Modeling of BLS and Census Bureau Time Series Using Frequency Specific Generalized Airline Models With GenAirNBB October 2008

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Abstract
Using software being developed by the National Bank of Belgium to estimate and select among Frequency Specific Models, we investigate how often these models are selected over the Box-Jenkins Airline model, which they generalize, for a set of BLS series and a set of Census Bureau series. Consequences for seasonal adjustment are considered, as is the stability under future data additions of the model selection procedure used to select among these models.

Key Words: Seasonal ARIMA models, seasonal adjustment, trend estimation

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1. Frequency Specific Models

We present results from a study of the applicability of the Frequency Specific Models (FSMs) of Aston, Findley, McElroy, Wills and Martin (2007) to two sets of seasonal economic time series, one set from the Bureau of Labor Statistics (BLS), the other from the U.S. Census Bureau. These models are generalizations of the most widely used seasonal ARIMA model, the (0,1,1)(0,1,1) or Airline model of Box and Jenkins (1970). They make possible improved modeling of series whose seasonal movements are dominated by frequency components with frequencies in a proper subset of the seasonal frequencies 1, 2, 3, 4, 5 and 6 cycles per year (for monthly data). Our study was done with a prototype of a versatile menu-driven program named GenAirNBB that is being developed for Internet distribution by the National Bank of Belgium. Its model-based seasonal adjustments shown are not official seasonal adjustments of any of the authors' agencies.

For a monthly seasonal time series Z_t, the Box-Jenkins Airline model is

$$(1-B)(1-B^{12})Z_t = (1-\theta B)(1-\Theta B^{12})\epsilon_t, \tag{1}$$

where ϵ_t is a zero-mean i.i.d. process with finite variance. Here B is the backshift operator; $BZ_t = Z_{t-1}$, $B^{12}\epsilon_t = \epsilon_{t-12}$, etc. The coefficients are constrained to satisfy $|\theta|, |\Theta| \leq 1$, with no loss of generality for Gaussian ϵ_t.

If $\Theta \geq 0$, as is typical for modeled macroeconomic time series, including all series in our study, (1) can be written as

$$\begin{aligned}
&(1-B)^2\left[(1+B)\prod_{j=1}^{5}\left(1-2\cos(\frac{2\pi j}{12})B+B^2\right)\right]Z_t \\
= \quad &(1-\theta B)(1-\Theta^{1/12}B)\left[(1+\Theta^{1/12}B)\prod_{j=1}^{5}\left(1-2\Theta^{1/12}\cos(\frac{2\pi j}{12})B+\left(\Theta^{1/12}\right)^2B^2\right)\right]\epsilon_t.
\end{aligned} \tag{2}$$

Thus, the same coefficient $\Theta^{1/12}$ applies in each factor associated with the suite of seasonal frequencies $j = 1, 2, \ldots, 6$ cycles/year. (Note that $1+\Theta^{1/12}B = 1-\Theta^{1/12}\cos(\frac{2\pi 6}{12})B$.) It also occurs in one of the two nonseasonal MA factors on the r.h.s. that is paired with a trend differencing operator $1-B$ on the l.h.s.

An extreme alternative to (2) is the 8-coefficient *frequency specific model* in which every occurrence of $\Theta^{1/12}$ in (2) is replaced by a different coefficient,

$$(1-B)^2\left[(1+B)\prod_{j=1}^{5}\left(1-2\cos(\frac{2\pi j}{12})B+B^2\right)\right]Z_t$$

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$$= \quad (1-\theta B)(1-c_0 B)\left[(1+c_6 B)\prod_{j=1}^{5}\left(1-2c_j\cos(\frac{2\pi j}{12})B+c_j^2B^2\right)\right]\epsilon_t. \tag{3}$$

with $|c_j| \leq 1, 0 \leq j \leq 6$. However, with monthly series of typical lengths, (3) has the undesirable property of usually having *spurious* unit estimates $c_j = 1$ for one or more $0 \leq j \leq 6$, falsely indicating perfectly predictable behavior at some of these frequencies. (Theoretical and empirical explanations for this phenomenon are discussed in Aston et al., 2007.) To overcome this deficiency, constraints can be placed on the coefficient vector

$$\mathbf{c} = \left(\begin{array}{cccccccc} \theta & c_0; & c_1 & c_2 & c_3 & c_4 & c_5 & c_6 \end{array}\right) \tag{4}$$

of (3). Aston et al. (2007) considered two types of constraints on (4). In their *3-coefficent* type (our focus), $c_1, \ldots, c_6$ have only *two distinct values*, denoted c_1 and c_2, and $c_0 = c_1$:

There are 6 models in which c_2 applies to a single frequency. E.g., the {1} model with $\mathbf{c} = (\theta \quad c_1; \quad c_2 \quad c_1 \quad c_1 \quad c_1 \quad c_1 \quad c_1)$.

There are 15 models in which c_2 applies to two frequencies. E.g., the {1,4} model with $\mathbf{c} = (\theta \quad c_1; \quad c_2 \quad c_1 \quad c_1 \quad c_2 \quad c_1 \quad c_1)$.

Finally, there are 20 models in which c_2 applies to three frequencies. E.g., the {1,2,4 model with $\mathbf{c} = (\theta \quad c_1; \quad c_2 \quad c_2 \quad c_1 \quad c_2 \quad c_1 \quad c_1)$.

Thus there are 41 3-coefficient models, and our notation for each is the set of seasonal frequencies associated with c_2.

Aston et al. (2007) also considered 4-coefficient specializations of (4) in which $c_j, 1 \leq j \leq 6$ have only two distinct values, denoted c_1 and c_2, and c_0 is *unconstrained*. These 4-coefficient models do not occur in our study. (They were not preferred over the selected 3-coefficient models for any series.)

2. Model Selection Criteria Generalizing AIC

Akaike's AIC. For a model for a time series, let $\widehat{\vartheta}$, $\dim \vartheta$, and $L(\widehat{\vartheta})$ denote respectively the maximum likelihood parameter vector, its dimension, and the associated maximum log-likelihood value.

The estimated model's AIC is defined by

$$AIC\left(\widehat{\vartheta}\right) = -2L(\widehat{\vartheta}) + 2\dim\vartheta.$$

If $AIC(\widehat{\vartheta}^A)$ and $AIC(\widehat{\vartheta}^F)$ denote the AIC values of the Airline model and an FSM, Akaike's Minimum AIC criterion (MAIC), see Konishi and Kitagawa (2007), says that the FSM is to be preferred if

$$AIC(\widehat{\vartheta}^A) > AIC(\widehat{\vartheta}^F).$$

MAIC's asymptotic Type I error probability with a single FSM F is achieved for a family $\mathcal{F}$ of several FSMs with the same number of coefficients as F by preferring the minimum AIC model in $\mathcal{F}$ over the Airline model when

$$AIC(\widehat{\vartheta}^A) > \min_{F\in\mathcal{F}} AIC(\widehat{\vartheta}^F) + \Delta^{\mathcal{F}},$$

holds for a certain $\Delta^{\mathcal{F}} > 0$. We call this criterion GMAIC.

For the 3-coefficient families $\mathcal{F}$ considered here, the simulation-based Table 1 of Aston et al. (2007) gives $\Delta^{\mathcal{F}} = 2.8$ for the family of 6 models with c_2 assigned to a single frequency and $\Delta^{\mathcal{F}} = 4.6$ for the family of all 41 3- coefficient models.

In Aston et al. (2007), this model selection procedure was applied to all 72 Census Bureau Manufacturing, Import and Export series modeled with the Airline model for production seasonal adjustment in 2004. FSMs were preferable for 21 series (29%). 17 of the preferable FSMs were 3-coefficient models. 18 preferable FSMs were invertible (i.e. all $|c_j| < 1$). At present, there is no justification for the use of MAIC for series with truly noninvertible models. Various special arguments used to prefer FSMs estimated as noninvertible are given in Aston et al. (2007).

3. New Results for Census Bureau and BLS Series

We first present summary FSM results obtained with GenAirNBB for series with the final year of data omitted. The full data span for the Census Bureau series runs from January 1992 through December 2007. The full data span for the BLS series runs from January 1993 through December 2004. The regressors (trading day, holiday, outlier) used with the FSMs are those used with the airline model. The precise sets of series and their GMAIC results, obtained by omitting the last year of data, are

- Among all 10 Census Bureau *Monthly Retail Trade and Food Service* series currently modeled for direct seasonal adjustment with the Airline model: FSMs (all 3-coefficient and invertible) are selected for 3 series (30%).
- Among all 52 BLS series modeled with the Airline model in the study of Scott, Tiller and Chow (2007): FSMs (all 3-coefficient, all but one invertible) are selected for **10** series (19%). The estimate $c_2 = 1$ causing the one FSM to be noninvertible is treated as spurious because the same FSM is the GMAIC model for the full series, where $c_2 = .90$.

Remark 1. If we remove from consideration the 17 BLS series with a seasonal factor range (max - min) smaller than the smallest seasonal factor range, 3.54 percent, of a BLS series for which an FSM is accepted, then the success rate for FSMs for BLS series becomes 10/35 (29%). (The smallest seasonal factor range among the 10 Census Bureau series is 31.14 percent.)

Next, going beyond issues considered in Aston et al. (2007), we examine the effect on model selection of extending each of the 13 series to include the final year of data that had been withheld. For the Airline model, this means applying to each full series X-12-ARIMA's implementation of the automatic model selection procedure of Gómez and Maravall (2000) (in which AIC in not used for ARIMA model selection) to check if a different seasonal ARIMA model is selected instead of the airline model. For the FSMs, it means checking if the FSM chosen for the shorter series still has the smallest AIC among the 41 FSMs fit to the full series. For the FSMs, we considered a model change for the extended series to be *negligible* when the minimum AIC among FSMs differed from the AIC of the initially chosen FSM by less than 1.0, see Burnham and Anderson (2004).

Summary Results From Adding Data to Series With an FSM Preferred

- For the 3 Census Bureau series: 0 changes from the Airline model; 0 FSM changes
- For the 10 BLS series: 1 change from the Airline model, to an ARIMA(1,1,1)(1,1,1)–whose AIC is less than the AIC of the GMAIC FSM, the same FSM chosen for the shorter series;
  4 FSM changes—**1** nonnegligible, from {1,4} to {1,2,4}. Details are presented below.

Remark 2. In seasonal adjustment practice, the model used is usually not changed from one year to the next unless model quality diagnostics (e.g. Ljung-Box Q statistics or spectrum diagnostics) indicate that it is inadequate for the extended series.

3.1 Details for Two Series

We finish by presenting results for two illustrative examples. In the tables, for the Airline model, $\Theta = c_1^{12} = c_2^{12}$. A Ljung-Box Q statistic, testing zero autocorrelation of the model residuals at a suite of lags with maximum lag at most 24, is counted as statistically significant if its p-value is below .05. The p-values of Q at maximum lags 12 and 24 are denoted p_{12} and p_{24} and are shown when significant. For the canonical seasonal adjustments of Hillmer and Tiao (1982), with an Airline model, values $\Theta \leq 0.35$ suggest variable seasonal patterns and result in seasonal adjustments with quite substantial smoothing. By contrast, values $\Theta \geq .80$ suggest quite stable seasonal patterns and result in only quite localized seasonal movement suppression, see Findley and Martin (2006). Correspondingly, for an FSM, when c_2^{12} is much smaller than Θ and c_1^{12} is not much larger than Θ, the canonical seasonal adjustment and trend (obtained from GenAirNBB) are noticeably smoother than the Airline model's, as is seen for the example series in Figures 2,3, 5 and 6 below for the last four years of the full series. In the nonseasonal MA factors, the closer the coefficients are to one, the more linear are the canonical trends. Small coefficients are associated with highly variable trends, see Figures 6 and 7.

3.1.1 A Census Bureau Food Service Series

Table 1. Model Results for Sales by Restaurants and Bars

Series End	Selected Model	θ	c_1	c_1^{12}	c_2^{12}	No. Sig. Qs	$p_{12}, p_{24} < 05$	AIC
12/06	Airline	.32	.97	.67	.67	11	$p_{12} \simeq .02$	-1013.7
12/06	{1}	.55	.98	.76	.22	0	-	-1020.8
12/07	Airline	.35	.97	.68	.68	7	-	-1093.4
12/07	{1}	.56	.98	.75	.24	1	-	-1099.4

The FSM reduces the number of significant Qs and has $c_2^{12} << \Theta$. Figure 1 shows that each calendar month's seasonal factors from the {1} model for the full series move much less smoothly and with greater range than the Airline model's, resulting in greater smoothness in the seasonal adjustment and trend in Figures 2 and 3.

3.1.2 A BLS Current Employment Series

Table 2. Model Results for Performing Arts and Spectator Sports Payroll Employment

Series End	Selected Model	θ	c_1	c_1^{12}	c_2^{12}	No. sig. Qs	p_{12}, p_{24} < 05	AIC
12/03	Airline	-.07	.96	.62	.62	21	$p_{12}, p_{24} \simeq .01$	2446.7
12/03	$\{1,4\}$	.01	.98	.78	.32	4	-	2439.7
12/04	Airline	-.07	.95	.55	.55	19	$p_{12}, p_{24} \simeq .01$	2688.9
12/04	$\{1,2,4\}$	.07	.99	.84	.34	1	-	2679.7

The FSMs reduce the number of significant Qs and have $c_2^{12} << \Theta$. Figure 4 shows that each calendar month's seasonal factors from the {1,2,4} model for the full series move less smoothly and with greater range than the Airline model's, resulting usually in greater smoothness in the seasonal adjustment and trend in Figures 5 and 6.

For the full series, the {1,4} model still has 4 significant Qs (at lags 4–7) and its AIC of 2681.8 is larger by 2.1 than the AIC of the {1,2,4} model Table 2, where it is seen that the latter model has 1 significant Q. Fig. 7. shows that this {1,2,4} model's trend has mostly smaller month-to-month changes in the final years than the {1,4} model's trend.

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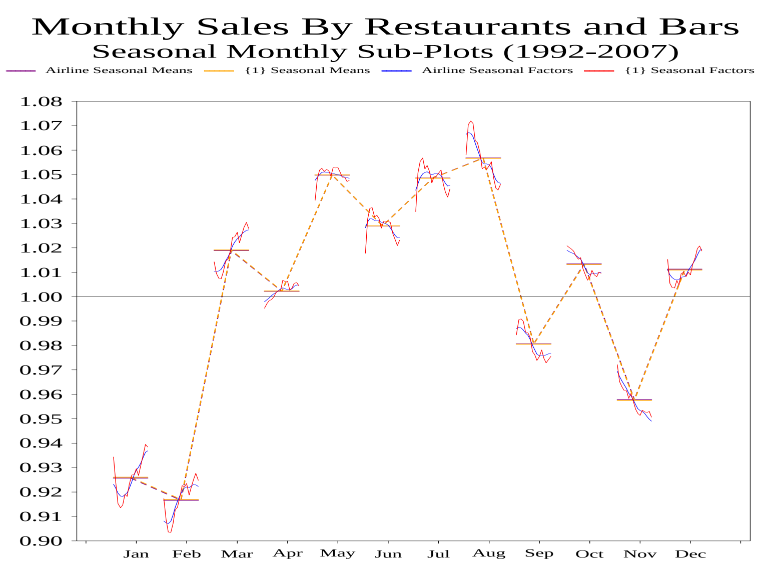

Figure 1: The {1} model's seasonal factors vary more and with greater ranges than the Airline model's over each calendar month, as might be expected from $c_2^{12} << \Theta$ and $c_1^{12} \approx \Theta$.

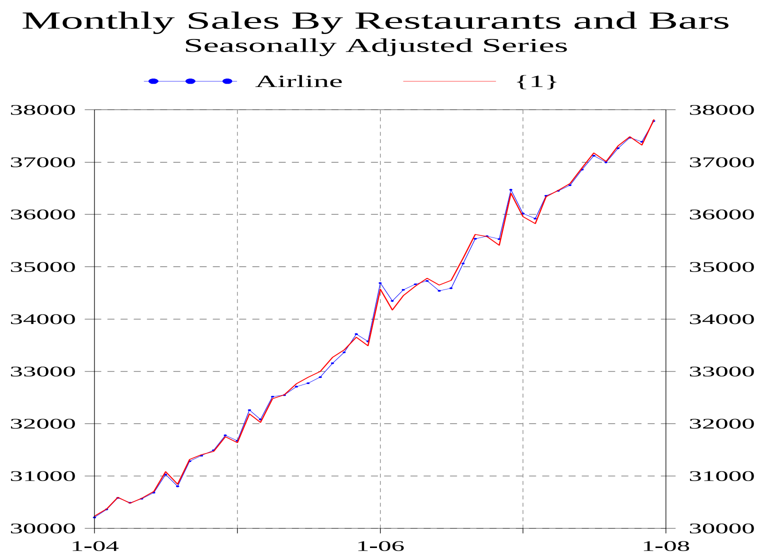

Figure 2: The {1} model's seasonal adjustment is mostly smoother than the Airline model's. (Shown for the last four years.)

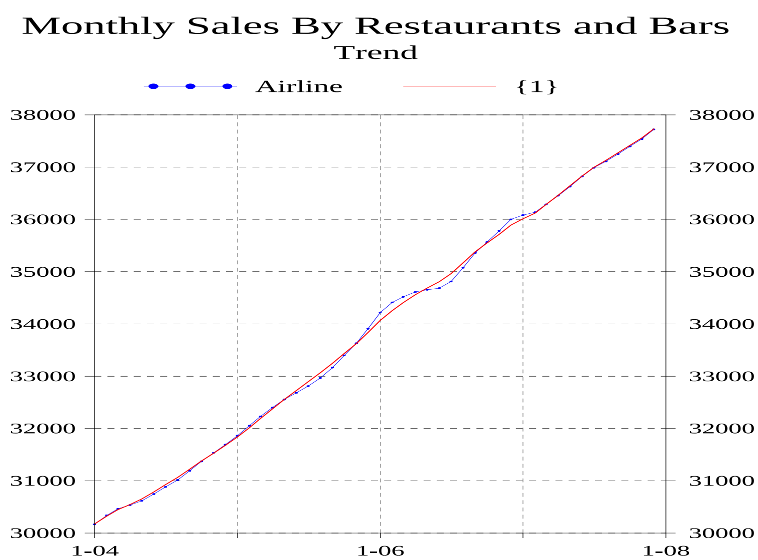

Figure 3: Relative to the {1} model's trend, the Airline model's trend has additional small oscillations of doubtful significance. (Shown for the last four years.)

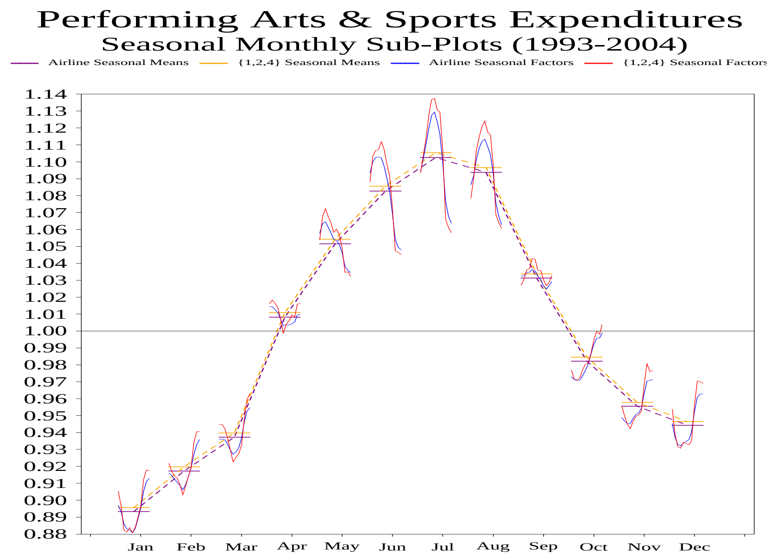

Figure 4: The {1,2,4} model's seasonal factors range more widely and slightly less smoothly than the Airline model's in each calendar month.

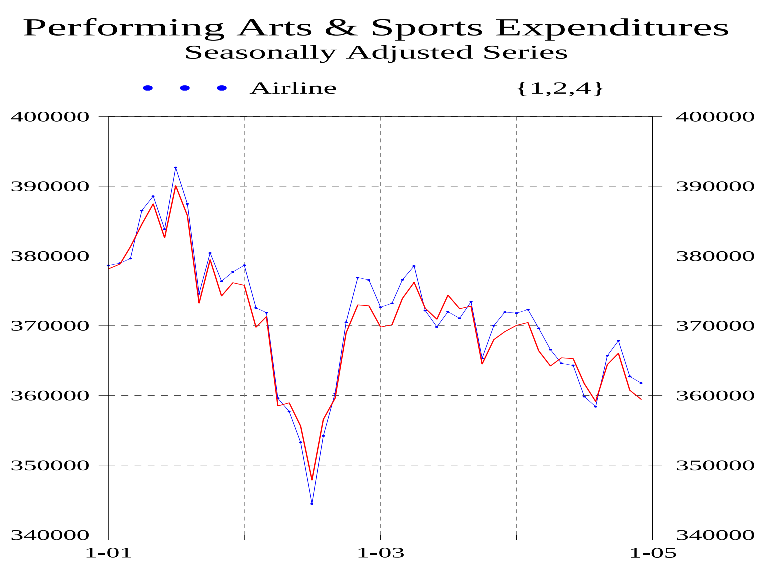

Figure 5: The {1,2,4} model's seasonal adjustment oscillates less widely than the Airline model's. (Shown for the last four years.)

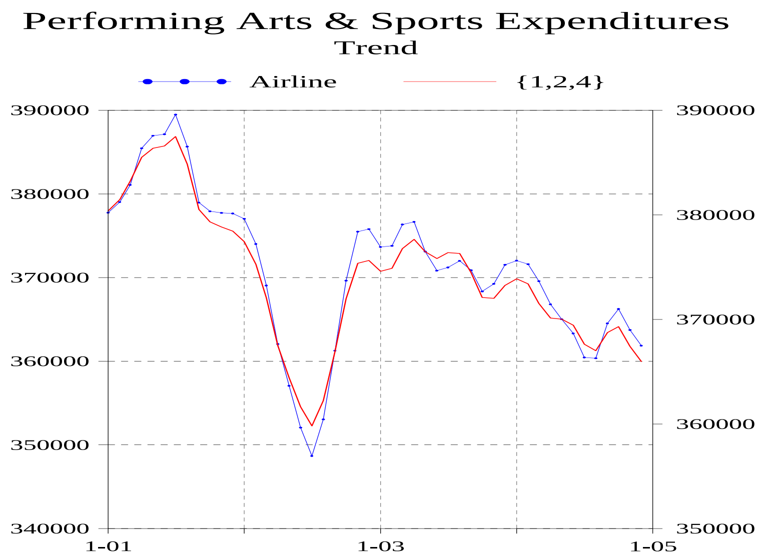

Figure 6: The {1,2,4} model's trend is smoother than the Airline model's, with month-to-month changes that are often noticeably smaller. (Shown for the last four years.)

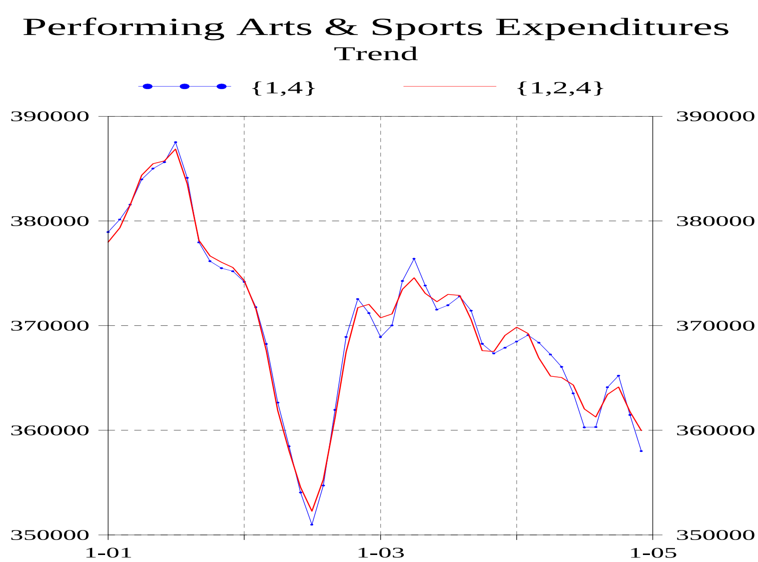

Figure 7: The {1,2,4} model's trend oscillates less widely than the {1,4} model's in the last years of the series, resulting in usually smaller month-to-month changes.