and in the index relative. This means that when there is price bouncing and the previous index level is high, the relative will be low. But then the share will be high. This creates a negative correlation between the weights and the relatives, which causes negative drift (if quantities moved elastically with price, the drift would be positive). This can be seen in Figure 7, where the Geometric Lowe is below even the C-CPI-U[24].

To get a meaningful breakdown, a long term Geometric Lowe index (LTGL) must be constructed

$$G_t^{LTGL} = \Pi_{\tau=1}^{t} \exp \left(\Sigma_{ia} \frac{Aggweight_{iaB} I^{LT,t}_{i,a,\tau-1}}{\Sigma_{ia} Aggweight_{iaB} I^{LT,t}_{i,a,\tau-1}} \ln d_{iat} \right) \tag{38}$$

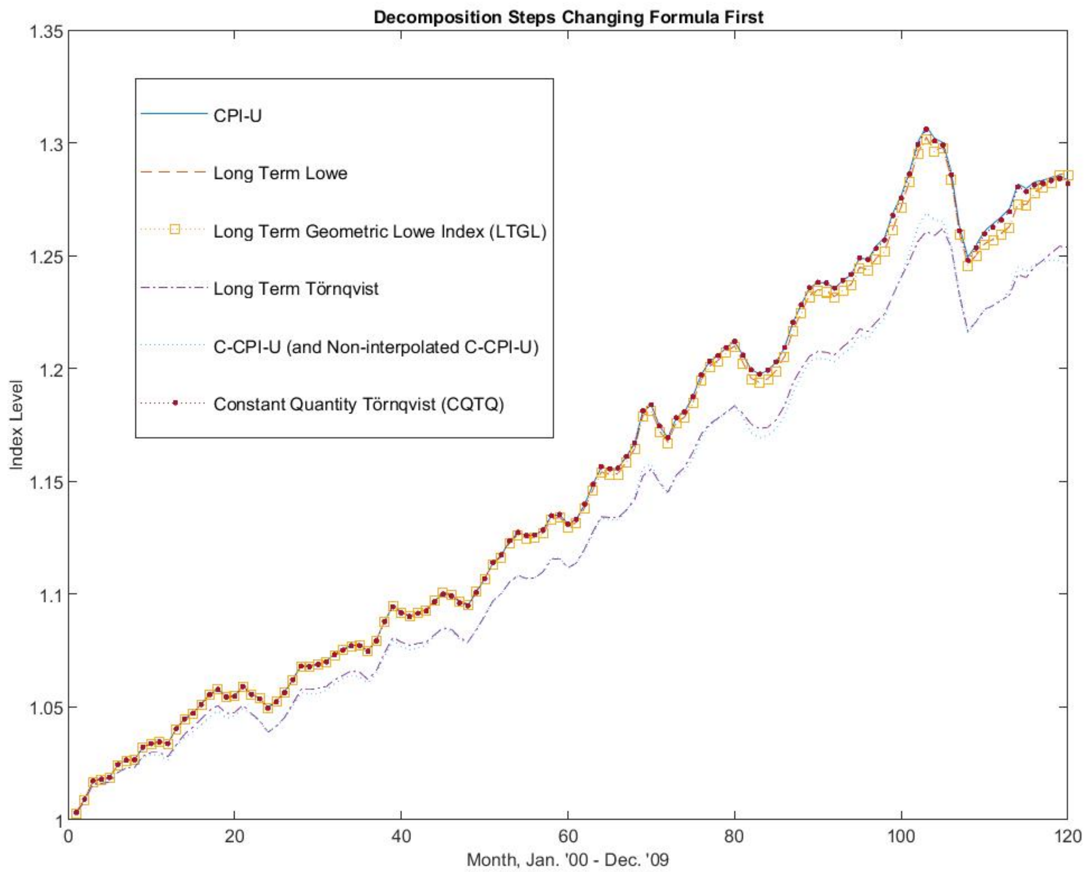
[24]Consistent with Figure 2, the C-CPI-U line is combined with the Non-interpolated C-CPI-U lines since the difference cannot be seen on the scale of the figure.

Figure 7:



Drift in the Lowe Constant Quantity Geometric Means Index
CPI-U
Lowe CQG Index
C-CPI-U
Index Level
Month, Jan. '00 - Dec. '09
1.35
1.3
1.25
1.2
1.15
1.1
1.05
1
20
40
60
80
100
120

Figure 8:



.

The long term Geometric Lowe index is graphed in Figure 8. There is still a little drift from first month, since for the first month in each series used to make the monthly index levels, if the previous index level is high, the month's relative will be low but the share will be high.

An alternative which requires no adjustment for resonance chain drift is to smooth out the weights in adjacent months by using the Törnqvist weights. Greenlees (2013) shows how the Lowe arithmetic mean can be approximated with a Törnqvist-like geometric mean which holds the quantities fixed, as in the Lowe index. This Constant Quantity Törnqvist (CQTQ) is simply the Törnqvist index with constant quantities in each period imposed. Greenlees (2014) showed that the CQTQ is in fact a second order logarithmic approximation to a Lowe or Laspeyres index. It uses the Lowe weight updating method, but takes an average of the Lowe shares over the current and previous months, like the Törnqvist index. The log CQTQ relative between t-1 and t, denoted $T^{CQ}_{t-1,t}$ for constant quantity (CQ), and T^{CG}_{t} for the level for period t, are

$$\ln T^{CQ}_{t-1,t} = \Sigma_{ia} \frac{1}{2} s_{i,a,B} \left(\frac{R_{i,a,B,t-1}}{L_{B,t-1}} + \frac{R_{i,a,B,t}}{L_{B,t}} \right) \ln R_{iat} \quad (39)$$

$$T^{CQ}_{t} = \Pi^{T}_{t=1} \exp \left(\Sigma_{ia} \frac{1}{2} s_{i,a,B} \left(\frac{R_{i,a,B,t-1}}{L_{B,t-1}} + \frac{R_{i,a,B,t}}{L_{B,t}} \right) \ln R_{iat} \right) \quad (40)$$

, where $R_{i,a,B,t}$ denotes the item-area index relative from the base period B to month t, and $L_{B,t}$ denotes the Laspeyres index of inflation from base period B to month t. The CQTQ updates the base period expenditure share $s_{i,a,B}$ for item i in area a by the relative inflation each item-area up to months t-1 and t, thus holding the implicit quantity constant.

If the index relatives bounce as in Figure 3 at the points B-C-G-H-I-J-K, so that a relatively high price is followed by a relatively low price, the effect on the first share will be the opposite of the effect on the second, so there will not be much correlation with the index relative. This can be seen in Figure 8, as the CQTQ is almost on top of the CPI-U, with a difference of 1.45% of the total.

Therefore there are two ways to do the first step.

One is to move to the long term Lowe index first. This is the same as step 1 before, with an 8.38% difference with the CPI-U.

The second step is to go to the long term Geometric Lowe index,

$$\Sigma_{t=1}^{T}\left[Lowe_t^{LT}-G_t^{LTGL}\right]$$

$$=\Sigma_{t=1}^{T}\left[\Pi_{\tau=1}^{t}\Sigma_{ia}\frac{Aggweight_{iaB}I_{i,a,\tau-1}^{LT,t}}{\Sigma_{ia}Aggweight_{iaB}I_{i,a,\tau-1}^{LT,t}}d_{iat}-\Pi_{\tau=1}^{t}\exp\left(\Sigma_{ia}\frac{Aggweight_{iaB}I_{i,a,\tau-1}^{LT,t}}{\Sigma_{ia}Aggweight_{iaB}I_{i,a,\tau-1}^{LT,t}}\ln d_{iat}\right)\right] \tag{41}$$

for a difference of 1.79%.

The third step is to move to the long term Törnqvist by inserting the Törnqvist share weights in place of the constant-quantity share weights, for a difference of 86.25%,

$$\Sigma_{t=1}^{T}\left[G_t^{LTGL}-T_t^{LT}\right]$$

$$=\Sigma_{t=1}^{T}\left[\Pi_{\tau=1}^{t}\exp\left(\Sigma_{ia}\frac{Aggweight_{iaB}I_{i,a,\tau-1}^{LT,t}}{\Sigma_{ia}Aggweight_{iaB}I_{i,a,\tau-1}^{LT,t}}\ln d_{iat}\right)-\Pi_{\tau=1}^{t}\Pi_{ia}d_{iat}^{s_{ia\tau}^{TQ}}\right] \tag{42}$$

.

The last step two steps are the same as steps 4 and five above, for 1.25% and 2.34% of the difference respectively.

The other way to change formula first is to move directly to the CQTQ first,

$$\Sigma_{t=1}^{T}\left[CPIU_t-T_t^{CQ}\right]$$

$$=\Sigma_{t=1}^{T}\left[\Pi_{\tau=1}^{t}\Sigma_{ia}\frac{Aggweight_{iaB}I_{i,a,\tau-1}}{\Sigma_{ia}Aggweight_{iaB}I_{i,a,\tau-1}}R_{ia\tau}-\Pi_{t=1}^{T}\exp\left(\Sigma_{ia}\frac{1}{2}s_{i,a,B}\left(\frac{R_{i,a,B,t-1}}{L_{B,t-1}}+\frac{R_{i,a,B,t}}{L_{B,t}}\right)\ln R_{iat}\right)\right] \tag{43}$$

. As one might expect, this makes little difference. In fact, this is only 1.45% of the difference, as mentioned above, similar to step 2 above and step 3 when changing the weights first.

Using this sequence of steps, the next step is to move to the non-interpolated Törnqvist, by using the Törnqvist share weights and allowing the implicit quantities to change,

$$\boldsymbol{\Sigma}_{t=1}^{T}\left[T_{t}^{CQ}-T_{t}^{nI}\right]$$

$$=\Sigma_{t=1}^{T}\left[\Pi_{t=1}^{T}\exp\left(\Sigma_{ia}\frac{1}{2}s_{i,a,B}\left(\frac{R_{i,a,B,t-1}}{L_{B,t-1}}+\frac{R_{i,a,B,t}}{L_{B,t}}\right)\ln R_{iat}\right)-\Pi_{\tau=1}^{t}\Pi_{ia}Rint_{ia\tau}^{s_{ia\tau}^{TQ}}\right] \qquad (44)$$

, for a whopping 96.22% of the total difference. The final step is the same as the final steps above, for 2.34%. In the first two methods, the moves to and from using the long term relatives summed to 8.38%+1.25% = 9.63%. This is roughly the difference between the weight effects for the first and third methods.

The results are robust to either of the three methods used, as in each case the large majority of the total difference is due to using share weights that allow the implicit quantities to change over long periods of time. The results are also robust to whether the weights or the formula is changed first. In both cases, allowing the implicit quantities to change and using long term relatives creates an index that is very close to the C-CPI-U. In fact the last method, moving to the CQTQ and then to the C-CPI-U, shows that the result that the majority of the difference is due to allowing quantity changes doesn't depend on using long term index relatives at all.

Appendix A shows more general results using different base months instead of just Dec. '99. The mean effects of the long term relatives is actually negative, at -1.18%. This effect varies between positive and negative for different initial months, and different lengths of time averaged over, but is always very small. Effectively, the long term C-CPI-U is on average the same as the C-CPI-U, only differing by small noise. Since the TQ weighted long term arithmetic index only differs from the long term C-CPI-U by the formula effect, around 3.5%, this means the TQ weighted long term arithmetic index is almost the same as the C-CPI-U (not counting the effects of interpolated relatives, which are not necessarily part of a COLI). That means that simply chaining at a monthly frequency and using long term relatives is 96.5% sufficient for measuring a COLI, without any formula assumptions. If such an index was used in place of the C-CPI-U, for many purposes it would be sufficient, providing a measure of a COLI while avoiding any theoretical issues surrounding it.

6 Conclusions/Discussion

All three approaches come to the same conclusion. The large majority of the difference between the CPI-U and the C-CPI-U is due to the weighting differences, which constrain quantities to be constant over long periods in the CPI-U but allow implicit quantities to change over long periods in the C-CPI-U. **I**t doesn't matter whether the weights are changed before the formula, the formula changed before the weights, or the formula first changed to a Törnqvist with constant quantities without any adjustment for chain drift. One month is a short enough period so that holding quantity or share weights constant makes little difference, but the effects add up if the quantity weights are held constant for an average of 36 months.

This study develops a new method to control for 'bad', or resonance, chain drift. The fact that the item-area expenditure shares can be effectively held constant for geometric mean index construction makes the long term relatives an effective solution to chain drift. This may not be the case for all contexts, and for other data, a different technique may be required. However, this can still provide an example of a general method that could be modified for another environment.

The findings of this paper extend to CPI formulas in general, not just the Laspeyres-type indexes and the Törnqvist. Every index formula in significant use is either an arithmetic mean, geometric mean, or some combination thereof. If resonance chain drift is controlled for, such as by using long term relatives, and if chaining is at the monthly frequency, it makes little difference whether an arithmetic or geometric mean is used in a context similar to the CPI over the time period studied, because prices change little over a single month and the price or index ratios are close to 1. Therefore, by generally explaining the cause of why the Lowe formula converges to the Törnqvist formula as the Lowe weights approach the Törnqvist weights, this paper shows that every relevant formula should converge at the monthly frequency. Also, since the Laspeyres formula is an upper bound on a COLI and it converges to a superlative Törnqvist index, it's hard to see how a relevant formula would not converge.

Chaining an index more frequently is basically interpolating within the range of the data. This is filling in the space between the current and previous period. Of course, interpolations converge as the range shrinks. The functional choice only matters if extrapolating outside of the range of the data, or if the interval must be long. While the change in the Divisia index yields the change in the cost of living for each point in time, it does not yield the correct COLI over a span of time unless consumer preferences are homothetic, which is unlikely. To study long time spans, a direct index over the entire interval is more accurate, and this could depend heavily on the formula assumption. Therefore the purpose of the index must be kept in mind, in determining how relevant the formula is. It may also be that additional precision is needed. After all, if tax brackets, social security, rent contracts and other payments are indexed to a CPI, even very small differences in the index can imply very large dollar amounts. That is why the BLS switched formulas for the initial C-CPI-U estimates.[25]

But if current inflation rates are the goal and the precision needed is on the scale of the difference between the CPI-U and the C-CPI-U, chaining at the monthly level, long term relatives and weights updated without holding quantities constant are sufficient for an index such as the CPI to effectively approximate a COLI. Therefore, the fact that the C-CPI-U is lower than the CPI-U does not depend on its specific form. In fact, if the assumptions used for approximating a COLI are undesirable, it isn't necessary to try to measure a COLI at all to get effectively the same results - the TQ weighted long term arithmetic index could be used instead.

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[25] See Klick (2018).

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9 Appendices

A Robustness to Different Initial Periods

All of the results reported above were generated using Dec. '99 as the initial period, and all the long term relatives were defined as the path from Dec. '99 to the current month according to (14). Therefore it is important to check whether the initial month affects the results. The same breakdowns were calculated using every month in the first 6 years of data, Dec. '99 - Dec. '05, as initial periods. The months used for each index went from the initial month to the same final month, Dec. '09. Months later than Dec. '05 were not used because the total length of the index would be too short for reliable results.[26] The simple means of the fractions listed in Table 1 over all 72 of the initial periods are reported in Table 2. The results are similar to Table 1.

[26] Fewer index months meant more variance from single month shocks, since the index levels are cumulative monthly changes. Since the index levels were more variable and there were fewer months to sum the differences in index levels, the results became unstable and unreliable.

Table 2: Mean Effects Across Initial Periods of each Step by Method of Breakdown as % of Total CPI-U vs. C-CPI-U Difference

	Method		
	Weights First	Formula First - Geomeans	Formula First - CQTQ
Replace Short Term Relatives with Long Term Relatives	6.29%	Same	NA
Weight Effect: Replace Constant Quantity Shares with Actual Shares	88.10%	88.33%	94.04%
Formula Effect: Replace Arithmetic Mean with Geometric Mean	3.64%	3.41%	2.82%
Replace Long Term Relatives with Short Term Relatives	-1.18%	Same	NA
Replace Non-interpolated Relatives with Interpolated Relatives	3.14%	Same	Same

B Equivalence of Continuously Chained Laspeyres and Long Term Laspeyres

To show this, first I will show that for continuous chaining, an arithmetic mean of price or index relatives is the same as the geometric mean. This is also shown in Diewert (1980). It is a well-known approximation to a logarithm that $\ln R \cong R - 1$, and $exp(R-1) \cong R$. Since all relatives are close to 1 as the chaining interval is small enough,[27] this means that at a given period in time, the geometric mean of relatives weighted by w_{ia} such that $\Sigma_{ia} w_{ia} = 1$ is

$$\begin{aligned} \lim_{R_{iat} \to 1} \exp\left(\Sigma_{ia} w_{ia} \ln R_{iat}\right) &= \exp\left(\Sigma_{ia} w_{ia}\left(R_{iat} - 1\right)\right) = \exp\left(\Sigma_{ia}\left(w_{ia} R_{iat} - w_{ia}\right)\right) \\ &= \exp\left(\Sigma_{ia} w_{ia} R_{iat} - 1\right) = \Sigma_{ia} w_{ia} R_{iat} \end{aligned} \tag{45}$$

which is the arithmetic mean.

Theorem 1 shows that an Laspeyres index using the long term relatives approaches the same limit with continuous chaining as a Laspeyres with the month index relatives. Therefore, using the long term relatives can avoid the spurious upward chain drift and approach the Divisia limit without diverging first as chaining becomes more frequent.

Proof. Proof of Theorem 1.

[27] Actual price changes are typically in discrete jumps, so at many points in time the relatives would not actually equal one. However, since these price changes would not occur at the same time, it can be assumed that the mass of price changes is small enough so that the relatives are not significantly different from one.

Because an arithmetic and geometric mean are interchangeable at continuous chaining,

$$
\begin{aligned}
\exp\left(\int_{\tau=0}^{t} \ln\left(\Sigma_{ia} s_{ia\tau} \tilde{R}_{ia\tau}\right) d\tau\right) &= \exp\left(\int_{\tau=0}^{t} \ln\left(\exp\left(\Sigma_{ia} s_{ia\tau} \ln \tilde{R}_{ia\tau}\right)\right) d\tau\right) \\
&= \exp\left(\int_{\tau=0}^{t} \left(\Sigma_{ia} s_{ia\tau} \ln \tilde{R}_{ia\tau}\right) d\tau\right) \\
&= \exp\left(\Sigma_{ia} \int_{\tau=0}^{t} s_{ia\tau} \ln \tilde{R}_{ia\tau} d\tau\right) \qquad (46)
\end{aligned}
$$

. With constant shares,

$$
\begin{aligned}
&= \exp\left(\Sigma_{ia} \int_{\tau=0}^{t} s_{ia.} \ln \tilde{R}_{ia\tau} d\tau\right) \qquad (47) \\
&= \exp\left(\Sigma_{ia} s_{ia.} \int_{\tau=0}^{t} \ln \tilde{R}_{ia\tau} d\tau\right) \qquad (48)
\end{aligned}
$$

. Similarly for the index using long term relatives,

$$
\begin{aligned}
\exp\left(\int_{\tau=0}^{t} \ln\left(\Sigma_{ia} s_{ia.} \tilde{d}_{ia\tau}\right) d\tau\right) &= \exp\left(\Sigma_{ia} s_{ia.} \int_{\tau=0}^{t} \ln \tilde{d}_{iat} d\tau\right) \qquad (49) \\
&= \exp\left(\Sigma_{ia} s_{ia.} \ln \tilde{d}_{iat} \int_{\tau=0}^{t} d\tau\right) \\
&= \exp\left(\Sigma_{ia} s_{ia.} t \ln \tilde{d}_{iat}\right)
\end{aligned}
$$

, where the continuous version of (15) is

$$\tilde{d}_{iat} = \exp\left(\frac{1}{t}\int_{\tau=0}^{t} \ln \tilde{R}_{iat} d\tau\right) \tag{50}$$

. Plugging this into the last line of (49) yields (47). ■

This is the case illustrated in Figure 3, where the chaining points move to B-C-D-E-F-etc., and then even closer together. Therefore both indexes are a chained long term index. Also, as mentioned, it has been shown that the continuous Laspeyres is equal to the Divisia index, since the Divisia index is simply a geometric mean version of a continuous Laspeyres.

Theorem 2 gives an analogous result for the Törnqvist index, but without requiring continuous chaining.

Proof. Proof of Theorem 2.

Rewriting the r.h.s. of (35) by switching the order of addition and bringing out the shares as

$$\exp\left(\Sigma_{ia} s_{ia.} \Sigma_{t=1}^{T} \ln d_{iat}\right)$$

and plugging in (15) yields

$$\begin{aligned} & \exp\left(\Sigma_{ia} s_{ia.} \Sigma_{t=1}^{T} \Sigma_{t=1}^{T} \frac{1}{T} \ln R_{iat}\right) \\ = \; & \exp\left(\Sigma_{ia} s_{ia.} \Sigma_{t=1}^{T} \ln R_{iat}\right) \end{aligned}$$

which can again be rewritten as the l.h.s. of (35) by again switching the order of addition and bringing out the shares. ■

C Stability of Expenditure Shares

To demonstrate how holding the shares constant makes only a small difference, Figure 9 compares the C-CPI-U to a geometric means index where the shares are held constant at the average share over the ten years for each item-area. The only difference between these indexes is that the C-CPI-U uses an average share between the current and previous month. The two indexes are very close relative to the CPI-U. While Greenlees and Williams (2010) do find that shares change in response to relative index level changes, these effects are clearly not large for the purpose of comparing the CPI-U to the C-CPI-U. The original initial estimates of the C-CPI-U also demonstrate this, because they held shares constant and were very close to the final C-CPI-U results.[28]

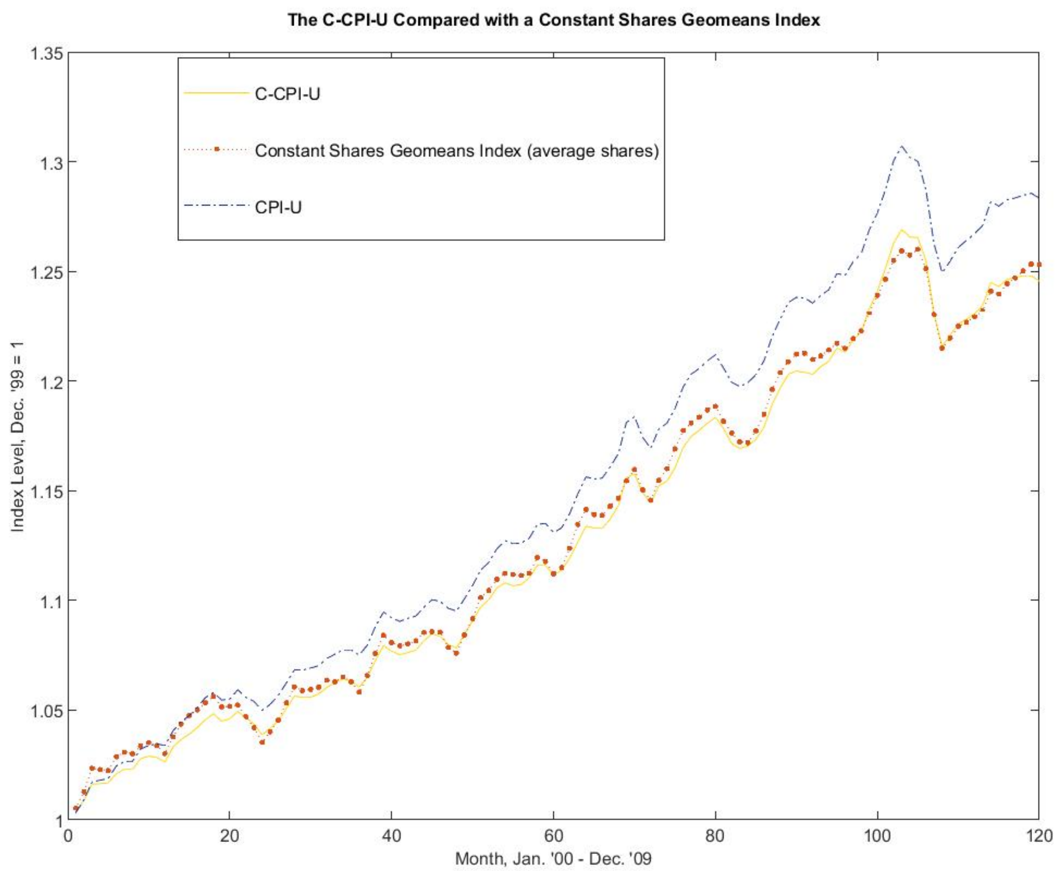
Except for sampling variation, this implies that the implicit quantity can be treated as moving roughly inversely proportionate to price, so that the shares change little. The fact that the TQ weights are averages of two month's shares doesn't matter if shares don't trend much. In fact it makes them even closer to being constant, which is even closer to the index in (7), and thus the Törnqvist weighted chained arithmetic mean index has even more upward chain drift than the Chained Laspeyres.[29]

[28] Because of the processing lag for publishing the final C-CPI-U, initial estimates were published that used a Geometric Young formula, holding the shares constant over the same period that the CPI-U's quantity weights are held constant. When the C-CPI-U was first planned, it was thought that there would be a significant downward bias in the initial C-CPI-U relatives compared to the final C-CPI-U relatives. This difference was to be measured over time as the average ratio between the initial and final C-CPI-U relatives. This adjustment factor would then be used to adjust the initial estimates to make them closer to the final relatives. However, when the adjustment factor turned out to be small, it was decided to drop the adjustment factor altogether. See Cage, Greenlees, and Jackman (2002).

Assuming that the expenditure shares are constant can still make a significant difference with the final C-CPI-U values for some purposes. The BLS has now changed from a constant shares geometric mean index for the initial estimates to a more flexible constant-elasticity-of-substitution function to make the initial values even closer to the final ones. See Klick (2018).

[29] The response of shares is prices is actually inelastic, shown in Greenlees & Williams (2010). Therefore, quantities move less than if the shares were constant, causing the drift pattern to be more pronounced when

Figure 9:



The C-CPI-U Compared with a Constant Shares Geomeans Index
C-CPI-U
Constant Shares Geomeans Index (average shares)
CPI-U
Index Level, Dec. '99 = 1
Month, Jan. '00 - Dec. '09
1.35
1.3
1.25
1.2
1.15
1.1
1.05
1
0
20
40
60
80
100
120

Drift in a geometric mean index, where the index is an expenditure share weighted geometric mean of item-area index relatives, is not caused by the same data pattern that causes drift in an arithmetic mean index. Instead of being caused by a correlation between quantities and index relatives, geometric mean index chain drift is caused by a correlation between the expenditure shares and the index relatives. Suppose that the expenditure share weight rose when the price rose, so that for a pattern like Figure 3, the expenditure share weights at points A, F, and H were relatively high, while for points B, G, and I they were relatively low. If a geometric mean index was chained at the intervals of A-B-F-G-H-I-J, it would have a high weight when the index relatives were falling, and vice versa when rising. Therefore it would have downward chain drift. Because the Törnqvist index uses an average expenditure share from the current and previous month, there is usually little drift in it. The short run variation in the item-area index relatives is only slightly correlated with the monthly Törnqvist weights, so that using a geometric mean removes the drift that the Chained Laspeyres has.

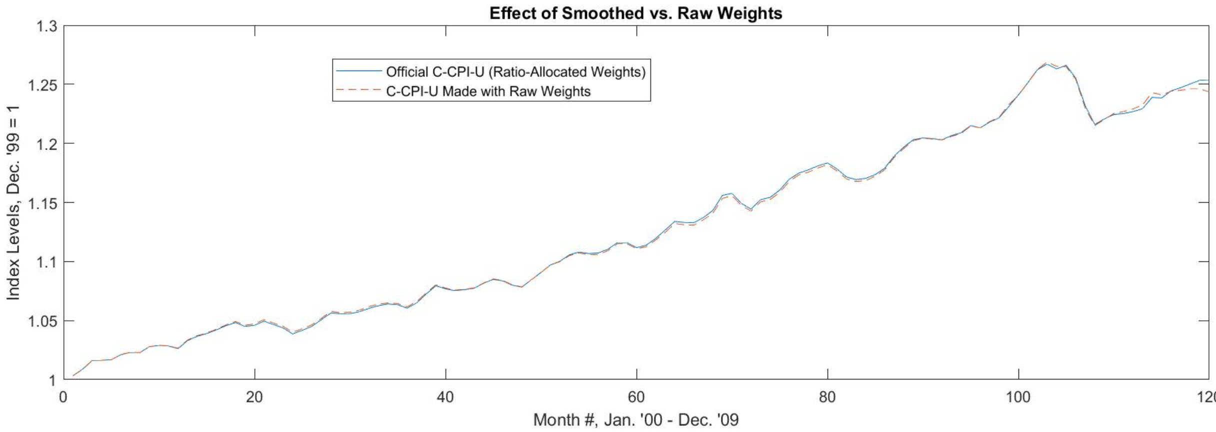
Therefore, the effect of formula on the chained Laspeyres index or TQ weighted Chained Arithmetic Mean index is not the same as on the CPI-U.

D Effect of Weight Smoothing in C-CPI-U

Figure 10 graphs the official C-CPI-U made with CE survey-measured weights that are smoothed with the ratio allocation process to compensate for the weight variance from the low sample size for the CE. Also, a C-CPI-U version that is made with the raw, unsmoothed

the average share is used.

Figure 10:



weights is graphed. As can be seen, the smoothing has very little effect on the index.