Improving the Preliminary Values of the Chained CPI-U

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John S. Greenlees[1]

[1]U. S. Bureau of Labor Statistics, 2 Massachusetts Avenue NE, Washington DC 20212

Abstract

This paper employs a constant-elasticity of substitution (CES) index formula to improve the accuracy of the preliminary values of the Chained Consumer Price Index for All Urban Consumers (C-CPI-U). Using the CES behavioural model, I present estimates of the overall extent of consumer response to relative price changes exhibited in Consumer Expenditure Survey data for 1999-2008. The associated parameter estimates are then used to develop CES forecasts of the final C-CPI-U index values. Simulations demonstrate that use of the CES approach over the last several years would have resulted in smaller index revisions between the preliminary and final C-CPI-U releases. Looking to the future, CES-based preliminary estimates could increase the usefulness of the C-CPI-U to government programs and other users.

Key Words: CPI; consumer; price; index; prediction; CES

1. Introduction

This paper employs a constant-elasticity of substitution (CES), or Lloyd-Moulton, index formula to improve the accuracy of the preliminary values of the superlative Chained Consumer Price Index for All Urban Consumers, or C-CPI-U. Simulations demonstrate that use of the CES approach over the last several years would have resulted in smaller index revisions. Looking to the future, CES preliminary estimates could increase the usefulness of the C-CPI-U in many potential applications.

The headline CPI, the CPI for All Urban Consumers (CPI-U), employs a form of the Lowe, or Modified Laspeyres, index structure. The aggregate US City Average All Items CPI-U is computed as an arithmetic average of lower-level indexes, with weights derived from consumer expenditures during a base period. Since 2002, the expenditure base period has been updated every two years, with each update introducing a new two-year base period.[i] Even with biennial updating, the CPI-U remains subject to the consumer substitution bias inherent in the Lowe structure. To address this concern, in 2002 the Bureau of Labor Statistics (BLS) introduced a new, supplemental index, the C-CPI-U. The Törnqvist formula used in the aggregation of the final C-CPI-U is designed to be a closer approximation to a cost-of-living index (COLI) than the Lowe formula used in the CPI-U. It uses actual consumer expenditure estimates from both the current and previous months to weight the basic indexes as a means of accounting for consumer substitution between item categories.[ii]

Monthly values of the C-CPI-U are published beginning with data for January 2000 (December 1999=100). Current values are released in the middle of each month along with the headline CPI-U and the CPI for Wage Earners and Clerical Workers (CPI-W), the latter of which is used widely for wage and benefits escalation. Because of unavoidable lags in the collection and processing of expenditure data, however, the C-CPI-U is subject to two annual revisions. The most recent final monthly values, for calendar year 2008, became available in February 2010. It is only in this final version of the index that the superlative Törnqvist formula is used in the aggregation of basic indexes. The preliminary monthly index values are computed using a weighted geometric mean formula with the weights corresponding to the same base period used in the CPI-U.

Because the Chained CPI uses a superlative formula and thereby reflects consumer response to changing relative prices, it has frequently been proposed as an alternative to the CPI-U or CPI-W for escalation purposes. The fact that its index values are subject to revision has been a major deterrence, however. The availability of nine calendar years of final C-CPI-U data and eight years of preliminary-to-final revisions provides an opportunity to determine whether the evidence would support a modification of the geometric mean formula used in the preliminary C-CPI-U, as a means of reducing the quantitative importance of index revisions.

I begin with a background discussion in Section 2, and follow it in Section 3 by estimating and comparing superlative and CES-based cost-of-living indexes for each annual period from 1999 through 2008. I present parameter estimates measuring the overall extent of substitution behavior in BLS consumer expenditure data, using a modified version of an econometric approach taken by Greenlees and Williams (2009). Those parameter estimates are then used in Section 4 to develop alternative CES forecasts of the final C-CPI-U index values. Section 5 provides an example of how the CES predictions could improve indexation processes based on the C-CPI-U. Section 6 concludes.

2. Background on Index Number Formulas

The central empirical issue of my paper is whether an operationally feasible CES formula can out-perform a geometric mean formula in providing accurate real-time preliminary estimates of the final Törnqvist C-CPI-U. This section reviews the different index number formulas involved in that evaluation. For reference, and to introduce notation, I begin with the Lowe formula used in the headline CPI-U.

As discussed in the international CPI manual published by the International Labour Office (ILO),[iii] a Lowe price index is distinguished from the familiar conceptual Laspeyres index by the separation of the weight reference (or expenditure base) period and price reference (or link) period.[iv] That is, let q_{tk} denote the total quantity purchased in period t of the k-th CPI item/area category, with p_{tk} denoting the corresponding basic index level. Let s_{tk} then indicate the associated expenditure share of that item/area category in total expenditure:

$$s_{tk} \equiv \frac{p_{tk} q_{tk}}{\sum_j p_{tj} q_{tj}} \tag{1}$$

Then the aggregate Laspeyres index between period 0 and period t is defined by:

$$IX_{t,0} \equiv \frac{\sum_k q_{0k} \, p_{tk}}{\sum_k q_{0k} \, p_{0k}} = \sum_k s_{0k} \left(p_{tk} \, / \, p_{0k} \right) \tag{2}$$

Construction of the Lowe index recognizes the operational lag in collecting and compiling expenditure shares, which then necessitates a lag between the expenditure base period b and the price reference period 0 in which those weights are introduced into the index. Writing the Lowe index in share form requires that those shares be “price-updated” to the link period. The price-updated share for the k-th item is given by

$$s_{0,bk}^{Lo} \equiv \frac{s_{bk} \; p_{0k} \big/ p_{bk}}{\sum_j s_{bj} \; p_{0j} \big/ p_{bj}} \tag{3}$$

These price-updated shares can be thought of as the shares that would be observed in period *0* if there were no changes in relative quantities purchased between periods *b* and *0*. Using the price-updated shares, the Lowe index between periods *0* and *t* is

$$IX_{t,0,b}^{Lo} \equiv \sum_k s_{0,bk}^{Lo}\ p_{tk} / p_{0k}$$

$$= \frac{\sum q_{bk}\, p_{tk}}{\sum_k q_{bk}\, p_{ok}} \tag{4}$$

The expenditure data used for CPI series weighting come from the Consumer Expenditure (CE) Survey, conducted for the BLS by the US Census Bureau. As noted earlier, the CPI-U weights are updated every two years. During 2008 and 2009, the CPI-U (as well as the CPI-W) employed the period 2005-2006 as its expenditure base period *b* and the period December 2007 as its link month *0*. Effective with data for January 2010, the periods *b* and *0* were updated to 2007-2008 and December 2009, respectively.

Standing in contrast to the Lowe index is the superlative Törnqvist formula used in the final C-CPI-U. The theory and advantages of superlative indexes were developed by Diewert (1976) and are discussed at length in the international CPI manual.[v] Sweden produces an approximation to a superlative CPI,[vi] and other countries have examined superlative CPI series computed retrospectively.

Final values of the C-CPI-U use a monthly-chained Törnqvist formula, employing estimated monthly expenditure weights from the CE survey. By employing weights from both the reference period and current period, the Törnqvist should provide a closer approximation to a true cost-of-living index between the two periods. The C-CPI-U formula for the change between months *t-1* and *t* is given by:

$$IX_{t,t-1}^{TQ} = \exp\left(\sum_k 0.5\left(s_{t-1,k} + s_{tk}\right) \ln\left(p_{tk} / p_{t-1,k}\right) \right) \tag{5}$$

It is well known that in the presence of consumer price-taking and utility-maximizing behavior the Laspeyres index provides an upper bound to the true cost-of-living index. That bounding result does not apply to the Lowe index. Research suggests, however, that under many reasonable conditions a Lowe index will tend to have an upward bias relative to the Laspeyres index and hence also to a target superlative or cost-of-living index.[vii] Consistent with that research, the annual increases in the CPI-U have exceeded those of the final C-CPI-U in every year for which the latter has been published.

Monthly CE weights for year *y* are not available for use until the beginning of year *y+2*, making it necessary that current C-CPI-U values be based on a preliminary formula that does not require current weighting information. As discussed in Cage *et al.* (2003), the preliminary C-CPI-U uses the same expenditure base period and link month as the CPI-U, but replaces the arithmetic Lowe form with a geometric mean. Moreover, the expenditure weights are not price-updated between the base and link periods. These changes lead to the following expression for month-to-month index change, similar to the Törnqvist formula except for the weights used:

$$IX_{t,t-1,b}^{Y} = \exp\left(\sum_k s_{bk} \ln\left(p_{tk} / p_{t-1,k}\right) \right) \tag{6}$$

The index can be alternatively expressed by

$$IX_{t,0,b}^{Y} = \exp\left(\sum_k s_{bk} \ln\left(p_{tk} / p_{0k}\right) \right) \tag{7}$$

This formula, known as a Geometric Young index, is consistent with a Cobb-Douglas consumer expenditure function, in which expenditure shares remain constant when prices change.[viii] Comparing equations (7) and (4), it can be seen that the Lowe and Geometric Young indexes will differ for two reasons: the functional form (arithmetic or geometric weighted mean) and the share weights (price-updated or not).

Recognizing the potential inaccuracy of the Cobb-Douglas assumption, the BLS included in its preliminary formulas a multiplicative factor λ, which could be used to adjust the forecast monthly changes higher or lower depending on whether λ was above or below unity. Over the forecast period the Geometric Young formula is then modified to take the form:

$$IX^{Y}_{t,t-1,b} = \lambda \exp\left(\sum_k s_{bk} \ln\left(p_{tk} / p_{t-1,k} \right) \right) \tag{8}$$

Lacking conclusive evidence to justify other values of λ, however, the BLS has thus far chosen to set λ=1 in each period, reducing the formula back to the Geometric Young form. As a consequence, the preliminary and final values of the C-CPI-U will coincide if consumer preferences are, in fact, Cobb-Douglas, or in the unlikely case that all CPI component indexes increase at the same rate. If neither condition holds, the preliminary values will be revised upward or downward when the CE data become available.

As indicated above, in February of each year *y+2* the BLS uses the CE expenditure data for year *y* to compute final C-CPI-U indexes for that year. It then uses the formula above to generate a forecast of revised "interim" indexes for the twelve months of year *y+1* as well as "initial" indexes for January and, subsequently, the remaining months of year *y+2*. The interim *y+1* indexes can differ from the initial *y+1* indexes they supersede for three reasons: (1) they will be linked to final rather than interim values of the December indexes for year *y*, (2) the expenditure base period used for the weights s_{bk} may be different from that used for the initial indexes, and (3) BLS may have changed the adjustment factor λ, although as was noted no such changes have yet been made.

Empirical analysis of simulated US superlative CPI series, and comparison of these to the CPI-U, goes back to Aizcorbe and Jackman (1993) and Shapiro and Wilcox (1997). BLS studies of the final C-CPI-U include Cage *et al.* (2003); Shoemaker (2005), who examines the statistical significance of differences between the C-CPI-U and CPI-U; Zadrozny (2008), who employs time-series methods to generate alternative preliminary estimates of final index changes; and Cage and Wilson (2009), who develop preliminary index series using forecasted monthly expenditures.

Okamoto (2001) and Lent and Dorfman (2009) have employed other data sets to focus on the problem of approximating true superlative indexes without the use of current-period expenditure data. Employing Japanese CPI data, Okamoto constructs "midpoint-year basket" indexes: annual indexes between years *s* and *t* using weights from period .5(*s*+*t*). Lent and Dorfman construct quarterly Laspeyres and Geometric Young indexes of airline fares using US Bureau of Transportation Statistics data on ticket prices. They then find weights such that the weighted averages of those two indexes approximate future movements in a superlative index.

In this paper I simulate the effect on the preliminary C-CPI-U estimates of replacing the Geometric Young formula by the constant-elasticity-of-substitution or CES formula. In the consumer price index context, if preferences take the CES form the resulting cost-of-living index is often referred to as the Lloyd-Moulton index,[ix] and in share form the index change from expenditure base period *b* to current period *t* is given by:

$$IX_{t,b}^{LM} = [\sum_k s_{bk} (p_{tk} / p_{bk})^{1-\eta}]^{1/(1-\eta)} \tag{9}$$

If the substitution parameter η equals zero the CES index reduces to the Laspeyres form, and it approaches the geometric mean form as η approaches unity. Because of its economy of parameters the CES or Lloyd-Moulton form has been used frequently in price index studies, such as Feenstra (1994), Shapiro and Wilcox (1997), Balk (1999), and Broda and Weinstein (2010).

Cage *et al.* (2007) fitted CES indexes to the final C-CPI-U using search techniques and demonstrated that for different time periods and levels of aggregation the closest approximations were consistently obtained by using CES substitution parameters between 0 and 1, that is, between corresponding Lowe and geometric mean indexes. Similarly, Shapiro and Wilcox (1997) fitted a CES index to an experimental superlative CPI by searching over values of η, with the best fit being at η =0.7. For this paper I take a very different approach; I make use of the fact that the Sato-Vartia index is exact for the CES preference system. That is, under the (strong) assumption that preferences do take the CES form, and given the availability of both current and base-period expenditure shares, the cost-of-living index can be calculated without knowing the substitution parameter η by computing the Sato-Vartia formula:

$$\ln IX_{t,0}^{SV} = \sum_k w_{t,0k} \ln\left(p_{tk} / p_{0k}\right) \tag{10}$$

This formula is very similar to the superlative Törnqvist except that the weights are the log-means of the reference and comparison period shares, defined by

$$w_{t,0k} \equiv \left(s_{tk} - s_{0k}\right)/\left(\ln s_{tk} - \ln s_{0k}\right) \tag{11}$$

and normalized to sum to unity over all cells k. Note that the Sato-Vartia is not a superlative index, because although it is exact for the CES cost function, the latter does not meet the conditions of a flexible functional form.[x]

I then employ a result by Feenstra and Reinsdorf (2003) that shows how the substitution parameter can be conveniently estimated consistent with the Sato-Vartia index. If the Sato-Vartia and Törnqvist indexes move very similarly over an estimation period, it makes sense to expect that a CES index with the Sato-Vartia's implied value of η could yield accurate forecasts of future Törnqvist levels.

3. Analysis of Substitution Behavior in BLS Expenditure Data

In the first part of this section I estimate annual price indexes using BLS data and several different price index formulas, with the primary goal being to determine the degree of similarity between superlative and Sato-Vartia indexes.[xi] The expenditure data used for my analyses are taken from the CE Survey, which as noted above provides all weights for the CPI-U and C-CPI-U. The data are drawn from the CPI expenditure weight database and thus are computed and classified in the same way as for the official indexes. In each period I have expenditure totals and basic indexes for 211 item categories and 38 areas, for a total of 8,018 cells.

I compute three different indexes between each of the adjacent years from 1999 through 2008. The two superlative indexes are the Fisher Ideal and the Törnqvist. Note that all these indexes are computed for analysis of demand behavior, not as operational alternatives to BLS practice. They are only feasible retrospectively, not in "real time," because in each case there is no lag between the price reference period and the end of the weight reference period. The annual log-change index estimates are shown in Table 1.

Table 1. Estimated Index Log-changes

Period		Index Formula		
Base	**Current**	**Fisher**	**Tornqvist**	**Sato-Vartia**
1999	2000	0.0287	0.0288	0.0289
2000	2001	0.0229	0.0231	0.0232
2001	2002	0.0126	0.0127	0.0128
2002	2003	0.0201	0.0201	0.0203
2003	2004	0.0238	0.0238	0.0240
2004	2005	0.0300	0.0302	0.0302
2005	2006	0.0304	0.0304	0.0305
2006	2007	0.0252	0.0253	0.0254
2007	2008	0.0358	0.0358	0.0359
Total 1999-2008		0.2294	0.2302	0.2311

Developing these estimates required making a decision about a single extreme outlier component index. As discussed in Greenlees and Williams (2009), one item-area index with a small weight fell by more than 99 percent between 1999 and 2000, while its associated annual expenditures increased slightly. Those values were used in the official CPI-U and C-CPI-U, a review having demonstrated that the underlying collected price data were correct.[xii] For the purposes here, however, it makes sense to eliminate that outlier index change so as not to distort the relationships among superlative index series and the conclusions regarding consumer substitution. I therefore recode the 1999-2000 index change to unity for that single item-area cell.

Consistent with expectations, the Table 1 results reveal that the annual log-changes in the two superlative series are extremely close together in every year, usually differing by less than .0001. Second, and more important for present purposes, the table shows that the Sato-Vartia changes based on the CES assumption are slightly higher than the superlative index changes except in one year, 2005, when the Törnqvist's log-change exceeds the Sato-Vartia's by approximately 0.00001. In total, the Sato-Vartia change is higher than the Törnqvist's by only 0.0009, or 0.01 percent per year. This closeness of the Sato-Vartia to the superlatives provides support for my use of the CES form to obtain a summary consumer substitution statistic for BLS data.

I next turn to examining the results of estimating the CES η parameter using the Feenstra-Reinsdorf (2007) regression approach. They show that a weighted, logarithmic regression of the change in expenditure shares s_k on the changes in component indexes p_k yields an accurate estimate of both η and the Sato-Vartia index IX^{SV}. With observations weighted in proportion to the log-means of the shares, the regression equation takes the form

$$dln\, s_k = -\alpha + \beta\, dln\, p_k + \varepsilon_k \qquad (12)$$

In (12) the disturbance terms ε_k are assumed to be mutually independent, and I use the notation $dln\, x_k$ to indicate $ln\, x_{tk} - ln\, x_{t-1,k}$.

Using price and share data from 1999 and 2000, for example, the regression yields α = 0.01047 and β = 0.36259. Feenstra and Reinsdorf show that β provides an estimate of $1-\eta$, while α will equal $(1-\eta)$ multiplied by the Sato-Vartia log-change between 1999 and

2000 (here, referencing Table 1, 0.36259 x 0.02887 = 0.01047). Table 2 presents the estimates of η along with the standard error of the $1-\eta$ coefficient estimate for each annual comparison.[xiii]

Table 2. Estimated Substitution Parameters

Period		CES substitution parameter	
Base	**Current**	**Estimate**	**Standard Error**
1999	2000	0.637	0.049
2000	2001	0.521	0.051
2001	2002	0.631	0.053
2002	2003	0.583	0.052
2003	2004	0.655	0.054
2004	2005	0.553	0.059
2005	2006	0.650	0.060
2006	2007	0.981	0.062
2007	2008	0.192	0.056

The annual results show a remarkable similarity from year to year, except for the last two years. The values of η vary only within a range of 0.521 to 0.655 from 2000 to 2006. In 2007, however, the regression coefficient on $dln\ p_k$ falls to only 0.019 and is not significantly different from zero (i.e., η not significantly different from unity). The reverse phenomenon is observed in the 2007-2008 changes; the $dln\ p_k$ coefficient of 0.808 is much higher than in any other year and implies a substitution term η of only 0.192. All the annual estimates, even for 2008, strongly reject the possibility of a zero substitution elasticity. Except for 2007, the implicit $\eta=0$ assumption underlying the Geometric Young preliminary CPI series is also strongly rejected.

Further examination of the data suggests that 2007 is the anomalous year. When I estimated equation (12) using price and expenditure share changes directly between the two years 2006 and 2008, I obtained a $dln\ p_k$ coefficient of 0.479, consistent with the range of η values for 2000 through 2006. It is possible that the 2006-2007 and 2007-2008 estimates resulted from one or more sharp category price changes during 2007 that for some reason were not reflected in consumer spending until 2008. I explored this possibility by re-estimating equation (12) several times with different expenditure components excluded. The components I excluded, in turn, were: Gasoline, which fluctuated widely in price between and within the last years of my study period; Other lodging away from home including hotels and motels, a historically volatile category with high sampling error; Care of invalids and elderly at home, a category that was re-assigned across CPI major groups in 2008 and therefore processed slightly differently in that year; Pets, pet products, and services, which had a sharp increase in estimated expenditure in 2007 when its CE reporting source was changed; and Owners' equivalent rent, the largest CPI item category. None of these exclusions, however, eliminated the phenomenon of a very high estimated elasticity in 2007 and a very low elasticity in 2008. Lacking any substantive explanation for the 2007 results, I conclude that consumers vary their purchase quantities significantly and inelastically on average across CPI categories, but that occasional anomalous years will occur.

It is easy to criticize the CES elasticity assumption. No one would seriously argue that the preferences of the representative consumer involve the same elasticity of substitution between apples and bananas as between apples and gasoline. Even more problematic is the idea that the same elasticity holds across metropolitan areas, for example between apples in Chicago and bananas in Boston. Fortunately, the latter point is of little importance quantitatively. As Cage *et al.* (2007) note, constraining substitution to be zero across areas has almost no effect on an aggregate superlative index, because the price change variance across item categories is so much greater than the variation across areas. The same holds in the data used here. An ANOVA decomposition of annual 2003-2004 index change by item and area, for example, shows that item category explains 18.4 of the 52.0 total sum of squared variation in $dln\ p_k$, whereas area explained only 0.3. In any event, for the purposes of this paper the issue is not whether the single-parameter CES model is true, but whether it can adequately model the behavior of a superlative index, and that will be determined in the forecast simulations presented in the next section.

The primary overall conclusion from the results of these index simulations and substitu-tion parameter estimates is that consumers vary their purchase quantities significantly but inelastically on average in response to relative price changes across the basic indexes of the CPI. Although this has long been an argument used against Laspeyres or Lowe index-es such as the CPI-U, Greenlees and Williams (2009) note that it also supports research on other methodological changes that would stop short of abandoning the fixed-basket nature of the CPI-U, such as more frequent expenditure weight revisions and alternative methods of updating expenditure weights between the base period and link month.

4. Initial and Interim Indexes using the CES Formula

The results of the previous section were derived retrospectively, using ten years of ex-penditure data and nine annual comparisons. In order to determine how the CES formula could have improved preliminary index estimates, however, I simulate forecast estimates using only the information that would have been available at the time the forecasts would have been made. For example, when the first official C-CPI-U values were released in July 2002, the latest CE expenditure data were for 2000, and those were used for the final 2000 C-CPI-U indexes. At the same time the first preliminary estimates were made, for the years 2001 and 2002. In order to obtain my alternative preliminary estimates for 2001 and 2002, I assume that the only results available for estimation of η were the estimates in the first row of Table 2, based on the annual changes between 1999 and 2000. Under that assumption, monthly projections forward from December 2000 could have been made by using the formula

$$IX^{LM}_{t,t-1} = \frac{[\sum_k s_{bk} (p_{tk} / p_{bk})^{1-\eta}]^{1/(1-\eta)}}{[\sum_k s_{bk} (p_{t-1,k} / p_{bk})^{1-\eta}]^{1/(1-\eta)}} \qquad (13)$$

where b is the base period 1999-2000, and η = 0.637. The base period 1999-2000 is used in the formula in order to be identical to the base period that was in effect in 2002 for the official CPI-U as well as for the BLS Geometric Young preliminary projections.

For subsequent years, additional data were available for estimating η. In February 2003, it would have been possible to pool the 1999-2000 and 2000-2001 price changes and ex-penditure shares to estimate an equation of the Feenstra-Reinsdorf form with two years of annual changes. A dummy variable *Y2001* for the second year is included to allow for different rates of inflation, while constraining η to be the same in both years. This yields the regression result:

$$dln\ s_k = -0.01200 + 0.41555\ dln\ p_k + 0.00234\ Y2001 + \varepsilon_k \qquad (14)$$

These updated parameters could have been used to generate a series of interim C-CPI-U values for 2002 and the initial values for 2003. Note that the updated estimate of $1\text{-}\eta$ from this equation is close to the average of the individual estimates from 2000 and 2001 in Table 2, and the estimates of overall index change one obtains by dividing the constant and year-dummy terms by the $dln\ p_k$ coefficient are also close to those in Table 1.
Successive pooled regressions of the form above yield the series of values of η shown in Table 3. The table also shows in which years those updated values would have been available for use in forecasting the C-CPI-U. The last year of final official C-CPI-U values are for 2008, and the most updated forecasts for that year would have been the interim values, based on expenditure data through 2007. Thus, the last value of η in Table 3 cannot be used in comparisons to final C-CPI-U data.

Table 3. Substitution Parameters for Predictions

Estimation Sample		Parameter	Used For	
From	To	Estimate	Initial	Interim
1999	2000	0.637	2002	2001
1999	2001	0.584	2003	2002
1999	2002	0.598	2004	2003
1999	2003	0.595	2005	2004
1999	2004	0.606	2006	2005
1999	2005	0.597	2007	2006
1999	2006	0.602	2008	2007
1999	2007	0.639	.	2008
1999	2008	0.589	.	.

For background in comparing the CES forecasts to the official Geometric Young preliminary values, Figure 1 shows the BLS initial C-CPI-U values along with the final index series. It is important to recall that each 12-month initial series begins at the tail of the previous year's interim index, not at the tail of the previous initial index. Both the initial and interim indexes are thus discontinuous 12-month projections. This is represented by the gaps in the initial index series in Figure 1. The figure further indicates that the initial forecasts have tended to be underestimates of the final values in each year.
Figure 2 and Table 4 analyze the relative performance of the CES model. Figure 2 is a monthly plot of the absolute initial prediction errors, that is, the absolute values of the differences between the final C-CPI-U and the initial forecasts of the two competing models.[xiv] The Official series in Figure 2 measures the absolute values of the differences between the lines in Figure 1, and the CES series is the corresponding differences between the CES initial forecasts and the final C-CPI-U. The CES initial forecasts are clearly superior except for a period in late 2006 and early 2007. Most noticeable is the period immediately following Hurricane Katrina, when gasoline prices rose sharply. For September 2005, the initial C-CPI-U index level was 114.7, but the final value was 115.6, an error in the initial of 0.9 percentage points. By contrast, the CES prediction is 115.3, roughly a third as large an underestimate. Two months later, after an equally sharp fall in gasoline prices, the initial C-CPI-U still significantly underestimated the final, whereas both the final value and the CES initial estimate were 114.9. In these periods, the Geometric Young's implicit model of Cobb-Douglas elasticity appears to have given too little weight to the gasoline price changes.

Figure 1. Initial and Final C-CPI-U Indexes

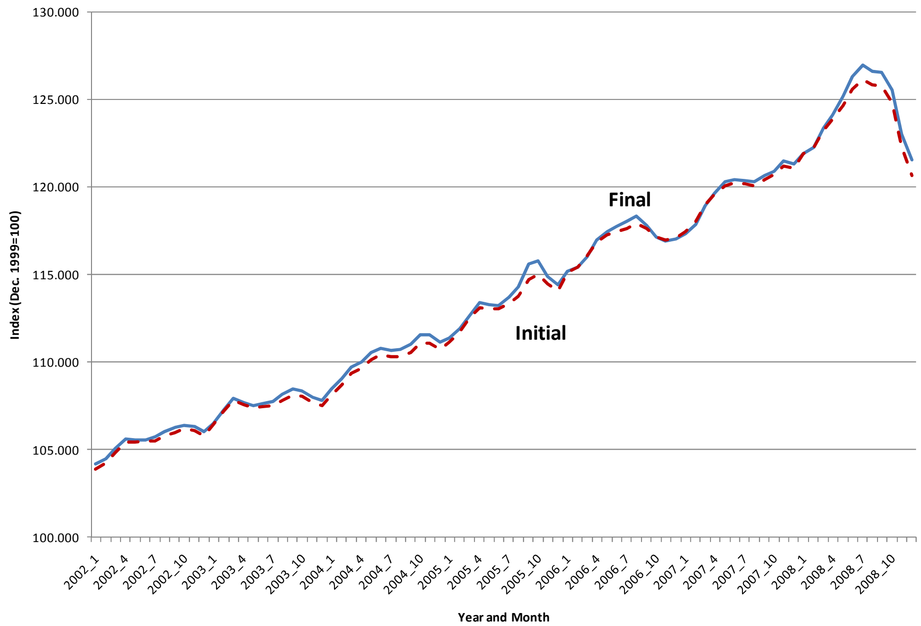
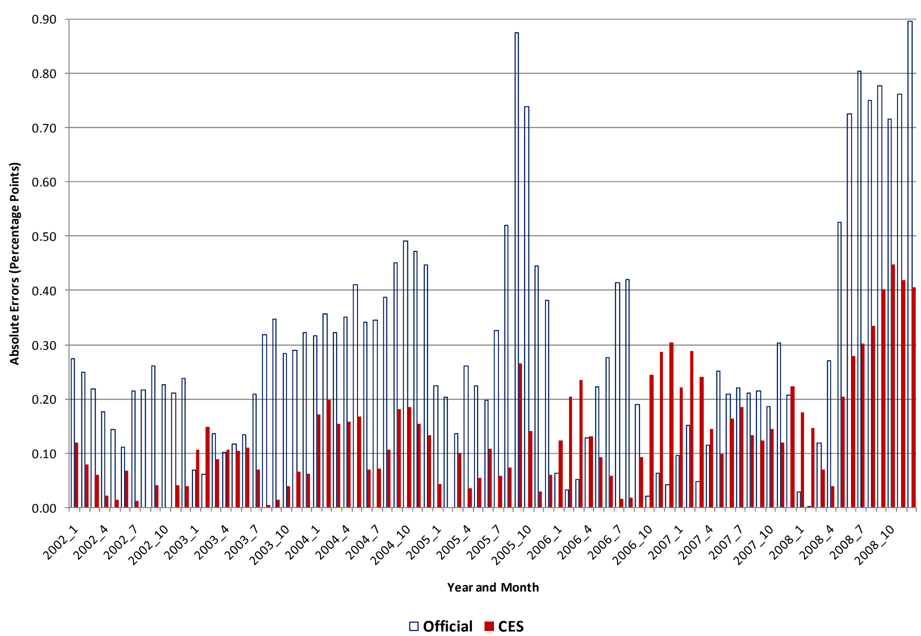


Figure 2. Absolute Errors of Initial Forecasts



Interim C-CPI-U values are projected out from the last known final for a shorter period (12 instead of 24 months), and the forecast errors are typically smaller than for initial estimates. The same pattern of superiority of the CES estimates occurs, however. Table 4 shows the mean errors, mean absolute errors, and root mean squared errors of the two approaches for both the initial and interim estimates of the final C-CPI-U index levels. The official predictions are significantly lower than the final values on average (using a standard z-test and ignoring auto-correlations), whereas the CES Lloyd-Moulton predictions are more nearly unbiased (the mean CES initial estimate is not significantly different from zero). The mean absolute errors and RMSEs of the official predictions are much higher.

Table 4. Summary of Prediction Results

Errors in Levels (percentage points)	Initials		Interims	
	Official	CES	Official	CES
Mean	-0.281	0.003	-0.150	-0.050
Mean Absolute	0.294	0.170	0.134	0.114
RMSE	0.361	0.169	0.270	0.180

5. Example of Indexation using the CES Model

Despite the fact that it is subject to two revisions, the C-CPI-U has often been recommended as an improved alternative to the CPI-U or CPI-W as a tool for indexation. In its annual *Budget Options* publication the Congressional Budget Office regularly calculates the revenue and spending impacts of moving to the BLS superlative index for tax and/or benefit adjustments.[xv] Alan Greenspan, as Chair of the Federal Reserve Board of Governors, advocated moving to the C-CPI-U, as did the recent Committee on National Statistics (CNSTAT) panel on the CPI.[xvi]

As described by the CNSTAT panel, in the presence of revisions the use of the C-CPI-U likely would involve computing a current annual inflation rate using the preliminary index values and adjusting that estimate by a measure of past revisions (i.e., indexation errors). To generate a realistic simulation of such a process, I assume a cost-of-living adjustment (COLA) regime in which an annual COLA is computed by comparing the average of the three third-quarter monthly index values for the current year to the same monthly values of the prior year. That COLA would then be applied to benefits at the beginning of the next year. This corresponds to the method used for Social Security and federal retirement COLAs, although it must be emphasized that those programs use the CPI-W for indexation rather than the CPI-U. Thus, the simulated COLAs in this section cannot be compared directly to actual Social Security or federal retirement COLAs to determine the impact of using a superlative series for indexation or a CES approach for preliminary index calculation. Moreover, it is important also to emphasize that the results below are presented only to demonstrate the relative performance of the CES-based preliminary estimates. No endorsement of the current COLA rule or advocacy of any alternative federal indexation process should be inferred on the part of either the author or the Bureau of Labor Statistics.

Denoting the initial, interim, and final index C-CPI-U values by *I*, *N*, and *F*, respectively, one can set the COLA percentage at time t using the following formula:

$$C_t = (I_t - N_{t-1}) + (N_{t-1} - I_{t-1}) \qquad (15)$$

This rule obviously can be simplified to be expressed as the change from the initial index estimate for year *t-1* to the initial estimate for period *t*.[xvii] Equation (15) is written as it is in order to highlight two components of the COLA. The first parenthesized term is the current (i.e., in period *t*) estimate of inflation from time *t-1* to time *t*, where all values are third-quarter average levels; I_t is the current estimate of the period *t* index level, and N_{t-1}

is the period-*t* estimate of the period *t-1* index level. The second term is an adjustment for the error in past COLA estimates. That this is true can be seen more clearly by decomposing that second term into two parts, the first consisting of the change in (update of) the estimate of period *t-1* inflation and the second equaling the change in the estimate of period *t-2* inflation. These are shown as the bracketed terms in the equation below:

$$C_t = (I_t - N_{t-1}) + \\ ([(N_{t-1} - F_{t-2}) - (I_{t-1} - N_{t-2})] + [(F_{t-2} - F_{t-3}) - (N_{t-2} - F_{t-3})]) \quad (16)$$

A special treatment must be given to the initial releases. In reality, the first C-CPI-U values were issued retrospectively, in 2002. That is, at that time it would have been too late to make a 2001 adjustment. For convenience, however, I will assume that the interim C-CPI-U values for 2001 were used for the 2001 adjustment. This does not distort my methodological comparisons and avoids having to assign an arbitrary value for the 2001 adjustment. Thus, using both the official and CES models I simulate series according to:

$$C_{2001} = N_{2001} - F_{2000} \ ,$$
$$C_{2002} = I_{2002} - N_{2001} \ ,$$
and
$$C_t = (I_t - N_{t-1}) + (N_{t-1} - I_{t-1}) \text{ for } t\text{=2003, \dots , 2007} \quad (17)$$

The results are shown in Table 5.[xviii] For 2003 and subsequent years I separately display the two terms in equation (17), the current inflation factor and the adjustment for past errors. The table again clearly demonstrates the superiority of the CES COLAs. From 2003 to 2008, the years for which we have final C-CPI-U values, the ex *post* revisions to the hypothetical COLAs using the BLS preliminary indexes are all positive, and in all but one of those years they are much larger in absolute value than the revisions to the hypothetical CES-based COLAs. The CES COLA revisions are always smaller than 0.1 percentage point, whereas the official preliminary values generate revisions greater than 0.1 in every year except 2007.

Table 5. Simulated Third Quarter to Third Quarter COLAs

		Official Preliminary Estimates			CES Preliminary Estimates		
Year	C-CPI-U Final	Current	Ex post Adjustment	Total	Current	Ex post Adjustment	Total
2001	2.16%	1.99%		1.99%	2.07%		2.07%
2002	1.27%	1.22%		1.22%	1.34%		1.34%
2003	1.99%	1.72%	0.22%	1.94%	1.94%	0.09%	2.03%
2004	2.47%	2.25%	0.13%	2.38%	2.42%	-0.08%	2.34%
2005	3.37%	3.03%	0.18%	3.21%	3.33%	0.06%	3.39%
2006	3.10%	3.08%	0.23%	3.32%	3.23%	-0.03%	3.20%
2007	2.01%	2.09%	0.03%	2.12%	2.21%	-0.09%	2.11%
2008	5.19%	4.60%	0.13%	4.73%	4.87%	-0.09%	4.78%
2009	.	-1.58%	-0.02%	-1.60%	-1.55%	-0.15%	-1.70%
2010	.	.	1.00%	.	.	0.56%	.
Mean			0.24%			0.03%	
RMSE			1.08%			0.61%	

For the two years 2009 and 2010, we have no final C-CPI-U indexes, but the required revisions to prior-year predictions can be computed. For 2009, these required revisions would be relatively small for the preliminary BLS indexes, because the 2007 data came so close to matching the Geometric Young model. In contrast, the 2010 revisions would be extremely large, because the 2008 data violated the geometric assumption so strongly.

Although the CES-based revisions are also large in 2010, the total 2009 and 2010 CES revisions are again much smaller than those based on the BLS preliminary values.

6. Conclusions and Further Issues

The analyses in this paper have confirmed, once again, that the consumer expenditure data underlying the US CPI imply consumer substitution away from goods and services with rising relative prices. This provides further evidence that Lowe index formulas such as the CPI-U yield higher inflation estimates than would a true cost of living index.

When analyzed using the Feenstra-Reinsdorf regression approach in Section 3, the data for 1999-2008 indicate that the average CES elasticity is between zero and unity. Although the annual elasticities are consistently closer to unity, Sections 4 and 5 demonstrate that employing those elasticity estimates offers significant improvement in forecasting the final superlative CPI values when compared to the current BLS based on an assumption of unitary elasticities. The methods I employ in this paper are based on data available at the time forecasts are made, and thus constitute an approach that the BLS could consider following in the future.

The need to revise past values of the C-CPI-U has limited its potential value as an indexation tool. This paper takes no stand on whether or how any government or private payments should be indexed for inflation. That caveat notwithstanding, an improvement in the methodology for preliminary values that regularly reduced the size of revisions could significantly enhance the value of the C-CPI-U to government programs and other users.

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[i] Information on all CPI procedures can be found in Bureau of Labor Statistics (2007).

[ii] For more details on the structure and development of the C-CPI-U, see Cage, Greenlees, and Jackman (2003).

[iii] ILO (2004) paragraphs 1.17-1.23.

[iv] The weight reference and price reference periods should not be confused with the index reference period, at which the index is set to 100.

[v] ILO (2004), for example in paragraphs 1.97-1.101. Besides the Törnqvist, well-known superlative indexes include the Fisher Ideal and Walsh formulas.

[vi] See Ribe (2005).

[vii] See, for example, ILO (2004), paragraphs 15.43-15.45, and Balk and Diewert (2003).

[viii] See, for example, Balk (2009). An ordinary Young index uses period-*b* shares, without price-updating, but does not use a geometric mean formula. See ILO (2004), paragraphs 1.35-1.40.

[ix] See ILO (2004), paragraphs 17.61-17.64.

[x] A superlative index is one that is exact for a cost function that with suitable coefficients can provide a second-order approximation to an arbitrary cost function. The CES does not satisfy that requirement. See Diewert (1976).

[xi] The results in this section are modified versions of those reported in Greenlees and Williams (2009).

[xii] The arithmetic mean Lowe formula used in the CPI-U was very robust to the outlier because of its small expenditure weight. The C-CPI-U was entirely unaffected because it is a monthly chained index that began in January 2000, after the steep price drop occurred.

[xiii] Lent and Dorfman (2009) demonstrate that η can alternatively be estimated by approximating a superlative index as a weighted average of an arithmetic (Laspeyres) and Geometric index. In the CE data used in this paper, the Lent/Dorfman and Feenstra/Reinsdorf methods yield very similar estimates of η.

[xiv] All the CPI data used in this paper are computed from full-precision index values rather than the rounded values used in BLS publications.

[xv] See, for example, Congressional Budget Office (2009), pp. 132-133, 147-148, and 186-187.

[xvi] Federal Reserve Board (2004), and Committee on National Statistics (2002), pp. 194-195.

[xvii] A similar process using the C-CPI-U is described in Congressional Budget Office (2010).

[xviii] As elsewhere in the paper, I ignore rounding effects. Indexation amounts are often specified to one decimal place only.