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ESTIMATED U.S. MANUFACTURING PRODUCTION CAPITAL AND TECHNOLOGY BASED ON AN ESTIMATED DYNAMIC STRUCTURAL ECONOMIC MODEL*

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ABSTRACT

Production capital and total factor productivity or technology are fundamental to understanding output and productivity growth, but are unobserved except at disaggregated levels and must be estimated before being used in empirical analysis. In this paper, we develop estimates of production capital and technology for U.S. total manufacturing based on an estimated dynamic structural economic model. First, using annual U.S. total manufacturing data for 1947-1997, we estimate by maximum likelihood a dynamic structural economic model of a representative production firm. In the estimation, capital and technology are completely unobserved or latent variables. Then, we apply the Kalman filter to the estimated model and the data to compute estimates of model-based capital and technology for the sample. Finally, we describe and evaluate similarities and differences between the model-based and standard estimates of capital and technology reported by the Bureau of Labor Statistics.*

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1. Introduction.

Time series of production capital and total factor productivity or technology, as we call it here, are fundamental to understanding output and productivity growth. Unfortunately, capital and technology are unobserved except at the most disaggregated levels of production units and capital components and must be estimated before being used in empirical analysis. Standard methods for estimating capital and technology are based largely on work by Jorgenson (1963) and Solow (1957). We develop alternative estimates of production capital and technology for U.S. total manufacturing based on an estimated dynamic structural economic model. First, using annual U.S. total manufacturing data from 1947-1997, we estimate by maximum likelihood a dynamic structural economic model of a representative production firm. In the estimation, capital and technology are completely unobserved or latent variables. Then, we apply the Kalman filter to the estimated model and the data to compute model-based estimates of capital and technology for the sample. Finally, we describe and evaluate similarities and differences between the model-based estimates and standard estimates of capital and technology, the latter reported by the Bureau of Labor Statistics.

We estimate aggregate capital for U.S. total manufacturing production capital (equipment and structures) in two major steps, a model-parameter estimation step followed by an unobserved-variable estimation step. In the first step, we specify and estimate by maximum likelihood a dynamic structural economic model of a representative production firm in an industry. We assume the firm solves a dynamic optimization problem, which is a standard convex adjustment cost problem except that adjustment costs on capital and technology are derived from a parsimoniously parameterized production function, rather than being stated directly as is usually done. We compute and incorporate the resulting optimal decision rules into the two estimation steps. We estimate the model's structural parameters without using any observations on capital or technology. We use only observations on prices and quantities of output, investment, research (short for "research and development"), labor, and materials inputs. We overcome the lack of capital and technology data by using a missing-data variant of the Kalman filter (Jones, 1980; Ansley and Kohn, 1985; Zadrozny, 1988, 1990) to compute the likelihood function and by using the overidentifying restrictions on reduced-form parameters in terms of structural parameters implied by optimal decision rules. The reduced-form equations of the estimated model imply correlations between unobserved capital and technology and the observed variables in the model. In the second step, we use these correlations in the Kalman filter to compute linear least squares estimates of capital and technology in terms of the observed variables of the model and their standard errors.

We now broadly review standard methods for estimating aggregate production capital and technology and the relative advantages of the present model-based estimation method.

Standard methods for estimating capital stock are usually based on iterating on a perpetual inventory equation (PIE), starting from a chosen initial value of capital. A basic PIE is $k_t = k_{t-1} + i_t - d_t$, where k_t denotes (usually unobserved) capital stock at the end of period t (the time interval [t-1,t] between moments t-1 and t), i_t denotes (usually observed) investment flow over period t, and d_t denotes (sometimes observed) discarded capital flow over period t. If d_t is unobserved, then, it could be set as $d_t = \phi_{k1}k_{t-1}$, where $\phi_{k1} \in (0,1)$ denotes one minus a constant geometric rate of capital depreciation, which results in the most commonly used PIE. More generally, a PIE could be a rational distributed lag (Jorgenson, 1966),

$$(1.1)\quad k_t = \phi_{k1}k_{t-1} + \ldots + \phi_{kp}k_{t-p} + \phi_{i0}i_t + \phi_{i1}i_{t-1} + \ldots + \phi_{iq}i_{t-q},$$

where the ϕ_k and ϕ_i parameters determine a possibly nongeometric depreciation schedule. While $k_t = \phi_{k1}k_{t-1} + \phi_{i0}i_t$ corresponds to a first-order autoregressive time-series model, a rational distributed-lag PIE corresponds to a higher-order autoregressive moving-average time-series model.

Aggregate production capital is also sometimes estimated as an index of service flows of capital components (equipment, structures, and other disaggregates) which are often estimated using Jorgenson's (1963) rental prices and are indexed using expenditure weights. In such cases, disaggregated data is usually used to estimate the aggregate capital and, aside from chosen initial values of capital stocks, the capital estimates depend on observed investment minus discards or on observed investment and a distributed-lag depreciation schedule.

The Solow residual (Solow, 1957) is the standard method for estimating technology in percentage change form: $d\tau_t = dq_t - \sum_{i=1}^{n} s_{it}dx_{it}$, where $d\tau_t$, dq_t, and dx_{it} denote percentage changes of technology, output, and production inputs, between periods t-1 and t, and s_{it} denotes the share of input costs of input i in period t. Because noisy or short-run variations in output are often not considered to be variations in technology and output displays significant noisy variations, the Solow residual is often first smoothed by some method before it is considered to be an acceptable estimate of technological change. For example, French (2000) smooths the Solow residual using the Kalman filter.

The model-based method has the relative advantages over standard methods that a detailed and larger econometric model has over a simpler and smaller econometric model: greater generality (fewer implicit or explicit restrictions), more details, and more implications. The disadvantages are the need for more data, a greater risk of model misspecification, and greater theoretical (mathematical) and computational complexity. Standard methods for estimating capital and technology, while not in theoretical conflict with each other, are computationally independent. The model-based method views capital and technology as the result of coordinated investment and research decisions which aim to solve a single and purposeful dynamic optimization problem. By constrast, in standard Solow-residual methods, technology is an unexplained residual. Whereas the model-based method explicitly includes adjustment costs, standard methods implicitly assume there are no adjustment costs. However, standard methods are nonparametric and, except for possibly having to specify and estimate a parametric capital depreciation schedule, are conceptually easier to understand and computationally easier to implement.

The model-based method uses the Kalman filter which simultaneously computes estimates of unobserved capital and technology and their standard errors, which quantify uncertainty about the estimates. In the model-based method, disturbances in capital's and technology's PIEs and Kalman-filter estimates of the disturbances are the basis for the standard errors. By contrast, standard methods cannot similarly produce standard errors of capital and technology estimates, because they do not include disturbances in capital's and technology's PIEs. As figures 3a-4a below show, reducing the standard error of capital's disturbance in its PIE (2.9) by 10^{-4} changes capital estimates from noisy, with many and large short-run variations and large standard-error bounds, to smoother ones, with fewer and smaller short-run variations and much smaller standard-error bounds.

Recently, economists have estimated technology as filtered estimates of an unobserved estimated exogenous process (Slade, 1989; French, 2000). The present paper goes further by treating capital and technology as <u>jointly generated</u> <u>endogenous</u> processes. We are unaware of filtering methods having been used similarly to estimate jointly generated endogenous capital and technology processes, although filtering methods have been used to estimate endogenous inflationary expectations (Burmeister and Wall, 1982; Hamilton, 1985; Zadrozny, 1997). Regression methods have been used to estimate GNP, aggregate capital, and other macroeconomic variables (Romer, 1989; Levy and Chen, 1994; Levy, 2000) but, in contrast to filtering methods, regression methods have more limited applicability and are less efficient. Unlike filtering methods, regression methods require the estimated variables to be observed in some periods and cannot exploit correlations at all leads and lags. The present model-based method for estimating capital and technology could be seen as an extension of Lucas (1967) based on more recently available computational methods. Finally, we note Jorgenson, Gollop, and Fraumeni (1987), Adams (1990), Griliches (1995), Caballero (1999), Nadiri and Prucha (2001), and references therein as more recent examples of work on production capital and technology.

The paper continues as follows. Section 2 specifies the model and explains how the representative firm's dynamic optimization problem is solved. Section 3 prepares the model for estimation of parameters, capital, and technology by assembling its equations as a vector autoregression (VAR) and, then, restating the VAR as a state representation. Section 3 also discusses the parameter identification and reconstructibility conditions underlying the estimations. Section 4 discusses the application to annual U.S. total manufacturing data for 1947-1997. It discusses sources and properties of the data, statistical and economic properties of the estimated model, and compares model-based estimates of capital and technology with standard estimates published by the Bureau of Labor Statistics. Section 5 contains concluding remarks. Some technical details are in the appendix.

2. Specification and Solution of the Model.

Following Zadrozny (1996), we describe an industry in terms of a representative firm (henceforth, "the firm"). Except for scale differences, firm- and industry-level variables are identical. Every period, t, the firm maximizes the expected present value of profits,

$$(2.1) \qquad v_t = E_t \sum_{k=0}^{\infty} \delta^k \pi_{t+k} ,$$

with respect to a feedback decision rule, where the maximization is subject to equations to be specified, E_t denotes expectation conditional on the firm's information in period t, $\delta \in (0,1)$ denotes a constant real discount factor, and $\pi_t = r_{qt} - (c_{qt} + c_{it} + c_{rt})$ denotes real profits equal to revenues minus costs, where c_{qt} denotes the cost of production and c_{it} and c_{rt} denote direct (nonadjustment) costs of investment in capital and research in technology. Throughout, a real value is a nominal (current dollar) value divided by some aggregate price index like the GDP deflator. The firm's optimization problem is stated precisely at the end of this section.

To obtain a competitive rational-expectations-equilibrium solution, following Lucas and Prescott (1971), we set revenues in π_t to the area under the inverse output-demand curve, as $r_{qt} = \int_{x=o}^{q_t} p_q(x, d_t)dx$, where $p_q(\cdot)$ is the inverse output-demand curve, q_t is the production of saleable output, and d_t is the output-demand state. Alternatively, when $r_{qt} = p_q(q_t, d_t)q_t$, the solution represents the monopoly equilibrium.

To obtain linear solution equations, which facilitate estimation and to which the Kalman filter can be applied, we specify r_{qt}, c_{qt}, c_{it}, and c_{rt} as quadratic forms (constant and linear terms can be ignored). Accordingly, we assume the industry's inverse output-demand curve is

(2.2) $$p_{qt} = -\eta q_t + d_t + \zeta_{pq,t},$$

where $\eta > 0$ is the constant slope parameter, d_t is the demand state generated by the second-order autoregressive (AR(2)) process

(2.3) $$d_t = \phi_{d1} d_{t-1} + \phi_{d2} d_{t-2} + \zeta_{d,t},$$

and $\zeta_{pq,t}$ and $\zeta_{d,t}$ are disturbances. Distributional assumptions on disturbances are stated in section 3.

To specify c_{qt}, we first assume that the firm uses capital (k), labor (ℓ), and materials (m), to produce saleable output (q), install investment goods (i), and conduct research activities (r) (subscript t is omitted sometimes). We assume that the "output activities," q, i, and r, are restricted according to the separable production function

(2.4) $$h(q,i,r) = \tau \cdot g(k,\ell,m),$$

where τ is the Hicks-neutral stock of technology. Although τ is also total-factor productivity, because $g(\cdot)$ and $h(\cdot)$ are indexes of inputs and outputs, we refer to τ as technology. If τ were capital augmenting or labor augmenting, the production function would be written as $h(q,i,r) = g(\tau k,\ell,m)$ or $h(q,i,r) = g(k,\tau\ell,m)$. More specifically, following Kydland and Prescott's (1982) treatment of the utility function, we assume $g(\cdot)$ and $h(\cdot)$ are the constant elasticity functions,

$$(2.5) \qquad g(k,\ell,m) = (\alpha_1 k^\beta + \alpha_2 \ell^\beta + \alpha_3 m^\beta)^{1/\beta},$$

$$h(q,i,r) = (\gamma_1 q^\rho + \gamma_2 i^\rho + \gamma_3 r^\rho)^{1/\rho},$$

where $\alpha_i > 0$, $\alpha_1 + \alpha_2 + \alpha_3 = 1$, $\beta < 1$, $\gamma_i > 0$, $\gamma_1 + \gamma_2 + \gamma_3 = 1$, and $\rho > 1$. CES = $(\beta-1)^{-1} < 0$ is the constant elasticity of substitution among inputs, and CET = $(\rho-1)^{-1} > 0$ is the constant elasticity of transformation among outputs. Including i and r in $h(\cdot)$ is a parsimonious way of specifying internal adjustment costs. The idea is that positive rates of investment and research use capital, labor, and materials resources, which could otherwise be used to produce more output, and that this trade-off sacrifices ever more output per unit increases in investment and research.

We need the adjustment costs to generate dynamic decision rules for the firm, which determine correlations among current and lagged variables, which are used to estimate unobserved variables in terms of observed variables. Adjustment costs are commonly specified as convex investment costs, which are incurred in addition to purchase costs of investment goods. Here "investment" means investment in production capital and research in technology. In the next step, we derive a quadratic approximation of the dual variable production cost function (DVPCF) from production function (2.4)-(2.5). The DVPCF includes convex, investment and research, adjustment costs. Thus, having already introduced investment and research purchase costs, $p_{it}i_t + p_{rt}r_t$, we obtain a conventionally structured specification of investment and research adjustment costs. Although the DVPCF is conventionally structured, it is unconventionally parameterized. We derive the DVPCF from (2.4)-(2.5) to ensure that structural parameters are identifiable. If we had specified a general DVPCF, subject only to symmetry, homogeneity, and curvature restrictions, it would have 28 free parameters, too many for the structural parameters to be identified, hence, estimated. The identification problem arises because 4 of 13 variables in the model are completely unobserved. The missing-data and identification problems are solved by specifying the DVPCF in terms of the 6 free parameters of (2.4)-(2.5). For recent reviews of the investment adjustment cost literature, see, for example, Caballero (1999) and Nadiri and Prucha (1999).

Mathematically, convex internal adjustment costs arise in (2.4)-(2.5) when, for given technology, τ, and inputs, (k,ℓ,m), the transformation surfaces of the outputs, (q,i,r), are concave to the origin. The adjustment costs are "convex" because the derived DVPCF is convex in (q,i,r). Hall's (1973) analysis shows that the division of the production function into two separate input and output parts, $g(\cdot)$ and $h(\cdot)$, is a necessary condition for the output transformation surfaces to be concave to the origin. Here, $\rho > 1$ is a necessary and sufficient condition for the transformation surfaces to be concave. The transformation surfaces become more curved, hence, adjustment costs increase, as ρ increases. Similarly, $\beta < 1$ is a necessary and sufficient condition for the input isoquants to be convex to the origin, and the isoquants become more curved, hence, input substitutability decreases, as β decreases algebraically.

Let $c_q = p_\ell \ell + p_m m$, where p_ℓ is the real hiring price of labor and p_m is the real purchase price of materials. Let $c_i = p_i i$ and $c_r = p_r r$, where p_i and p_r are the real purchase prices of investment and research goods and services. Because ℓ and m are variable (not subject to adjustment costs) and k and τ are quasi-fixed (subject to adjustment costs), we refer to c_q as the variable cost and to $c_i + c_r$ as the fixed cost. Let $c_q(w)$ denote the dual variable cost function: given $w = (w_1, \ldots, w_7)^T = (q, i, r, k, \tau, p_\ell, p_m)^T$ (superscript T denotes transposition), $c_q(w)$ = minimum of $p_\ell \ell + p_m m$, with respect to ℓ and m, subject to production function (2.4)-(2.5).

In the standard approach to multifactor productivity analysis (Bureau of Labor Statistics, 1997), all inputs are treated symmetrically, as variable flows. Accordingly, c_q would include all input costs as $c_q = p_k k + p_\tau \tau + p_\ell \ell + p_m m$, where p_k and p_τ are rental prices of capital and technology stocks, obtained using appropriate versions of Jorgenson's (1963) formula for converting investment purchase prices into capital rental prices. Although energy is often treated as a separate input, we merge it with materials, so that m_t denotes an aggregate of energy and materials inputs. Jorgenson's formula is based on more restrictive assumptions, notably that all inputs are variable. In this paper, we instead work with the purchase prices of investment and research because this allows greater flexibility for handling adjustment costs in the firm's dynamic optimization problem. It is the explicit solution of this problem that generates the identifying conditions that allow us to estimate the structural parameters of the model in the face of unobserved capital and technology.

The constant term in π does not affect optimal decisions in the approximate linear-quadratic dynamic optimization problem. Linear terms in π contribute only an additional constant term to the optimal decision rule, which is removed by mean adjustment of the data. Therefore, ignoring constant and linear terms, $c_q(w_t) \cong (1/2)w_t^T \cdot \nabla^2 c_q(w_0) \cdot w_t$, where $\nabla^2 c_q(w_0)$ denotes the Hessian matrix of second partial derivatives of c_q evaluated at $w = w_0$. $\nabla^2 c_q(w_0)$ is stated in the appendix, for $w_0 = (1, 1, 1, 1, 1, \alpha_2, \alpha_3)^T$, a value which results in the simplest expression for $\nabla^2 c_q(w_0)$. This choice of w_0 works acceptably in the application in section 4. Therefore,

$$(2.6) \qquad \pi_t = -(1/2)\eta q_t^2 + q_t(d_t + \zeta_{pq,t}) - (1/2)w_t^T \cdot \nabla^2 c_q(w_0) \cdot w_t - p_{it} i_t - p_{rt} r_t.$$

$\nabla^2 c_q(w_0)$ is symmetric and ideally $(1/2)w_t^T \cdot \nabla^2 c_q(w_0) \cdot w_t$ should inherit the following properties from the exact $c_q(w)$ function, for all values of w: (i) linear homogeneity in (q,i,r,k); (ii) convexity in (q,i,r,k); (iii) strict convexity in (q,i,r), (q,i,k), (q,r,k), and (i,r,k); (iv) linear homogeneity in (p_ℓ, p_m); and (v) strict concavity in p_ℓ and p_m. In fact, $w_t^T \cdot \nabla^2 c_q(w_0) \cdot w_t$ satisfies homogeneity restrictions (i) and (iv) for $w = w_0$ and curvature restrictions (ii), (iii), and (v) for all w.

The difference between $(1/2)w_t^T \cdot \nabla^2 c_q(w_0) \cdot w_t$ and the translog cost function (Christensen, Jorgenson, and Lau, 1971, 1973) is that $\nabla^2 c_q(w_0)$ is not stated in logs of variables and that its elements are tightly restricted in terms of the parameters of the model, whereas the translog cost function is stated in logs of variables and its elements are unrestricted except for the homogeneity, convexity, and concavity restrictions. The present model could be specified in logs of variables, but the results should be similar because the data are standardized prior to estimation. As noted above and discussed more below, estimating parameters without any capital and technology data and, then, estimating the unobserved capital and technology requires having sufficient identifying parameter restrictions on the cost function. Although we do not know and would have difficulty determining the full set of identified cost-function parameterizations, we do know that the general translog cost function is not in this set.

We assume that p_i, p_r, p_ℓ, and p_m are exogenous to the industry and are generated by the AR(2) processes

$$(2.7)\qquad p_{it} = \phi_{pi,1}p_{i,t-1} + \phi_{pi,2}p_{i,t-2} + \zeta_{pi,t},$$

$$p_{rt} = \phi_{pr,1}p_{r,t-1} + \phi_{pr,2}p_{r,t-2} + \zeta_{pr,t},$$

$$p_{\ell t} = \phi_{p\ell,1}p_{\ell,t-1} + \phi_{p\ell,2}p_{\ell,t-2} + \zeta_{p\ell,t},$$

$$p_{mt} = \phi_{pm,1}p_{m,t-1} + \phi_{pm,2}p_{m,t-2} + \zeta_{pm,t},$$

where $\zeta_{pi,t}$, $\zeta_{pr,t}$, $\zeta_{p\ell,t}$, and $\zeta_{pm,t}$ are disturbances. Processes (2.7) need not be stationary. A constant-coefficient autoregressive process is stationary or asymptotically stable if and only if its characteristic roots are less than one in absolute value. For example, the p_{it} process is stationary if and only if the roots, λ_1 and λ_2, which solve the characteristic equation, $\lambda^2 - \phi_{pi,1}\lambda - \phi_{pi,2} = 0$, are less than one in absolute value. The only restriction that we need on processes (2.7) in order to solve the firm's dynamic optimization problem is that $|\bar{\lambda}| < 1/\sqrt{\delta}$, where $|\bar{\lambda}|$ is the largest absolute characteristic root of any equation in processes (2.7).

We assume that capital accumulates according to the continuous-time law of motion

$$(2.8)\qquad \partial k(s)/\partial s = -f_k \cdot k(s) + i(s) + \tilde{\zeta}_k(s),$$

where $f_k > 0$ is a depreciation parameter and $\tilde{\zeta}_k(s)$ is a continuous-time disturbance. Integrating equation (2.8) over the sampling period $s \in [t-1,t)$, on the assumption that $i(s)$ is constant in $[t-1,t)$, we obtain the discrete-time capital law of motion,

$$(2.9)\qquad k_t = \phi_{k1}k_{t-1} + \phi_{i0}i_t + \zeta_{kt},$$

where $\phi_{k1} = \exp(-f_k)$, $\phi_{i0} = [(1-\exp(-f_k)]/f_k$, and $\zeta_{kt} = \int_{s=0}^{1} \exp[-f_k(1-s)]\tilde{\zeta}_k(t-1+s)ds$ is the implied discrete-time disturbance. It is customary to specify (2.9) directly, where $\phi_{i0} \equiv 1$. However, this specification understates the depreciation of investments undertaken early in a sampling period compared to those undertaken later in the period. The problem could be avoided by treating ϕ_{k1} and ϕ_{i0} as separate parameters, but this specification is less natural and introduces an additional parameter. Thus, assuming that $\zeta_{kt} \sim \text{NIID}(0, \sigma_k^2)$, we parameterize (2.9) in $\phi_{k1} \in (0,1)$ and $\sigma_k^2 > 0$, where $\phi_{i0} = (\phi_{k1}-1)/\ln(\phi_{k1})$. Similarly, we obtain the discrete-time technology law of motion

$$\tau_t = \phi_{\tau 1}\tau_{t-1} + \phi_{r0}r_t + \zeta_{\tau t}, \tag{2.10}$$

parameterized in $\phi_{\tau 1} \in (0,1)$ and $\sigma_\tau^2 > 0$, where $\phi_{r0} = (\phi_{\tau 1}-1)/\ln(\phi_{\tau 1})$ and $\zeta_{\tau t} \sim \text{NIID}(0, \sigma_\tau^2)$.

Equations (2.9)-(2.10) imply geometrical depreciation, in which most of capital and technology's depreciation occurs in early periods of their use. A rational-distributed-lag (RDL) specification (Jorgenson, 1966) could describe more general depreciation patterns, in particular, in which most depreciation occurs in late periods of use. A RDL could also include gestation lags as additional sources of capital and technology fixity. However, the need for parsimonious parameterization precludes RDL capital and technology equations, at least for the present data. Most RDLs could also be derived from underlying continuous-time specifications (Zadrozny, 1988).

The model's structural components have now been specified. It remains to explain how to solve the firm's dynamic optimization problem and how to assemble specified laws of motion and solved optimal decision rules into a system of linear simultaneous equations that are the equilibrium equations of the model.

To simplify the dynamic optimization problem, we eliminate q_t by maximizing π_t with respect to q_t. Because q_t is not a control variable in the laws of motion of k_t or τ_t, conditional on i_t and r_t being at their optimal values, the optimal value of q_t is given by maximizing π_t with respect to q_t. The first-order condition, $\partial\pi_t/\partial q_t = 0$, yields the output supply rule

$$q_t = -(c_{11} + \eta)^{-1}(c_{12}i_t + c_{13}r_t + c_{14}k_t + c_{15}\tau_t + c_{16}\,p_{\ell t} + c_{17}p_{mt} - d_t) + \zeta_{qt}, \tag{2.11}$$

where $(c_{11}, \ldots, c_{17})$ is the first row of $\nabla^2 c_q(w_0)$ and, for statistical reasons, ζ_{qt} is an added disturbance.

In addition to adding $\zeta_{pq,t}$ to output-demand curve (2.2) and ζ_{qt} to output supply rule (2.11), we also add disturbances to labor and materials decision rules (2.12)-(2.13) so that each of the 13 variables in the model has its own disturbance. Although the disturbances are added for statistical reasons, to ensure that the variables in the model have a nonsingular joint probability distribution, as usual they represent our specification errors or the firm's decision errors, or some combination of both.

Similar elimination of ℓ_t and m_t from the dynamic optimization problem is justified because ℓ_t and m_t are not control variables in the laws of motion of k_t or τ_t. Optimal values of ℓ_t and m_t, conditional on q_t, i_t and r_t being at their optimal values, are recovered using Shepard's lemma (a special case of the envelope theorem; Diewert 1971, p. 495),

$$(2.12)\qquad \ell_t = \partial c_{qt}/\partial p_{\ell t} = c_{61}q_t + c_{62}i_t + c_{63}r_t + c_{64}k_t + c_{65}\tau_t + c_{66}p_{\ell t} + c_{67}p_{mt} + \zeta_{\ell t},$$

$$(2.13)\qquad m_t = \partial c_{qt}/\partial p_{mt} = c_{71}q_t + c_{72}i_t + c_{73}r_t + c_{74}k_t + c_{75}\tau_t + c_{76}p_{\ell t} + c_{77}p_{mt} + \zeta_{mt},$$

where $(c_{61}, \ldots, c_{67})$ and $(c_{71}, \ldots, c_{77})$ are the 6th and 7th rows of $\nabla^2 c_q(w_0)$ and, for statistical reasons, $\zeta_{\ell t}$ and ζ_{mt} are added disturbances.

Optimality of labor and materials decision rules (2.12) and (2.13) also depends on $c_{qt} = (1/2)w_t^T \cdot \nabla^2 c_q(w_0) \cdot w_t$ being a good dual representation of production function (2.4)-(2.5). It is easy to derive decision rules for ℓ_t and m_t from the exact cost function implied by (2.4)-(2.5). However, such rules are nonlinear in variables, which complicates parameter estimation and filtering. Whether exact or approximate rules are used for decisions on ℓ and m, the approximate linear-quadratic dynamic optimization problem remains unchanged.

To solve the remainder of the firm's dynamic optimization problem, we restate it as a linear optimal regulator problem. We define the 2×1 control vector $u_t = (i_t, r_t)^T$ and the 14×1 state vector $x_t = (k_t, \tau_t, p_{it}, p_{rt}, p_{\ell t}, p_{mt}, d_t, k_{t-1}, \tau_{t-1}, p_{i,t-1}, p_{r,t-1}, p_{\ell,t-1}, p_{m,t-1}, d_{t-1})^T$. We assemble laws of motion (2.2)-(2.3) of output demand, (2.7) of input prices, (2.9) of capital, and (2.10) of technology, as the state equation

$$(2.14)\qquad x_t = Fx_{t-1} + Gu_t,$$

$$F = \begin{bmatrix} F_1 & F_2 \\ I_7 & 0_{7\times7} \end{bmatrix}, \quad G = \begin{bmatrix} G_0 \\ 0_{12\times2} \end{bmatrix},$$

where $F_1 = \text{diag}[\phi_{k1}, \phi_{\tau1}, \phi_{pi,1}, \phi_{pr,1}, \phi_{p\ell,1}, \phi_{pm,1}, \phi_{d1}]$, $F_2 = \text{diag}[0, 0, \phi_{pi,2}, \phi_{pr,2}, \phi_{p\ell,2}, \phi_{pm,2}, \phi_{d2}]$, $G_0 = \text{diag}[\phi_{i0}, \phi_{\tau0}]$, I_m is the $m\times m$ identity matrix, and $0_{m\times n}$ is the $m\times n$ zero matrix. We suppress disturbances in equation (2.14) because the regulator problem is certainty equivalent. We use output-supply rule (2.11) to eliminate q_t from π_t and write π_t as the quadratic form

$$(2.15) \qquad \pi_t = u_t^T R u_t + 2u_t^T S x_{t-1} + x_{t-1}^T Q x_{t-1}.$$

The matrices R, S, and Q are stated in the appendix in terms of η and the elements of $\nabla^2 c_q(w_0)$.

The regulator problem maximizes expected present value, (2.1), stated in terms of the quadratic form (2.15), with respect to the feedback matrix K in the linear decision rule $u_t = Kx_{t-1}$, subject to the state equation (2.14). Under concavity, stabilizability, and detectability conditions (Kwakernaak and Sivan, 1972), we compute the optimal K matrix by solving a discrete-time algebraic matrix Riccati equation using a Schur decomposition method (Laub, 1979). Finally, we write the investment-research decision rule as

$$(2.16) \qquad u_t = Kx_{t-1} + (\zeta_{it}, \zeta_{rt})^T,$$

where, for statistical reasons, $(\zeta_{it}, \zeta_{rt})^T$ is an added 2×1 disturbance vector.

3. Estimation of the Model and of Capital and Technology.

To estimate the model's parameters by maximum likelihood and, then, to estimate unobserved capital and technology, in both steps using the Kalman filter, we express the reduced form of the model in a state representation. To this end, we collect the variables of the model in the 13×1 vector $y_t = (p_{qt}, q_t, \ell_t, m_t, i_t, r_t, k_t, \tau_t, p_{it}, p_{rt}, p_{\ell t}, p_{mt}, d_t)^T$ and their disturbances in the 13×1 vector $\zeta_t = (\zeta_{pq,t}, \zeta_{qt}, \zeta_{\ell t}, \zeta_{mt}, \zeta_{it}, \zeta_{rt}, \zeta_{kt}, \zeta_{\tau t}, \zeta_{pi,t}, \zeta_{pr,t}, \zeta_{p\ell,t}, \zeta_{pm,t}, \zeta_{dt})^T$. We assume that the disturbances are mutually independent, normally distributed, stationary processes, such that the first 6 disturbances are AR(1) processes and the last 7 disturbances are serially independent. That is, we assume ζ_t =

$(I_{13} - \Theta L)^{-1}\varepsilon_t$, where $\varepsilon_t \sim \text{NIID}(0,\Sigma_\varepsilon)$, L is the lag operator, $\Theta = \text{diag}(\theta_{pq}, \theta_q, \theta_\ell, \theta_m, \theta_i, \theta_r, 0, 0, 0, 0, 0, 0, 0)$, where the θ's $\in$ (-1,1), and $\Sigma_\varepsilon = \text{diag}(\sigma^2_{pq}, \sigma^2_q, \sigma^2_\ell, \sigma^2_m, \sigma^2_i, \sigma^2_r, \sigma^2_k, \sigma^2_\tau, \sigma^2_{pi}, \sigma^2_{pr}, \sigma^2_{p\ell}, \sigma^2_{pm}, \sigma^2_d)$.

The equations which form the basis of the parameter and capital-technology estimation are (2.2), (2.3), (2.7), (2.9)-(2.13), and (2.16), or more concisely, (2.2), (2.11)-(2.14), and (2.16). These 13 scalar-level equations constitute the complete set of linear simultaneous equations which, for given values of parameters, past variables, and current and past disturbances, determine unique values of the 13 variables of the model. Concisely, the equations are

(3.1) $$A_0 y_t = A_1 y_{t-1} + A_2 y_{t-2} + (I_{13} - \Theta L)^{-1}\varepsilon_t,$$

where the elements of A_0, A_1, and A_2 are stated in the appendix in terms of η, ϕ's, elements of $\nabla^2 c_q(w_0)$, and elements of K. Rewriting (3.1), we obtain the reduced-form VAR(2) process

(3.2) $$y_t = B_1 y_{t-1} + B_2 y_{t-2} + \xi_t,$$

where A_0 is nonsingular for admissible values of parameters, $B_1 = A_0^{-1}(A_1 + \Theta A_0)$, $B_2 = A_0^{-1}(A_2 - \Theta A_1)$, $\xi_t = A_0^{-1}\varepsilon_t \sim \text{NIID}(0,\Sigma_\xi)$, and $\Sigma_\xi \sim A_0^{-1}\Sigma_\varepsilon A_0^{-T}$. Because the input-price equations map unchanged into equation (3.2), they are both structural and reduced-form equations.

A complete state representation comprises a state equation, which expresses the dynamics of the model, and an observation equation, which accounts for how variables in the model are observed. Corresponding to state equation (2.14), we write reduced-form equation (3.2) as state equation

(3.3) $$z_t = \overline{F} z_{t-1} + \overline{G}\xi_t,$$

$$\overline{F} = \begin{bmatrix} B_1 & B_2 \\ I_{13} & 0_{13\times 13} \end{bmatrix}, \quad \overline{G} = \begin{bmatrix} I_{13} \\ 0_{13\times 13} \end{bmatrix},$$

where $z_t = (y_t^T, y_{t-1}^T)^T$ is a 26×1 state vector and $\overline{F}$ is a 26×26 state-transition matrix. Associated with the state equation is the observation equation

(3.4) $$\bar{y}_t = \bar{H}_t z_t,$$

where $\bar{y}_t$ is the vector of variables observed in period t and $\bar{H}_t$ is a time-varying observation matrix.

Because $\bar{H}_t$ is completely flexible in assuming any values in any dimensions, including the null matrix if no observations are available, observation equation (3.4) can account for any pattern of missing data. For most sampling periods in the present application, $\bar{H}_t = [J, 0]$, where $J = I_{13}$ with rows of unobserved variables deleted and 0 is the identically dimensioned zero matrix. Thus, when variables 4, 7, 8, and 13 are unobserved, $J = I_{13}$ with rows 4, 7, 8, and 13 deleted and $0 = 0_{9\times 13}$. Also, $\bar{H}_t$ accounts for observations on different observed variables starting and ending in different periods. We call the Kalman filter applied to such a state representation the missing-data Kalman filter (MDKF).

The likelihood function is computed for maximum likelihood estimation (MLE) as follows. Let $\tilde{y}_t = \bar{y}_t - E[\bar{y}_t | \bar{Y}_{t-1}]$ denote the innovation vector, where $\bar{Y}_t = (\bar{y}_t^T, \ldots, \bar{y}_1^T)^T$ denotes the vector of observations through period t, and let $\Omega_t = E[\tilde{y}_t \cdot \tilde{y}_t^T]$ denote the innovation covariance matrix. In general, reduced-form disturbance vector, ξ_t, and innovation vector, $\tilde{y}_t$, coincide only when all variables are observed throughout the sample. Then, except for terms independent of parameters, -2 $\times$ log-likelihood function of the sample, $\bar{Y}_N$, is given by

(3.5) $$L(\vartheta, \bar{Y}_N) = \sum_{t=1}^{N} [\ln|\Omega_t| + \tilde{y}_t^T \Omega_t^{-1} \tilde{y}_t],$$

where ϑ is the 39×1 vector of structural parameters, which partitions as $\vartheta = (\vartheta_1^T, \vartheta_2^T)^T$, where $\vartheta_1 = (\delta, \alpha_1, \alpha_2, \gamma_1, \gamma_2, \sigma_{pq}^2, \sigma_{\ell}^2, \sigma_m^2)^T$ is 8×1 and $\vartheta_2 = (\phi_{pi,1}, \phi_{pr,1}, \phi_{p\ell,1}, \phi_{pm,1}, \phi_{pi,2}, \phi_{pr,2}, \phi_{p\ell,2}, \phi_{pm,2}, \sigma_{pi}^2, \sigma_{pr}^2, \sigma_{p\ell}^2, \sigma_{pm}^2, \theta_{pq}, \theta_q, \theta_{\ell}, \theta_m, \theta_i, \theta_r, \eta, \beta, \rho, \phi_{k1}, \phi_{\tau 1}, \phi_{d1}, \phi_{d2}, \sigma_q^2, \sigma_i^2, \sigma_r^2, \sigma_k^2, \sigma_{\tau}^2, \sigma_d^2)^T$ is 31×1. We used the MDKF to compute $L(\vartheta, \bar{Y}_N)$ accurately and quickly in the MLE. See Anderson and Moore (1979), Zadrozny (1988, 1990), and references therein for further details. For

ϑ_1 set according to identifying restrictions, $L(\vartheta, \overline{Y}_N)$ was minimized with respect to ϑ_2 using a trust-region method (Moreʹ et al., 1980).

Because the 39 parameters in ϑ are not identified without further a priori restrictions, we imposed the following 8 identifying restrictions on ϑ_1 to ensure that ϑ_2 is identified and estimatable. We set δ = .935, which corresponds to a real interest rate of δ^{-1} - 1 = .0695. In production function (2.4)-(2.5), we set $\alpha_1 = \alpha_2 = \alpha_3 = \gamma_1 = \gamma_2 = \gamma_3 = 1/3$. It seems we need to set one disturbance variance for each unobserved variable. Capital (k_t), technology (τ_t), and the output-demand state (d_t) are actually unobserved and, after some experimentation to obtain an acceptable estimated model, materials (m_t) was also treated as unobserved. However, it turned out that setting $\sigma^2_{pq} = \sigma^2_{\ell} = \sigma^2_m \cong 10^{-10}$ was sufficient to identify the unrestricted and estimated parameters. We set the 3 variances to small positive values, rather than exactly to zero, because doing so resulted in more accurate computations using the MDKF. We considered other identifying restrictions (e.g., of δ and the α's and γ's), but all of them resulted in approximately the same reduced-form parameter estimates, hence, in approximately the same estimates of capital and technology.

The mapping from structural to reduced-form parameters is too complicated to try to derive necessary or sufficient conditions on parameters under which parameters are identified. Here, the parameters in ϑ_1 were set directly, which rendered the parameters in ϑ_2 identified and estimatable. Analytical determination of the boundaries of identification is unnecessary, because after terminating at an estimate, the MLE program checks for identification numerically by checking whether the Hessian matrix of $L(\vartheta, \overline{Y}_N)$ is numerically positive definite. In practice, we can at best choose a set of "reasonable" identifying restrictions on ϑ_1, attempt to estimate ϑ_2 under these restrictions, and, if the MLE computations converge and the Hessian matrix is positive definite, then, we consider the chosen restrictions to be sufficient for identification.

Identifying restrictions could alternatively be considered calibrations based on other information, either particular economic theories or data sets or general notions or experience. Because the model makes predictions for all of its variables, the identifiable parameter space depends not just on the model's structure, but on the extent of unobserved variables, and expands as more unobserved variables become observed. In such case, previously unidentified parameters, which could only be set or calibrated, become identified and can be estimated. The practical challenge is imposing sufficient identifying restrictions to compensate for unobserved variables.

Because input prices are assumed to be generated exogenously by univariate AR(2) processes (2.7), the processes can be estimated (asymptotically) efficiently and individually using ordinary least squares (OLS), which is much simpler than estimating simultaneously all parameters in ϑ_2 using MLE. Thus, in the application, first, we estimated input-price parameters using OLS and, then, conditional on these estimates, estimated the remaining parameters in ϑ_2 using MLE. The resulting set and estimated parameters and maximized likelihood are denoted by $\hat{\vartheta}$ and $L(\hat{\vartheta}, \overline{Y}_N)$.

Two separate general identification conditions must be satisfied in order to estimate the structural parameters and, then, to estimate capital and technology for given estimated parameters. The first parameter identification condition is the standard condition that the Hessian matrix of 2nd-partial derivatives of $L(\vartheta, \overline{Y}_N)$ with respect to ϑ_2 is positive definite at set $\hat{\vartheta}_1$ and estimated $\hat{\vartheta}_2$. The condition for estimating capital and technology is that state representation (3.3)-(3.4) is reconstructible at $\hat{\vartheta}$. Briefly, let $R_t = [\overline{H}_1^T, \ \overline{F}^T \overline{H}_2^T, \ \ldots, \ (\overline{F}^{t-1})^T \overline{H}_t^T]^T$, where $\overline{F}$ and $\overline{H}_t$ denote the state-transition and observation matrices in state representation (3.3)-(3.4) at $\hat{\vartheta}$. Reconstructibility holds if R_t has full column rank for a sufficiently large t. See Kwakernaak and Sivan (1972), Anderson and Moore (1979), and references therein for further details. Both the parameter identification and reconstructibility conditions were verified numerically in the application.

4. Estimation Results.

4.1. Description of the Data.

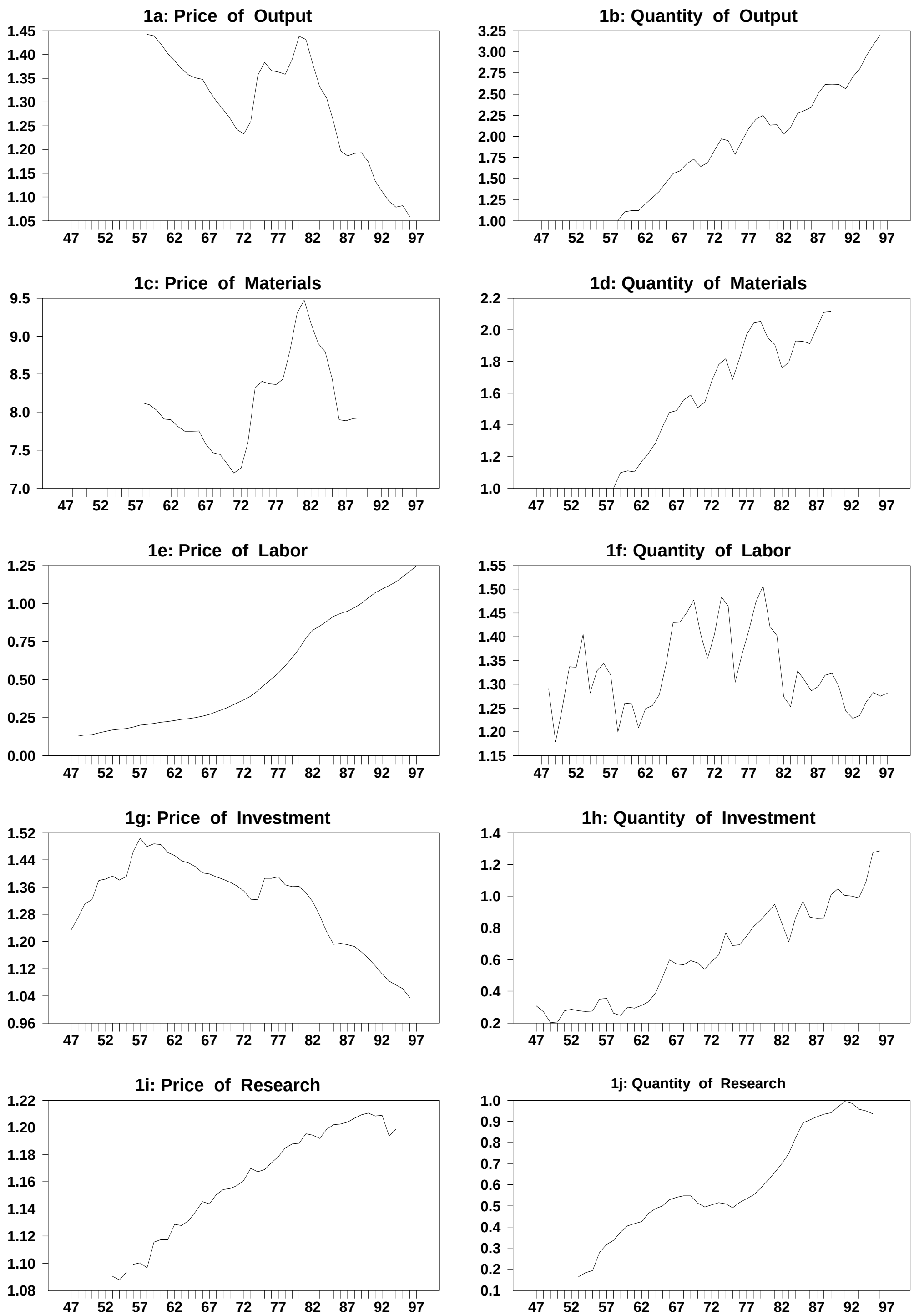
We used annual U.S. total manufacturing data on prices and quantities of output and inputs for 1947-1997. Investment and GDP-deflator data were obtained from the Bureau of Economic Analysis (BEA), research data from the National Science Foundation (1998), and all other data from the Bureau of Labor Statistics (BLS). All data were obtained in annual form, even though all except research price and quantity are also available monthly or quarterly, seasonally adjusted or not. All data were previously released to the public and are not confidential. Thus, we obtained data on 10 of the 13 variables in the model: p_{qt} and q_t for 1958-1996, $p_{\ell t}$ and ℓ_t for 1948-1997, p_{it} and i_t for 1947-1996, p_{rt} and r_t for 1953-1995, p_{mt} for 1958-1996, and m_t for 1958-1989.

Except for labor quantity measured by the number of production workers, all prices and quantities were computed as indexes based on given nominal price indexes, real quantity indexes, and nominal expenditures. Real quantities were computed as nominal expenditures divided by nominal price indexes and nominal prices were computed as nominal expenditures divided by real quantity indexes. Then, all given or computed nominal price indexes were converted to real form by dividing them by the GDP deflator.

Resulting real prices and real quantities of U.S. total manufacturing output and inputs are depicted in figure 1. For viewing convenience, the data were first standardized and were then shifted up and rescaled to lie between 0 and 4. The graphs suggest the following brief economic interpretation: increasing demand for output driven partly by a declining real price of output induced manufacturers to increase production capacity. Increasing quantities of investment and research built increasing stocks of capital and technology, hence, increased production capacity. As the price of labor increased, manufacturers used approximately the same labor input and more materials, capital, and technology inputs to produce more output, which resulted in increased productivity.

Initially, we considered total hours worked (total production workers multiplied by average hours worked per worker) as an alternate labor input measure. The graph of total hours worked (not shown) is very similar to that of total production workers in figure 1f. The main difference is that total hours worked is a somewhat noisier series. We chose total production workers as the labor input because it resulted in a slightly better fitting, but insignificantly different, estimated model. Choosing total production workers as the labor input caused the R^2s of output price and quantity, investment, and research to increase by .01 to .02 and that of labor to increase by .16 (throughout, an R^2 refers to the reduced-form equation of a variable). Choosing total production workers (a "stock" concept) instead of total hours worked (a "flow" concept) is theoretically more consistent with adjustment costs arising from production function (2.4)-(2.5).

Figure 1: U.S. Total Manufacturing Prices and Quantities of Output and Inputs, 1947-1997



Initially, we estimated the model using the data described above, but this resulted in a nearly zero R^2 for labor. The problem appeared to be misspecification of materials in the production function. The production function's form and the model's simulations indicate symmetrical roles for labor and materials, while the data in figures 1b and 1d show that the time path of materials matches closely that of output, not that of labor. The solution options were: (i) drop materials price and quantity from the analysis; (ii) assume materials quantity is in fixed proportions to the output good; or (iii) keep materials price and quantity in the model, as they are, continue to use materials price data in the parameter and capital and technology estimation, but treat materials quantity as unobserved. Options (i) and (ii) would be implemented implicitly by measuring the output good as value added instead of shipments and dropping materials as a production input. We chose option (iii), which required only that the materials quantity column in the data matrix be filled in with the missing-value indicator. Therefore, in the reported final estimates, materials quantity was treated as unobserved, along with unobserved capital, technology, and output-demand state.

4.2. Statistical Properties of the Estimated Model.

Table 1 reports OLS estimates of input-price-process parameters in ϑ_2. The table reports estimated coefficients, their absolute t ratios in parentheses, implied absolute characteristic roots, R^2, Ljung-Box Q statistics for testing absence of residual autocorrelations at lags 1-10, and their p values in parentheses. Estimated equations fit expectedly for level-form data, having $R^2 \geq .90$. Residuals show no significant autocorrelations, having p values of $Q > .25$. Except for the clearly stationary materials price equation and the possibly nonstationary research price equation, the investment price and labor price equations have borderline unit roots. All estimated characteristic roots satisfy the growth condition $|\bar{\lambda}| < 1/\sqrt{\delta}$, which is necessary for solving the firm's dynamic optimization problem. Although a cointegration analysis might seem appropriate, we did not attempt it because the input-price equations are needed only to provide input-price forecasts for the firm's dynamic optimization problem and they appear to do this adequately.

Table 1: Ordinary Least Squares Estimates of Input-Price Process Parameters in ϑ_2

Var.	Parameter Estimates			Fit Statistics	
	$\hat{\phi}_{\cdot,1}$	$\hat{\phi}_{\cdot,2}$	$\lvert\bar{\lambda}\rvert$	R^2	Q
p_i	1.45 (11.1)	-.441 (3.30)	1.02	.971	5.64 (.933)
p_r	.652 (4.01)	.282 (1.81)	.949	.979	4.67 (.968)
p_ℓ	1.88 (24.8)	-.883 (11.3)	1.01	.999	14.8 (.254)
p_m	1.49 (9.79)	-.617 (4.06)	.785	.903	9.13 (.692)

Columns 2-6 display estimates of $\phi_{\cdot,1}$ and $\phi_{\cdot,2}$, their absolute t statistics in parentheses, implied maximum absolute characteristic roots (solutions of $\lambda^2 - \hat{\phi}_{\cdot,1}\lambda - \hat{\phi}_{\cdot,2} = 0$), R^2, Ljung-Box Q statistics for testing absence of residual autocorrelations at lags 1 to 10, and their p values in parentheses.

Table 2 reports MLE of the remaining structural parameters in ϑ_2, conditional on OLS estimates of the input-price-process parameters, and fit statistics of the resulting estimated reduced-form equations of observed endogenous variables. The ML-estimated parameters in ϑ_2 are individually insignificant and their questionable numerical standard errors, based on the inverse information matrix, are not reported. Possibly more accurate standard errors could be computed using bootstrapping (Efron and Tibshirani, 1993). More importantly, the ML-estimated parameters in ϑ_2 are jointly identified in the sense that the Hessian matrix of $L(\vartheta, \overline{Y}_N)$ with respect to ϑ_2, evaluated at set and estimated parameters, $\hat{\vartheta}$, is numerically positive definite. But, most importantly, a likelihood-ratio test, discussed in detail below, does not reject overidentifying restrictions on the parameters. Reduced-form equations of observed endogenous variables fit expectedly for level-form data: moderate ($\cong$ .50) R^2 for labor and high (> .90) R^2 for other endogenous variables reflect labor's noisiness and the other variables' smoothness. Estimated 5 of 6 residual autocorrelation parameters in θ are near one, which raises the question of whether residual autocorrelations or the economic part of the model account for most of the sample variations of observed endogenous variables. However, reestimation with all θs set to zero produced $R^2_{pq} = .918$, $R^2_q = .879$, $R^2_\ell = .436$, $R^2_i = .772$, and $R^2_r = .944$ for the reduced-form equations, which means that the economic part of the model accounts for most of the sample variations of endogenous variables.

Table 2: Maximum Likelihood Estimates of Remaining Structural Parameters in ϑ_2

Production Function Parameters
$\hat{\beta} = -9.14$ (CES = -.099), $\hat{\rho} = 267.$ (CET = .004)
Output-Demand Curve Parameters
$\hat{\eta} = .605$, $\hat{\phi}_{d1} = 1.39$, $\hat{\phi}_{d2} = -.518$
Capital and Technology Equation Coefficients
$\hat{\phi}_{k1} = .589$, $\hat{\phi}_{i0} = .774$, $\hat{\phi}_{\tau 1} = .161$, $\hat{\phi}_{r0} = .459$
Residual Autocorrelation Coefficients
$\hat{\theta}_{pq} = .999$, $\hat{\theta}_q = .914$, $\hat{\theta}_\ell = .999$, $\hat{\theta}_m = .999$, $\hat{\theta}_i = .840$, $\hat{\theta}_r = .920$
Structural Disturbance Standard Deviations
$\hat{\sigma}_q = .417$, $\hat{\sigma}_i = .514$, $\hat{\sigma}_r = .362$, $\hat{\sigma}_k = .994$, $\hat{\sigma}_\tau = .055$, $\hat{\sigma}_d = .465$
Reduced-Form Equation Fit Statistics
$R^2_{pq} = .945$, $R^2_q = .948$, $R^2_\ell = .498$, $R^2_i = .926$, $R^2_r = .957$
$Q_{pq} = 10.8$, (.378); $Q_q = 5.96$, (.819); $Q_\ell = 5.97$, (.818); $Q_i = 18.6$, (.158); $Q_r = 21.4$ (.019)

Ljung-Box Q statistics are as in table 1.

The following likelihood ratio (LR) test of the overall validity of the overidentifying restrictions on the structural parameters is the key statistical test in the analysis. We obtained several different estimates of ϑ_2 based on several different restrictions on ϑ_1. In each case, ϑ_1 restrictions and ϑ_2 estimates implied approximately the same maximized likelihood values and reduced-form parameter estimates, hence, approximately the same Kalman-filter-based capital and technology estimates. Reduced-form parameters are identified by the model form and the data, but structural parameters are identified only with additional restrictions. The fact that structural parameters have particular set and estimated values is less important here, because the goal is to obtain Kalman-filter-based capital and technology estimates, which depend only on the data, the model form, and the reduced-form parameter values. The LR test is the key statistical test in the analysis because the Kalman-filter-based capital and technology estimates can be considered empirically valid if and only if the estimated reduced form is empirically valid and this occurs if and only if the overidentifying restrictions are not rejected for set and estimated structural parameter values.

$L(\hat{\vartheta}, \overline{Y}_N) = N{\cdot}\ln|\hat{\Omega}_N|$, where $\hat{\Omega}_N = (1/N)\sum_{t=1}^{N}\tilde{y}_t\tilde{y}_t^T$ and $\tilde{y}_t$ denotes innovations of y_t evaluated at $\hat{\vartheta}$, so that $LR = N(\ln|\hat{\Omega}_{N,R}|-\ln|\hat{\Omega}_{N,U}|)$, where $\hat{\Omega}_{N,R}$ and $\hat{\Omega}_{N,U}$ denote $\hat{\Omega}_N$ based on restricted and unrestricted innovations, i.e., from maximizing the likelihood function with the model's restrictions, respectively, imposed and relaxed. The MDKF automatically produces restricted innovations as part of MLE. We obtained unrestricted innovations as follows. We performed the test using the subsample 1960-1990 because only during this period were observations available for all 9 observed variables. For this period, the observation matrix, $\overline{H}_t$, is time invariant and given by $H = [J, 0_{9\times13}]$, where $J = I_{13}$ with rows 4, 7, 8, and 13 deleted. Then, combining state and observation equations (3.3)-(3.4), we obtained the infinite autoregressive representation of $\overline{y}_t$ and its finite p-lag approximation,

$$(4.1) \qquad \overline{y}_t = \Phi_1\overline{y}_{t-1} + \ldots + \Phi_p\overline{y}_{t-p} + \tilde{\tilde{y}}_t,$$

where the residual $\tilde{\tilde{y}}_t$ is an approximation of the innovation $\tilde{y}_t$. We want to test the <u>economic</u> restrictions of the model, excluding the zero restrictions implied by exogeneity and mutual independence of input-price processes (2.7). Therefore, except for these zero restrictions, we considered the Φ's to be free parameters. For p = 2, we estimated the endogenous-observed-variable equations of (4.1) by applying OLS to data for 1960-1990 and reestimated the exogenous-input-price equations using the shorter sample. The resulting residuals were insignificantly serially correlated and were used to compute $\hat{\Omega}_{N,U}$.

Under the null hypothesis that the overidentifying restrictions are valid, LR is distributed asymptotically $\chi^2(\kappa)$ as $N \to \infty$, where κ denotes the number of overidentifying restrictions. LR rejects the null hypothesis when it exceeds critical value c_α for significance level α. The period 1960-1990 implies the small values N = 31 and N/κ = .15, for κ = 118. For such situations, Sims (1980, p. 17, fn. 18) suggested replacing N with $N - \nu$ in LR, where, in this case, ν is the number of estimated parameters divided by the number of observed endogenous variables. Thus, $N - \nu$ = 31 - (143/9) = 15.1 and κ = 118, imply LR = 142, with a p value of 6.66% so that the overidentifying restrictions are not rejected at a conventional 5% significance level. Although unit roots, discussed at the end of this section, could modify the test results, it seems unlikely that accounting for their effects would change the nonrejection of the test result to strong rejection.

4.3. Economic Properties of the Estimated Model.

Because the estimates of capital and technology depend critically on the economic model, to be confident in the estimates we should be confident in the economic properties of the model. Therefore, we present and briefly discuss some structural variance decompositions (Sims, 1986) and impulse responses of the estimated model.

We begin by explaining how the variance decompositions are computed. Let $M = I_{13}$ with columns 1, 3, and 4 deleted. Then, combining state and observation equations (3.3)-(3.4), we obtain the structural infinite moving-average representation of $\overline{y}_t$ in terms of the structural disturbance vector, ε_t,

$$(4.2) \qquad \overline{y}_t = \Psi(L)\varepsilon_t = \left(\sum_{i=0}^{\infty} \Psi_i L^i\right)\varepsilon_t = \sum_{i=0}^{\infty} \Psi_i \, \varepsilon_{t-i},$$

where $\Psi_i = J\begin{bmatrix} B_1 & B_2 \\ I_{13} & 0_{13\times13} \end{bmatrix}^i \begin{bmatrix} I_{13} \\ 0_{13\times13} \end{bmatrix} M$ and J is defined as in (4.1). M has been introduced to delete the three structural disturbances, $\varepsilon_{pq,t}$, $\varepsilon_{\ell t}$, and ε_{mt}, whose variances are set to near zero. Let $E[\bar{y}_{t+k} | \bar{Y}_t]$ denote the k-step-ahead forecast of $\bar{y}_{t+k}$, let $\tilde{y}_{t,k} = \bar{y}_{t+k} - E[\bar{y}_{t+k} | \bar{Y}_t]$ denote the forecast error, and let $V_k = E\tilde{y}_{t,k}\tilde{y}_{t,k}^T$ denote the error covariance matrix, given by

$$(4.3) \qquad V_k = \sum_{i=0}^{k-1} \Psi_i \Sigma_\varepsilon \Psi_i^T .$$

We decompose the k-step-ahead forecast-error variances of the 8 endogenous variables and their sum in terms of the 10 estimated structural disturbance variances. That is, we decompose $v_{k,ii}$, for i = 1, ..., 8, and $\sum_{i=1}^{8} v_{k,ii}$, where $v_{k,ii}$ denotes the (i,i) diagonal element of V_k, in terms of σ_j^2, for j = 2, 5, 6, ..., 13. Let $s_{k,i,j}$ and $\bar{s}_{k,j}$ denote the fractions of $v_{k,ii}$ and $\sum_{i=1}^{8} v_{k,ii}$ due to σ_j^2; let $\Sigma_\varepsilon^{1/2}$ denote the square-root of Σ_ε, obtained by replacing the diagonal elements of Σ_ε with their positive square roots; let e_i denote the 13×1 vector with one in position i and zeroes elsewhere; and, let $\bar{e}$ denote the 13×1 vector with ones in the first 8 positions and zeros elsewhere. Then, for i = 1, ..., 8 and j = 2, 5, 6, ..., 13, the percentage variance decompositions of $v_{k,ii}$ and $\sum_{i=1}^{8} v_{k,ii}$ are given by

$$(4.4) \qquad s_{k,i,j} = e_i^T\Big(\sum_{i=0}^{k} \Psi_i \Sigma_\varepsilon^{1/2} e_j e_j^T \Sigma_\varepsilon^{1/2} \Psi_i^T\Big) e_i \Big/ e_i^T\Big(\sum_{i=0}^{k} \Psi_i \Sigma_\varepsilon \Psi_i^T\Big) e_i ,$$

$$(4.5) \qquad \bar{s}_{k,j} = \bar{e}^T\Big(\sum_{i=0}^{k} \Psi_i \Sigma_\varepsilon^{1/2} e_j e_j^T \Sigma_\varepsilon^{1/2} \Psi_i^T\Big) \bar{e} \Big/ \bar{e}^T\Big(\sum_{i=0}^{k} \Psi_i \Sigma_\varepsilon \Psi_i^T\Big) \bar{e} .$$

Table 3 shows the structural decompositions of k = 10 year ahead forecast-error variances. Rows 2-9 show decompositions of variances of endogenous variables; row 10 shows the decomposition of the sum of variances of endogenous variables. For example, elements 2, 3, 4, 7, and 11 in row 2 indicate that, according to the estimated model, 12.8, 5.2, 7.0, 5.5, and 66.8 percent of the variance of p_q is, respectively, accounted for by the variances of disturbances of output, investment, research, price of investment, and output demand or σ_q^2, σ_i^2, σ_r^2, σ_{pi}^2, and σ_d^2. Because the model is estimated using standardized data, the decompositions are unit free. Different restrictions of disturbance variances in ϑ result in different scales of responses.

Table 3: Structural Variance Decomposition of the Estimated Model

	σ_q^2	σ_i^2	σ_r^2	σ_k^2	σ_τ^2	σ_{pi}^2	σ_{pr}^2	$\sigma_{p\ell}^2$	σ_{pm}^2	σ_d^2
$s_{10,pq,j}$	12.8	5.2	7.0	.74	.14	5.5	.29	1.1	1.1	66.8
$s_{10,q,j}$	35.2	14.2	19.2	.20	.38	15.2	.81	3.0	3.0	8.8
$s_{10,\ell,j}$	7.9	4.0	.40	79.2	1.5	.29	.04	.40	5.7	.60
$s_{10,m,j}$	7.9	4.0	.40	79.4	1.5	.29	.04	.28	5.6	.60
$s_{10,i,j}$	.00	62.8	4.8	.51	.41	15.9	.78	3.2	3.6	8.0
$s_{10,r,j}$	.00	.01	58.7	.03	.50	20.0	1.1	4.2	4.5	10.9
$s_{10,k,j}$	.00	22.6	1.9	63.7	.09	5.8	.30	1.1	1.4	3.1
$s_{10,\tau,j}$	.00	.01	56.6	.03	4.3	19.1	1.1	4.0	4.4	10.5
$\bar{s}_{10,j}$	5.7	17.6	8.3	46.2	.52	7.4	.39	1.5	2.7	9.6

Rows 2-9 give percentage decompositions of 10-step-ahead forecast-error variances of the 8 endogenous variables in terms of the variances of the 10 estimated structural disturbances. Row 10 gives percentage decomposition of the sum of the variances of the 8 endogenous variables. Each row's numbers sum to 100.

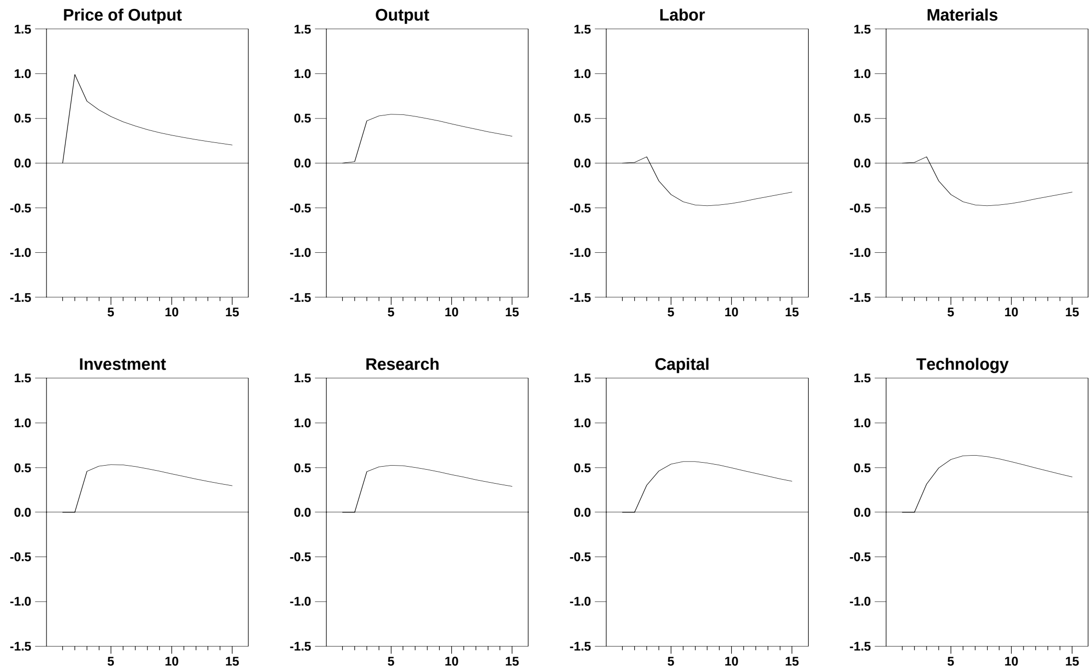
All disturbances except technology, price of research, and price of labor disturbances account for significant fractions (say, > 5%) of individual (rows 2-9) and overall (row 10) variations of endogenous variables. Interestingly, investment and capital disturbances account for slightly more of individual variations and signficiantly more of overall variations than do research and technology disturbances, which counters the real-business-cycle premise that technology disturbances are the primary source of variations of variables. Possibly, this could be because the research data here reflect a minor portion of the actual research by U.S. manufacturing firms and because the perpetual-inventory technology equation misspecifies the correct pattern of technology depreciation. Overall, in row 10 the decompositions indicate that investment, capital, price of investment, price of materials, and output demand disturbances are the leading sources of variations of the 8 endogenous variables.

The simulations in figure 2, respectively, display the dynamic adjustment-cost behavior in the model in response to unit impulses in output-demand and technology disturbances. The simulations in figure 2a match the general interpretation of figure 1. The simulations depict responses to a unit one-period shock (impulse) to the output-demand state in period 1, starting from an initial long-run equilibrium represented by the origin. The estimate $\hat{\eta}$ = .605 implies a moderately sloped output-demand curve. The estimates $\hat{\beta}$ = -9.14 and $\hat{\rho}$ = 267 imply CES = -.099 and CET = .004, hence, low input substitutability and very high adjustment costs on capital and technology. High adjustment costs imply a steep marginal-cost-of-production curve. Therefore, after the output-demand shock occurs, the price of output rises sharply but output increases only slightly. Initially, the extra output is produced using additional freely-adjusted labor and materials inputs and pre-shock stocks of capital and technology. Because the shocked demand state declines moderately slowly, firms have an incentive to increase their production capacities. Thus, they increase their investment and research rates and substitute capital and technology for labor and materials. Eventually, all variables return to the origin.

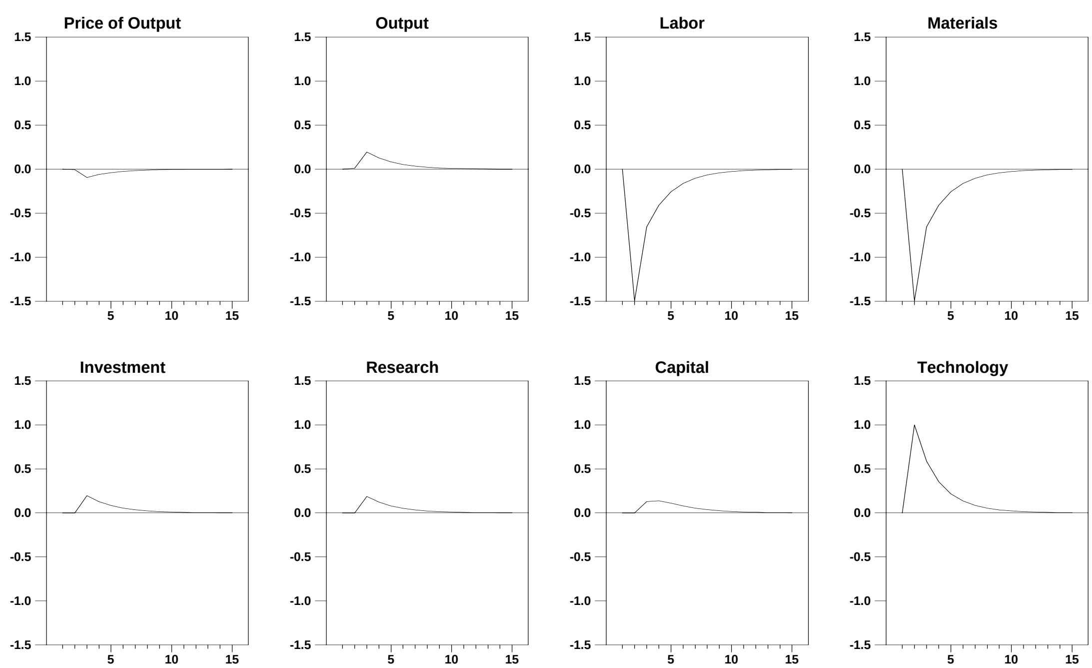
Figure 2b depicts responses to a unit one-period shock to technology in period 1, again starting from an initial long-run equilibrium at the origin. In figure 2b, output-demand conditions remain unchanged so there is little change in price or quantity of output. The shock mainly causes technology to be substituted for labor and materials until the windfall addition to technology has depreciated fully. Again, eventually all variables return to the origin.

Figure 2: Impulse Responses of the Estimated Model

a: To an Impulse in the Output-Demand Disturbance



b: To an Impulse in the Technology Disturbance



4.4. Model-Based and Standard Capital and Technology Estimates.

We applied the missing-data Kalman filter (MDKF) to the estimated model and data and, thereby, computed filtered state estimates, $\hat{z}_{t|t}$, and their error covariance matrices, $E(z_t-\hat{z}_{t|t})(z_t-\hat{z}_{t|t})^T$, for 1958-1997. Then, we picked elements 7 and 8 of $\hat{z}_{t|t}$ as the model-based estimates of capital and technology, $\hat{k}_{t|t}$ and $\hat{\tau}_{t|t}$, and square roots of diagonal elements 7 and 8 of the error covariance matrix as their estimated standard errors. Figures 3-4 depict model-based and standard estimates of (production) capital and technology for U.S. total manufacturing industries for 1958-1997. Solid lines depict model-based estimates and their 2-standard-error confidence bounds. For capital, dashed lines depict weighted sums of BLS stock estimates of equipment, structures, inventories, and land, based on nonstochastic perpetual inventory equations (PIE). In addition, BLS produces service-flow estimates and BEA produces stock estimates of equipment, structures, inventories and land, but weighted sums of these estimates are very similar to the BLS stock estimates and are, thus, not depicted or considered further. For technology, dashed lines depict BLS estimates of total factor productivity (TFP) based on Solow residuals. BLS capital stock and TFP estimates are graphed in figures 3-4 as examples of standard capital and technology estimates.

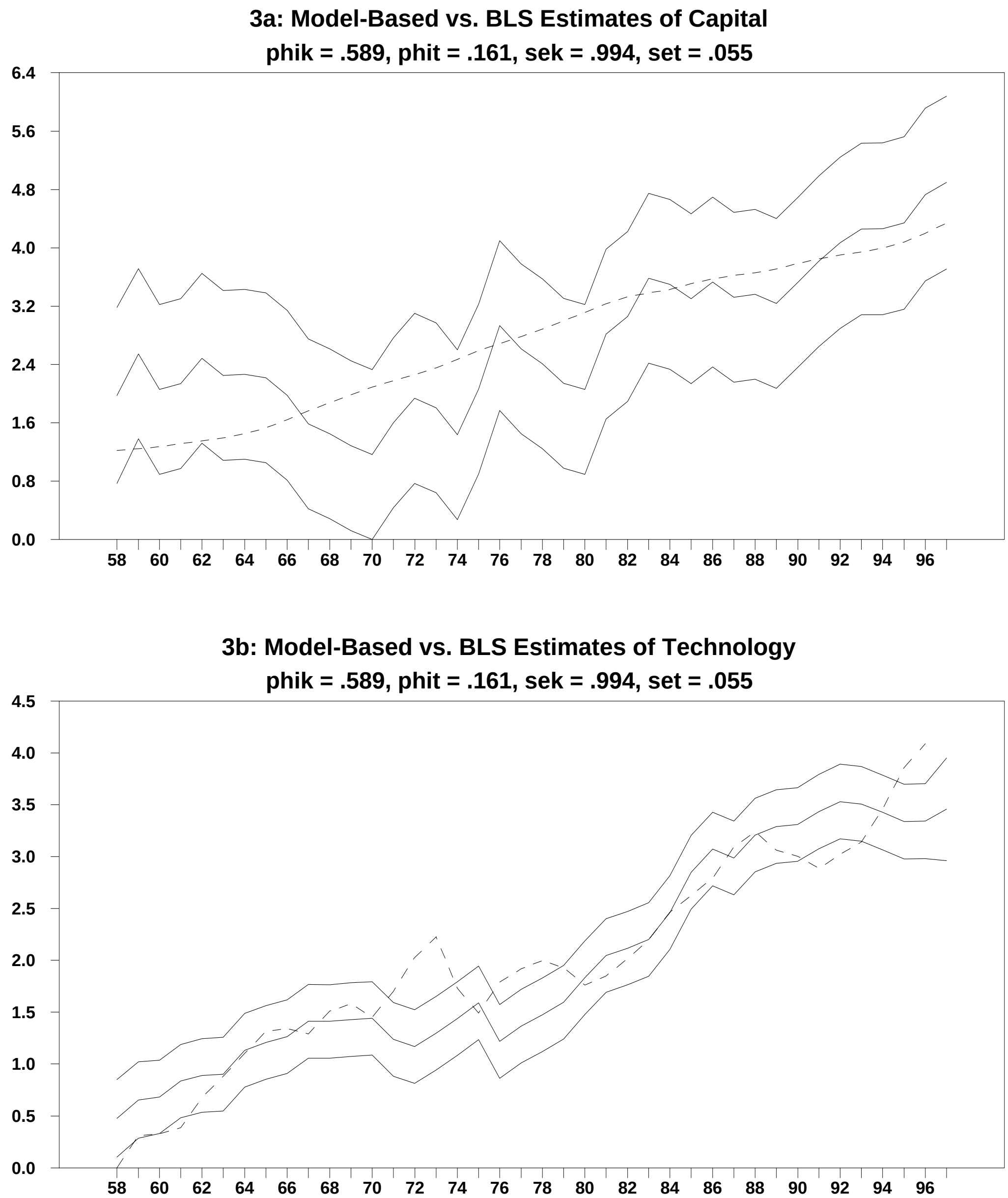
Because MLE is tractable generally only when data are scaled similarly, the data were standardized prior to estimation, by subtracting sample means and dividing by sample standard deviations. To make standard (BLS) capital (stock) and technology (TFP) estimates and their standard errors comparable with model-based ones, we standardized all estimates and their standard errors. Also, to make the estimates and standard-error confidence bounds look more sensible by being positive, before graphing them, we shifted them all up by the same amount. However, because the graphed values are arbitrary, vertical differences between them should not be interpreted as percentage differences. The graphs start in 1958 because output, a critical determinant of the estimates, is first available in 1958.

Figure 3 shows that 1958-1997 trends of model-based and standard capital and technology estimates are broadly similar, which makes them mutually reinforcing. However, being produced by government agencies and commonly used, standard estimates might be considered the "truer" ones. The intention here is not to challenge this view but to consider alternative capital and technology estimates based on an estimated dynamic structural economic model which has the following key features: the variables of primary interest, capital and technology, are endogenous in the model; production firms solve an explicitly considered dynamic optimization problem; resulting dynamics of endogenous variables arise naturally from elementary structural components, in particular, adjustment costs from the CES-CET production function; the model is identified and estimated using real (not simulated) data.

Suppose "short run" means cycles with periodicities less than about 5.6 years long (about the average business cycle length in the U.S. after World War II) and "long run" means longer cycles. Some short-run variations of capital and technology, either model-based or standard, are correlated with and, hence, may be considered explained by large known events such as the Vietnam War (1965-73) or oil-price shocks (1973, 1979). Remaining unexplained short-run variations may, then, be considered random noises. Figure 3 shows that the model-based capital estimates have more and larger noisy short-run variations than the model-based technology estimates. Consequently, the model-based capital estimates appear to be more uncertain than the model-based technology estimates, an interpretation supported by the standard errors produced by the MDKF. In figure 3, the two-standard-error confidence bounds of the model-based capital and technology estimates are, respectively, about 1.01 and .036, which says that the model-based capital estimates are about 28 times more uncertain than the model-based technology estimates. Being in unit-free standardized form, the model-based capital and technology estimates and their confidence bounds are comparable.

Do investment and capital disturbances or do research and technology disturbances better account for variations of endogenous variables in the 1990s, in particular, output (Gordon, 2000; Oliner and Sichel, 2000; Stiroh, 2001)? Nonrejection of the estimated model's overidentifying restrictions by the likelihood-ratio test suggests that the estimated model is an econometrically acceptable explanation of the data for 1947-1997. Variance decompositions of the estimated model in table 3 indicate that investment and capital disturbances account for slightly more of individual variations in endogenous variables (rows 2-9) and significantly more overall (row 10) than do research and technology disturbances, which counters the real-business-cycle premise that research and technology disturbances are the primary source of variations in variables.

Figure 3: Estimated Model-Based and Standard Capital (Stock) and Technology (TFP) Estimates for U.S. Total Manufacturing, 1958-1997



Model-based capital and technology estimates are based on $\hat{\vartheta}$, including $\hat{\sigma}_k$ = .994 and $\hat{\sigma}_\tau$ = .055. Solid lines depict model-based capital and technology estimates and 2-standard-error confidence bounds produced by the Kalman filter. Dashed lines depict standard capital (stock) and technology (total factor productivity) estimates produced by BLS.

Time-series properties of standard capital estimates depend entirely on time-series properties of investment and on capital depreciation in its perpetual inventory equation. For given time-series properties of investment, standard capital estimates should be smoother and more trendlike when capital depreciates more slowly. Time-series properties of model-based capital and technology estimates likewise depend on time-series properties of investment and research and on depreciation rates, but also on Kalman-filter estimates of disturbances in stochastic PIEs. For example, Kalman-filter estimates of capital, based on equation (2.9), are

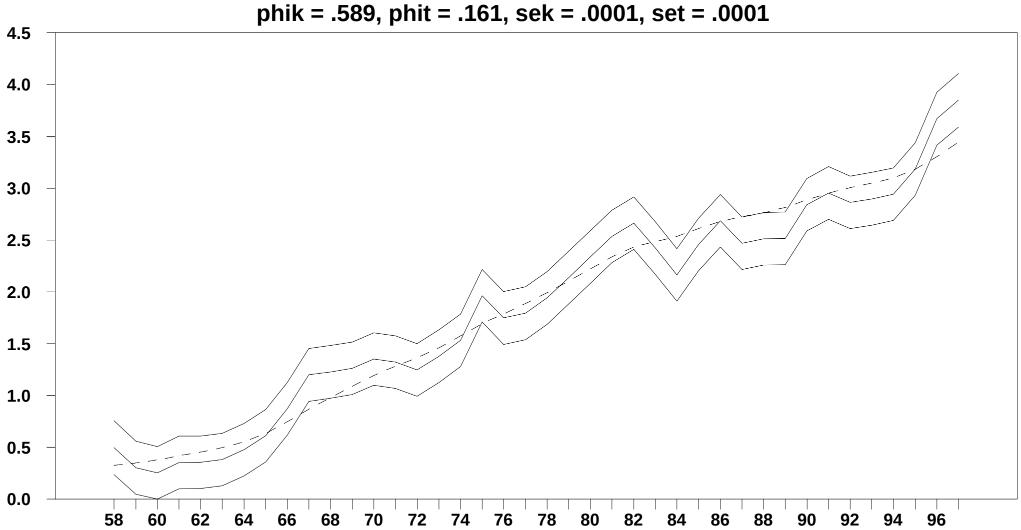
$$(4.6) \qquad \hat{k}_{t|t} = \phi_{k1}\hat{k}_{t-1|t} + \phi_{i0}i_t + \hat{\zeta}_{k,t|t},$$

where $\hat{k}_{s|t}$ denotes expected k_s conditional on $\overline{Y}_t$. Thus, time-series properties of model-based capital and technology estimates also depend partly on time-series properties of estimated disturbances, $\hat{\zeta}_{k,t|t}$.

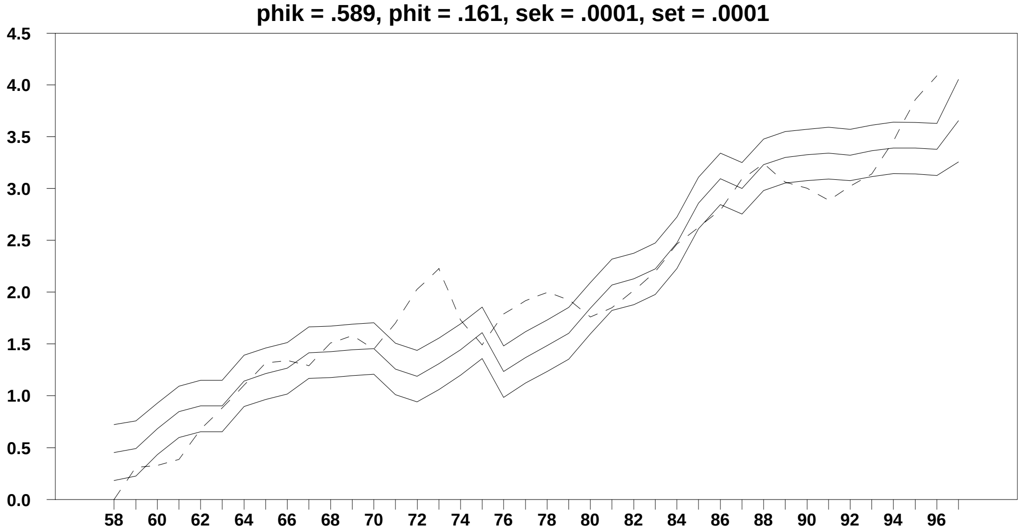
To consider how much noise the relatively large estimated standard deviation of the capital disturbance, $\hat{\sigma}_k = .99$, passes to the model-based capital estimates through equation (4.6), we recomputed the capital and technology estimates for virtually no capital and technology disturbances, for $\hat{\sigma}_k = \hat{\sigma}_\tau = .0001$ and the other parameters left at their set and estimated values. Thus, going from figure 3a to 4a, the sample average of estimated standard errors of model-based capital estimates declines 5-fold, from 1.03 to .205. Reducing $\hat{\sigma}_k$ and $\hat{\sigma}_\tau$ in the move from figure 3a to 4a does not reduce capital's standard error proportionately, because it also depends on unchanged standard deviations of other variables' disturbances. Going from figure 3a to 4a, short-run variations of capital estimates also decline 5-fold, causing the estimates to become more trend-like and to conform better to the BLS estimates. Going from figure 3b to 4b, causes the sample average of the estimated standard error of model-based technology estimates to decline only slightly, from .089 to .060, and, correspondingly, for the technology estimates to change little.

Figure 4: Alternative Model-Based and Standard Capital (Stock) and Technology (TFP) Estimates for U.S. Total Manufacturing, 1958-1997

4a: Model-Based vs. BLS Estimates of Capital
phik = .589, phit = .161, sek = .0001, set = .0001



4b: Model-Based vs. BLS Estimates of Technology
phik = .589, phit = .161, sek = .0001, set = .0001



Alternative model-based estimates are based on $\hat{\vartheta}$, except for $\hat{\sigma}_k = \hat{\sigma}_\tau = .0001$. Solid lines depict model-based capital and technology estimates and 2-standard-error confidence bounds produced by the Kalman filter. Dashed lines depict standard capital (stock) and technology (total factor productivity) estimates produced by BLS.

Estimated annual capital and technology depreciation rates of $1-\hat{\phi}_{k1} = .39$ and $1-\hat{\phi}_{\tau1} = .96$ are very high, in particular, compared to Jorgenson and Stephenson's (1967) implied annual capital depreciation rate of .11. To check whether systemwide MLE caused the high estimated depreciation rates, we reestimated capital and technology equations (2.9)-(2.10) in terms of their underlying continuous-time parameters using nonlinear least squares (NLS) and the model-based and BLS capital-stock and technology estimates as data. Although NLS estimates of $\hat{\phi}_{k1}$ and $\hat{\phi}_{\tau1}$ in table 4 differ somewhat from ML estimates in table 2, they are very similar for model-based and BLS data. As usual, estimated equation fit depends on dependent variable noisiness, so that the estimated capital equation fits better with BLS data ($R_k^2 = .891$) than with model-based data ($R_k^2 = .730$) and vice versa for the technology equation. Although MLE $\hat{\phi}_{k1} = .589$ and $\hat{\phi}_{\tau1} = .161$ might seem low, they work econometrically, because, along with the other set and estimated parameter values, they imply an unrejected estimated model and model-based capital and technology estimates whose trends conform to the standard estimates.

For given depreciation rates, investment, and disturbance variances, model-based capital and technology estimates are smoother to the extent that reduced-form characteristic roots are near one. A reduced-form characteristic root is an eigenvalue of transition matrix $\overline{F}$ of state equation (3.3). There are 26 such roots: 2 exogenous roots from demand-state process (2.3), 8 exogenous roots from input-price processes (2.7), 6 exogenous roots from residual autocorrelations, and 10 endogenous roots. The dynamic optimization problem suggests that one or more endogenous roots should be near one when β is negatively large, so that the input-substitution elasticity CES $\cong$ 0, and ρ is positively large, so that adjustment costs are high and the output-transformation elasticity CET $\cong$ 0. The estimates in tables 1-2 imply that 8 of 16 exogenous roots are within .02 of one and table 2 indicates that $\hat{\beta} = -9.14$, CES = -.099, $\hat{\rho} = 275$, and CET = .004, so that more than one endogenous root should be and is near one. To this extent, model-based capital and technology estimates should be smoother and more trendlike.

Table 4: Nonlinear Least Squares Estimates of Capital and Technology Equations

Capital Equation			
Capital Data	$\hat{\phi}_{k1}$	$\hat{\phi}_{i0}$	R^2_k
Model-Based Estimate	.336 (22.2)	.608 (9.14)	.730
BLS Estimate	.363 (78.1)	.629 (29.3)	.981
Technology Equation			
Technology Data	$\hat{\phi}_{\tau1}$	$\hat{\phi}_{r0}$	R^2_τ
Model-Based Estimate	.376 (118.)	.638 (42.2)	.992
BLS Estimate	.323 (50.8)	.599 (21.7)	.945

Columns 2-3 show estimated discrete-time parameters, ϕ, implied by estimated underlying continuous-time parameters, f. Absolute t statistics in parenthesis are based on linear approximations of the nonlinear mappings from f to ϕ.

5. Conclusion.

The paper has developed and applied an economic model-based method for estimating unobserved or latent stocks of production capital and technology or total factor productivity of U.S. total manufacturing industries for 1958-1997. The method involves estimating a dynamic structural economic model and computing estimates and standard errors of capital and technology by applying the Kalman filter to the estimated model and the data. Although, standard methods for estimating capital and technology are appealing in their theoretical and computational simplicity, they are unnecessarily restrictive in important respects, for example, ignore adjustment costs.

The estimated model accounts for the 1958-1997 data in the sense that overidentifying restrictions are not rejected and suggests that investment and capital disturbances better account for variations of endogenous variables than do research and technology disturbances because they account for significantly more of overall variations (table 3, row 10). Figure 3 also suggests visually that in the 1990s above average capital growth, not above average technology growth, better accounts, in particular, for above average output growth. Trends of model-based and standard estimates of capital and technology for 1958-1997 are broadly similar and, therefore, reinforce each other. Model-based capital estimates are much noisier than standard estimates and vice versa for technology. Model-based capital estimates are about 10 times more uncertain than model-based technology estimates in terms of estimated standard errors.

The paper has the specific objective of comparing standard capital and technology estimates with model-based capital and technology estimates computed by applying the Kalman filter to an estimated model. The key economic features of the particular estimated model used here are a demand-supply (partial) equilibrium in an output market in which a static output-demand curve is specified directly and a dynamic output-supply curve is derived from the solution of a representative production firm's dynamic optimization problem arising from adjustment costs. The paper suggests at least the following four extensions which could be considered in the future.

1. Capital and technology equations might be specified as rational distributed lags, with gestation lags and nongeometrical depreciation rates. For example, if in equation (1.1) $\phi_{k1} = ... = \phi_{kp} = 0$, $\phi_{i0} = ... = \phi_{ir} = 0$, and $\phi_{i,r+1} = ... = \phi_{iq} = 1$, then, a unit of investment gestates for r periods, becomes a unit of productive capital for the next q-r periods, and becomes fully depreciated thereafter.

2. Capital embodiment of technology has been considered frequently, recently by Greenwood, Hercowitz, and Krusell (1997). Here, if the two identification conditions discussed at the end of section 3 hold, as they do in the application, then, the Kalman filter automatically and implicitly disembodies technology from capital. However, the question of the degree to which capital embodies technology is unlikely to be resolved theoretically using other specifications, only empirically using more accurate research data. A cross-sectionally disaggregated analysis is unlikely to resolve this question either, because disaggregated research data are also often inaccurate and incomplete. Research inputs and outputs may simply be inherently difficult to measure because research is largely a mental process.

3. Capital and technology are usually only partly utilized and somewhat misallocated in any period. Both the standard and the present model-based capital and technology estimates treat capital and technology as fully utilized and optimally allocated. Including capital and technology utilization and misallocation in the analysis would require including capital and technology utilization rates and capital and technology market valuations as variables in the model and data on them in the empirical analysis.

4. The analysis could be extended in several ways to a general equilibrium. Input prices are now treated as exogenous and determined by autoregressions (2.7), with a purely statistical role and no particular economic meanings. Input prices could also be partly determined by endogenously determined research and technology. Accordingly, more of the above average output growth in the 1990s would be attributed to research and technology and less to investment and capital if research and technology growth reduced prices of investment and, thus, increased demand for investment and capital. Also, the static output-demand curve could be extended to a dynamic, economically motivated, output-demand curve analogous to the dynamic output-supply curve. Such extensions should inform about how much the present model-based capital and technology estimates depend on the particular model used here.

Appendix: Statement of Cost, Profit, and Reduced-Form Parameters.

Because $\nabla^2 c_q(w_0)$ is symmetric, it suffices to state its upper triangular part. Let c_{ij} denote element (i,j) of $\nabla^2 c_q(w_0)$. Then, for $w_0 = (1, 1, 1, 1, 1, \alpha_2, \alpha_4)^T$, we have:

$c_{11} = \gamma_1(1-\gamma_1)(\rho-1) + \gamma_1^2\alpha_1(1-\beta)/(1-\alpha_1)$

$c_{12} = -\gamma_1\gamma_2[\rho-1 + \alpha_1(1-\beta)/(1-\alpha_1)]$

$c_{13} = -\gamma_1\gamma_3[\rho-1 + \alpha_1(1-\beta)/(1-\alpha_1)]$

$c_{14} = -\gamma_1\alpha_1(1-\beta)/(1-\alpha_1)$

$c_{15} = -\gamma_1(1-\alpha_1\beta)/(1-\alpha_1)$

$c_{16} = \gamma_1/(1-\alpha_1)$

$c_{17} = \gamma_1/(1-\alpha_1)$

$c_{22} = \gamma_2(1-\gamma_2)(\rho-1) + \gamma_2^2\alpha_1(1-\beta)/(1-\alpha_1)$

$c_{23} = -\gamma_2\gamma_3[\rho-1 + \alpha_1(1-\beta)/(1-\alpha_1)]$

$c_{24} = -\gamma_2\alpha_1(1-\beta)/(1-\alpha_1)$

$c_{25} = -\gamma_2(1-\alpha_1\beta)/(1-\alpha_1)$

$c_{26} = \gamma_2/(1-\alpha_1)$

$c_{27} = \gamma_2/(1-\alpha_1)$

$c_{33} = \gamma_3(1-\gamma_3)(\rho-1) + \gamma_3^2\alpha_1(1-\beta)/(1-\alpha_1)$

$c_{34} = -\gamma_3\alpha_1(1-\beta)/(1-\alpha_1)$

$c_{35} = -\gamma_3(1-\alpha_1\beta)/(1-\alpha_1)$

$c_{36} = \gamma_3/(1-\alpha_1)$

$c_{37} = \gamma_3/(1-\alpha_1)$

$c_{44} = \alpha_1(1-\beta)[1 + \alpha_1(2-\alpha_1)/(1-\alpha_1)]$

$c_{45} = -\alpha_1 + \alpha_1(2-\alpha_1-\beta)/(1-\alpha_1)$

$c_{46} = -\alpha_1/(1-\alpha_1)$

$c_{47} = -\alpha_1/(1-\alpha_1)$

$c_{55} = (2-\alpha_1-\alpha_1\beta)/(1-\alpha_1)$

$c_{56} = -1/(1-\alpha_1)$

$c_{57} = -1/(1-\alpha_1)$

$c_{66} = -\alpha_3/[\alpha_2(1-\alpha_1)(1-\beta)]$

$c_{77} = -\alpha_2/[\alpha_3(1-\alpha_1)(1-\beta)]$.

Next, we state elements of 2×2, 2×14, and 14×14 coefficient matrices R, S, and Q, which define quadratic form (2.15). Because R and Q are symmetric, we state only their upper-triangular parts. R_{ij}, S_{ij}, and Q_{ij} denote (i,j) elements of the matrices. To eliminate the common factor 1/2, we scale π_t up by 2, which is allowed because optimal decisions are invariant to the scale of π_t. For simplicity, we state only nonzero elements of R, S, and Q, so that all unstated elements are zero. Thus, setting $c_0 = (\eta+c_{11})^{-1}$, we have

$R_{11} = c_0c_{12} - c_{22}$

$R_{12} = c_0c_{12}c_{13} - c_{23}$

$R_{22} = c_0c_{13}^2 - c_{33}$

$S_{11} = c_0c_{12}c_{14} - c_{24}$

$S_{12} = c_0c_{12}c_{15} - c_{25}$

$S_{13} = -1$

$S_{15} = c_0c_{12}c_{16} - c_{26}$

$S_{16} = c_0c_{12}c_{17} - c_{27}$

$S_{17} = -c_0c_{12}$

$S_{21} = c_0c_{13}c_{14} - c_{34}$

$S_{22} = c_0c_{13}c_{15} - c_{35}$

$S_{24} = -1$

$S_{25} = c_0c_{13}c_{16} - c_{36}$

$S_{26} = c_0c_{13}c_{17} - c_{37}$

$S_{27} = -c_0c_{13}$

$Q_{11} = c_0c_{14}^2 - c_{44}$

$Q_{12} = c_0c_{14}c_{15} - c_{45}$

$Q_{15} = c_0c_{14}c_{16} - c_{46}$

$Q_{16} = c_0c_{14}c_{17} - c_{47}$

$Q_{17} = -c_0c_{14}$

$Q_{22} = c_0c_{15}^2 - c_{55}$

$Q_{25} = c_0c_{15}c_{16} - c_{56}$

$Q_{26} = c_0c_{15}c_{17} - c_{57}$

$Q_{27} = -c_0c_{15}$.

Finally, we state structural coefficient matrices A_k, for k = 0, 1, 2. Let $A_{k,i,j}$ and $K_{i,j}$, respectively, denote elements (i,j) of A_k and K, the optimal investment-research feedback matrix. As before, only nonzero elements are stated. Also, because all diagonal elements of A_0 are one, they are not stated. Proceeding row-wise across the matrices,

$A_{0,1,2} = \eta$

$A_{0,1,13} = -1$

$A_{0,2,5} = c_0c_{12}$

$A_{0,2,6} = c_0c_{13}$

$A_{0,2,7} = c_0c_{14}$

$A_{0,2,8} = c_0c_{15}$

$A_{0,2,11} = c_0c_{16}$

$A_{0,2,12} = c_0c_{17}$

$A_{0,2,13} = -c_0$

$A_{0,3,2} = -c_{16}$

$A_{0,3,5} = -c_{26}$

$A_{0,3,6} = -c_{36}$

$A_{0,3,7} = -c_{46}$

$A_{0,3,8} = -c_{56}$

$A_{0,3,11} = -c_{66}$

$A_{0,3,12} = -c_{67}$

$A_{0,4,2} = -c_{17}$

$A_{0,4,5} = -c_{27}$

$A_{0,4,6} = -c_{37}$

$A_{0,4,7} = -c_{47}$

$A_{0,4,8} = -c_{57}$

$A_{0,4,11} = -c_{67}$

$A_{0,4,12} = -c_{77}$

$A_{0,7,5} = -\phi_{i0}$

$A_{0,8,6} = -\phi_{r0}$

$[A_{1,5,7}, \ldots, A_{1,5,13}] = [K_{1,1}, \ldots, K_{1,7}]$

$[A_{1,6,7}, \ldots, A_{1,6,13}] = [K_{2,1}, \ldots, K_{2,7}]$

$[A_{1,7,7}, \ldots, A_{1,13,13}] = [\phi_{k1}, \phi_{\tau 1}, \phi_{pi,1}, \phi_{pr,1}, \phi_{p\ell,1}, \phi_{pm,1}, \phi_{d1}]$

$[A_{2,5,7}, \ldots, A_{2,5,13}] = [K_{1,8}, \ldots, K_{1,14}]$

$[A_{2,6,7}, \ldots, A_{2,6,13}] = [K_{2,8}, \ldots, K_{2,14}]$

$[A_{2,7,7}, \ldots, A_{2,13,13}] = [0, 0, \phi_{pi,2}, \phi_{pr,2}, \phi_{p\ell,2}, \phi_{pm,2}, \phi_{d2}]$.

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