

Scan A7015

R G Wilson, J

Wden, feel free to
tear into individual
sheets

Mira >
lots to enter please!

Answers

→ A7015
A7365-73
A5277
A5114

13 July 1992

Vf94

Neil James Alexander Sloane
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AT&T Bell Telephone Laboratories Inc.
Murry Hill, New Jersey 07974
201-582-3000, ext. 2005

Subject: A Handbook of Integer Sequences

Dear Dr. Sloane,

%N Smallest k such that $\phi(n+k) \sim \phi(k)$

Please consider the following thirty-some odd sequences, most to the

first 101 terms for inclusion in your second edition of the above. "It is

easy to prove that for any given natural number k the equation $\phi(n+k) = \phi(n)$

has at least one solution in the natural numbers n ." page 231. There-

fore the following sequence: 1, 4, 3, 8, 5, 24, 5, 13, 9, 20, 7, 48, 13, 16,

13, 26, 17, 52, 19, 37, 21, 44, 13, 96, 25, 34, 27, 32, 13, 124, 17, 52, 33,
41, 19, 104, 35, 52, 37, 65, 25, 123, 17, 73, 39, 92, 41, 183, 35, 76, 39,
68, 53, 156, 35, 64, 57, 116, 41, 248, 61, 73, 61, 104, 65, 144, 67, 82, 41,
140, 37, 208, 73, 124, 65, 104, 37, 267, 65, 109, 81, 143, 83, 241, 85, 148,
87, 143, 37, 365, 41, 184, 61, 188, 55, 219, 97, 97, 91, 152, 101, ...

%R A51 840 %A rgw. %O 1, 2

"... have proved that for every natural number $k \leq 2 \cdot 10^{58}$ the equation

$\phi(n+k) = \phi(n)$ has at least two solutions in natural numbers n ." page 232.

Therefore the following sequence: 3, 7, 5, 14, 9, 34, 7, 16, 15, 26, 11, 68,

39, 28, 15, 32, 33, 72, 25, 40, 35, 56, 17, 101, 45, 37, 45, 56, 29, 152,
31, 61, 39, 56, 35, 144, 37, 61, 39, 74, 41, 128, 35, 88, 45, 161, 47, 192,
49, 82, 51, 74, 95, 216, 43, 97, 75, 203, 59, 304, 91, 88, 63, 122, 117,
194, 129, 112, 51, 146, 71, 288, 117, 148, 73, 119, 55, 292, 73, 130, 135,
146, 225, 246, 133, 172, 95, 146, 89, 372, 65, 259, 93, 194, 89, 339, 123,
112, 99, 164, 143, ...

The difference between the above two series is the following sequence:

2, 3, 2, 6, 4, 10, 2, 3, 6, 6, 4, 20, 26, 12, 2, 6, 16, 20, 6, 3, 14, 12, 4,
5, 20, 3, 18, 24, 16, 28, 14, 9, 6, 15, 16, 40, 2, 9, 2, 9, 16, 5, 18, 15,
6, 69, 6, 9, 14, 6, 12, 6, 42, 60, 8, 33, 18, 87, 18, 56, 30, 15, 2, 18, 52,
50, 62, 30, 10, 6, 34, 80, 44, 24, 8, 15, 18, 25, 8, 21, 54, 3, 142, 5, 48,
24, 8, 3, 52, 7, 24, 75, 32, 6, 34, 120, 26, 15, 8, 12, 42, ...

So:
For $k \geq 1$,
give smallest
soln

The
next
smallest
soln

The
jump

New
sequence
to be
entered

A7015

#1

As long as we are on this train of thought, then the next logical sequence is the third occurrence, and it is as follows: 15, 8, (3540000),

16, 15, 36, 21, 19, (100000), 35, 27, 72, 51, 34, 17, 38, 35, 73, 57, 52, (100000), 73, 23, 109, 75, 52, 55, 68, 51, 180, 39, 64, 45, 68, 75, 146, 49, 64, 45, 80, 111, 148, 43, 91, 51, 182, 65, 202, 147, 100, 57, 104, 123, 219, 55, 112, 91, 232, 177, 325, 93, 109, 105, 128, 183, 219, 201, 136, 57, 152, 111, 292, 175, 238, 75, 122, 77, 312, 79, 148, 145, 152, 243, 256, 153, 194, 99, 176, 119, 386, 91, 322, 135, 329, 95, 366, 273, 178, 117, 185, 237, ...

Continuing the next sequence is the fourth occurrence, and it is as follows: 104, 10, (3540000), 20, 21, 39, 45, 25, (100000), 100, 33, 78, 63,

41, 21, 50, 39, 82, 225, 55, (100000), 77, 69, 111, 99, 89, (100000), 82, 87, 194, 93, 76, 55, 74, 105, 164, 111, 73, 65, 95, 123, 153, 85, 112, 55, 184, 77, 218, 315, 130, 63, 178, 159, 246, 91, 133, 95, 266, 357, 360, 183, 124, 115, 152, 195, 244, 429, 148, 69, 155, 153, 303, 219, 259, 85, 128, 123, 327, 111, 157, 165, 164, 249, 296, 195, 247, 135, 182, 155, 456, 273, 353, 155, 365, 171, 369, 291, 181, 135, 200, 267, ...

And the final sequence is the fifth occurrence, and it is as follows:

164, 26, (3540000), 35, 15556, 43, 75, 28, (100000), 130, 45, 86, 75, 56, (100000), 56, 51, 102, 273, 70, (100000), 80, 99, 136, 105, 91, (100000), 112, 93, 208, 117, 100, (50000), 119, 111, 181, 399, 76, 105, 104, 153, 168, 129, 146, 63, 200, 135, 222, 525, 175, 85, 182, 429, 268, 99, 136, (50000), 290, 429, 369, 207, 190, (50000), 200, 255, 264, 441, 169, 115, 170, 213, 327, 651, 281, 105, 146, 125, 372, 231, 160, (50000), 224, 261, 306, 219, 286, 145, 185, 267, 482, 357, 364, 195, 376, 285, 384, 385, 193, 165, 260, 303, ...

In the same vein but a different function 'Sum of the Divisors' the

following sequence is the first occurrence for which $\sigma(n+k) = \sigma(n)$: 14, 33,

382, 51, 6, 20, 10, 15, 14, 21, 28, 35, 182, 24, 26, 30, 142, 40, 34, 42, 20, 57, 135, 70, 30, 99, 42, 66, 406, 88, 56, 60, 54, 93, 24, 105, 248, 147, 44, 63, 30, 80, 435, 114, 52, 196, 310, 140, 40, 105, 92, 160, 66, 120, 140, 105, 88, 352, 154, 224, 118, 177, 60, 117, 78, 220, 182, 135, 8786, 96, 112, 210, 752, 135, 92, 294, 110, 365, 735, 126, 126, 204, 60, 270, 102, 105, 254, 165, 78, 264, 88, 195, 174, 440, 114, 280, 138, 168, 124, 210, 316, ...

A7365
#2 ✓

Smallest k such that $\sum \sigma(u+k) \sim \sum \sigma(u)$ $\%R$ ASI 840.
%0 1,1

For the second occurrence the following sequence: 206, 54, (1935, 66,

46, 155, 62, 69, 16, 174, 154, 104, 782, 33, 62, 55, 238, 60, 158, 51, 30, 85, 231, 143, 46, 150, 48, 159, 496, 181, 58, 110, 35562, 96, 42, 130, 302, 246, 56, 84, 54, 135, 602, 123, 70, 205, 658, 165, 66, 132, 158, 198, 406, 180, 166, 132, 102, 852, 376, 315, 188, 224, 76, 120, 526, 232, 795, 186,

24885, 120, 945, 260, 862, 280, 130, 352, 190, 459, 1034, 147, 144, 748,
 184, 370, 166, 390, 358, 228, 114, 352, 130, 267, 11842, 736, 170, 330, 686,
 231, 154, 255, 658,

And the difference between the two produces the following sequence:

192, 21, 11553, 15, 40, 135, 52, 54, 2, 153, 126, 69, 600, 9, 36, 25, 96,
 20, 124, 9, 10, 28, 96, 73, 16, 51, 6, 93, 90, 73, 2, 50, 35508, 3, 18, 25,
 54, 99, 12, 21, 24, 55, 167, 9, 18, 9, 348, 25, 26, 27, 66, 38, 340, 60, 26,
 27, 14, 500, 222, 91, 70, 47, 16, 3, 448, 12, 613, 51, 16099, 24, 833, 50,
 110, 145, 38, 58, 80, 94, 299, 21, 18, 544, 124, 100, 64, 285, 104, 63, 36,
 88, 42, 72, 11668, 296, 56, 50, 548, 63, 30, 45, 342,

On page 234, "for every natural number s there exists a natural number m such that the equation $\phi(n) = m$ has precisely s solutions in natural numbers. We do not know the answer to this question even in the simple case of $s=1$ As was shown by V. L. Klee Jr. [3], there are no such numbers $m \leq 10^{400}$." Restating the original series and then continuing it, I present:

1, 2, 4, 8, 12, 32, 36, 40, 24, 48, 160, 396, 2268, 704, "

312, 72, 336, 216, 936, 144, 624, 1056, 1760, 360, 2560, 384, 288, 1320,
 3696, 240, 768, 9000, 432, 7128, 4200, 480, 576, 1296, 1200, 15936, 3312,
 3072, 3240, 864, 3120, 7344, 3888, 7220, 1680, 4992, 17640, 2016, 1152,
 6000, 12288, 4752, 2688, 3024, 13680, 9984, 1728, 1920, 2400, 7560, 2304,
 22848, 8400, 29160, 5376, 3360, 1440, 13248, 11040, 27720, 21840, 9072,
 38640, 9360, 81216, 4032, 5280, 4800, 4608, 16896, 3456, 3840, 10800, 9504,
 18000, 23520, 39936, 5040, 26208, 27360, 6480, 9216, 2880, 26496, 34272,
 23328, 28080,

% Smallest k such that $\phi(x)=k$ has n solutions.

However, just as there are solutions for $\phi(n) = m$, so are there values of m to which $s = 0$ or restated, there are no solutions in m . Beiler, page 91. They begin: 14, 26, 34, 38, 50, 62, 68, 74, 76, 86, 90, 94, 98, 114,

118, 122, 124, 134, 142, 146, 152, 154, 158, 170, 174, 182, 186, 188, 194,
 202, 206, 214, 218, 230, 234, 236, 242, 244, 246, 248, 254, 258, 266, 274,
 278, 284, 286, 290, 298, 302, 304, 308, 314, 318, 322, 326, 334, 338, 340,
 350, 354, 362, 364, 370, 374, 376, 386, 390, 394, 398, 402, 404, 406, 410,
 412, 414, 422, 426, 428, 434, 436, 446, 450, 454, 458, 470, 472, 474, 482,
 484, 488, 494, 496, 510, 514, 516, 518, 526, 530, 532, 534,

Mira, this gives lots more terms of
 A5277 - please enter them
 in cat 25

This ought to be the analog of A7368 - #6 on next page - check!

Mira, what is this? A book? #3? #4? A7374

Or if you like the above divided by two so as to save some room: 7, 13,

17, 19, 25, 31, 34, 37, 38, 43, 45, 47, 49, 57, 59, 61, 62, 67, 71, 73, 76,
 77, 79, 85, 87, 91, 93, 94, 97, 101, 103, 107, 109, 115, 117, 118, 121, 122,
 123, 124, 127, 129, 133, 137, 139, 142, 143, 145, 149, 151, 152, 154, 157,
 159, 161, 163, 167, 169, 170, 175, 177, 181, 182, 185, 187, 188, 193, 195,
 197, 199, 201, 202, 203, 205, 206, 207, 211, 213, 214, 217, 218, 223, 225,
 227, 229, 235, 236, 237, 241, 242, 244, 247, 248, 255, 257, 258, 259, 263,
 265, 266, 267, 269, ...

%N $\phi(x) \sim n$ has exactly 2 solutions.

%R ASI 840.

The equation $\phi(n) = m$ has just two solutions: 1, 10, 22, 28, 30, 46,

%A rgw.

52, 54, 58, 66, 70, 78, 82, 102, 106, 110, 126, 130, 136, 138, 148, 150,
 166, 172, 178, 190, 196, 198, 210, 222, 226, 228, 238, 250, 262, 268, 270,
 282, 292, 294, 306, 310, 316, 330, 342, 346, 358, 366, 372, 378, 382, 388,
 418, 430, 438, 442, 462, 466, 478, 490, 498, 502, 506, 508, 522, 546, 556,
 562, 568, 570, 580, 586, 598, 606, 618, 630, 642, 646, 652, 658, 676, 682,
 690, 708, 718, 726, 738, 742, 750, 772, 786, 796, 808, 810, 812, 822, 826,
 838, 852, 856, 858, ...

The equation $\phi(n) = m$ has just three solutions: 2, 44, 56, 92, 104,

116, 140, 164, 204, 212, 260, 296, 332, 344, 356, 380, 392, 444, 452, 476,
 524, 536, 564, 584, 588, 620, 632, 684, 692, 716, 744, 764, 776, 836, 860,
 884, 932, 956, 980, 1004, 1016, 1112, 1124, 1136, 1172, 1196, 1284, 1292,
 1304, 1316, 1352, 1364, 1416, 1436, 1484, 1544, 1592, 1616, 1644, 1652,
 1676, 1704, 1712, 1724, 1772, 1812, 1820, 1880, 1892, 1940, 1952, 1964,
 2036, 2060, 2124, 2172, 2180, 2192, 2204, 2216, 2288, 2300, 2324, 2360,
 2372, 2384, 2432, 2444, 2456, 2516, 2564, 2604, 2612, 2636, 2732, 2744,
 2844, 2852, 2876, 2892, 2900, ...

The equation $\phi(n) = m$ has just four solutions: 4, 6, 18, 42, 100, 162.

184, 208, 328, 424, 460, 468, 486, 492, 616, 636, 664, 688, 700, 712, 784,
 820, 900, 904, 1020, 1060, 1072, 1168, 1240, 1264, 1276, 1288, 1300, 1356,
 1360, 1384, 1404, 1458, 1480, 1528, 1672, 1740, 1768, 1864, 1896, 1900,
 1908, 2008, 2028, 2032, 2148, 2196, 2220, 2224, 2248, 2296, 2328, 2332,
 2344, 2380, 2500, 2508, 2568, 2584, 2620, 2628, 2704, 2860, 2868, 2872,
 3012, 3180, 3184, 3204, 3220, 3232, 3256, 3288, 3304, 3352, 3424, 3460,
 3544, 3580, 3624, 3820, 3904, 3912, 3916, 3948, 4068, 4120, 4180, 4308,
 4344, 4360, 4384, 4420, 4422, 4432, 4632, ...

The equation $\phi(n) = m$ has just five solutions: 8, 20, 220, 272, 300,

368, 416, 456, 500, 656, 732, 848, 876, 1092, 1160, 1212, 1236, 1328, 1376,
 1424, 1568, 1624, 1716, 1808, 2144, 2244, 2336, 2420, 2460, 2480, 2528,
 2556, 2768, 3056, 3080, 3252, 3320, 3344, 3536, 3560, 3612, 3728, 3732,
 3900, 4016, 4020, 4064, 4260, 4448, 4496, 4520, 4688, 4692, 5100, 5168,
 5232, 5340, 5360, 5408, 5512, 5744, 5840, 5984, 6036, 6132, 6156, 6200,
 6320, 6368, 6380, 6464, 6508, 6636, 6704, 6848, 7088, 7212, 7248, 7536,

7700, 7808, 7932, 8004, 8120, 8240, 8600, 8720, 8768, 8864, 9012, 9276,
 9320, 9488, 9536, 9560, 9728, 9800, 9824, 9940, ...

On page 235, "It is not known whether for every natural number k there exists a natural number m for which the equation $\sigma(x) = m$ has precisely k solutions in natural numbers x . This follows from the conjecture H (...). It can be proved that if m denotes the least of the numbers for which $\sigma(x) = m$ has precisely k solutions, then" : "1, 12, 24, 96, 72, 168, 240, 432, 360, 504, 576, 1512, 1080, 1008, 720, 2304, 3600, 5376, 2160, 1440," But this quoted series is incorrect. The problem with the above sequence is not that 432 does not have eight solutions for $\sigma(n) = 432$ (they being 230, 238, 255, 321, 355, 371, 391 & 431.), but that 432 is not the first number to possess this trait. The number 336 is, with the eight solutions being 132, 140, 182, 188, 195, 249, 287 & 299. Also 2520 is the number with 19 solutions and both 2160 and 1440 have one more solution that they are credited with, although they still retain the distinction of being the first. (2160 has as its 20 solutions: 870, 918, 920, 952, 1074, 1246, 1298, 1334, 1335, 1431, 1438, 1479, 1595, 1615, 1795, 1883, 1969, 2033, 2047 & 2059.) (1440 has as its 21 solutions: 552, 570, 594, 616, 790, 826, 874, 885, 957, 958, 969, 1015, 1045, 1077, 1195, 1253, 1343, 1349, 1357, 1363 & 1439.) Let me restate the series correctly from the beginning:

A7368
 #6

1, 12, 24, 96, 72, 168, 240, 336, 360, 504, 576, 1512, 1080, 1008, 720,
 2304, 3600, 5376, 2520, 2160, 1440, 10416, 13392, 3360, 4032, 3024, 7056,
 6720, 2880, 6480, 10800, 13104, 5040, 6048, 4320, 13440, 5760, 18720, 20736,
 19152, 22680, 43680, 28080, 26208, 14400, 16128, 25200, 11520, 8640, 78120,
 18144, 21600, 62208, 35280, 97200, 62496, 142848, 10080, 15120, 55440,
 44640, 66960, 38880, 24192, 42336, 98496, 52416, 17280, 97920, 64512, 46080,
 63360, 123120, 25920, 54720, 117936, 231840, 45360, 20160, 127680, 57600,
 43200, 75600, 200880, 48384, 228096, 158400, 147840, 131328, 215040, 334800,
 275184, 172368, 196992, 133920, 142560, 34560, 30240, 368640, 72576,
 (392212), ...

Smallest m such that $\# \sigma(x) \sim m$ has exactly
 $\# n$ solutions.
 % R AS1 840.

#sigma (x) $n = n$ # has no solution.

%R ASL 840.

A7369

However, just as there are solutions for $\sigma(n) = m$, so are there values of m to which $s = 0$ or restated, there are no solutions in m . They begin:

#7

2, 5, 9, 10, 11, 16, 17, 19, 21, 22, 23, 25, 26, 27, 29, 33, 34, 35, 37, 41, 43, 45, 46, 47, 49, 50, 51, 52, 53, 55, 58, 59, 61, 64, 65, 66, 67, 69, 70, 71, 73, 75, 76, 77, 79, 81, 82, 83, 85, 86, 87, 88, 89, 92, 94, 95, 97, 99, 100, 101, 103, 105, 106, 107, 109, 111, 113, 115, 116, 117, 118, 119, 122, 123, 125, 129, 130, 131, 134, 135, 136, 137, 139, 141, 142, 143, 145, 146, 147, 148, 149, 151, 153, 154, 155, 157, 159, 161, 163, 165, 166, ...

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#sigma (x) $n = n$ # has exactly 1 solution.

The equation $\sigma(n) = m$ has just one solutions: 1, 3, 4, 6, 7, 8, 13,

#8

14, 15, 20, 28, 30, 36, 38, 39, 40, 44, 57, 62, 63, 68, 74, 78, 91, 93, 102, 110, 112, 121, 127, 133, 138, 150, 158, 160, 162, 164, 171, 174, 176, 183, 194, 195, 198, 200, 204, 212, 217, 222, 230, 242, 255, 256, 258, 260, 266, 278, 282, 284, 296, 300, 304, 306, 307, 314, 318, 330, 332, 338, 348, 350, 352, 354, 363, 364, 368, 374, 380, 381, 396, 398, 400, 402, 410, 414, 422, 458, 462, 464, 465, 474, 476, 488, 494, 496, 500, 508, 510, 511, 512, 518, ...

A7371

The equation $\sigma(n) = m$ has just two solutions: 12, 18, 31, 32, 54, 56,

#9

80, 98, 104, 108, 114, 124, 126, 128, 132, 140, 152, 156, 182, 186, 210, 264, 272, 280, 308, 320, 342, 378, 390, 392, 399, 403, 408, 416, 440, 444, 448, 492, 522, 532, 570, 572, 594, 608, 630, 632, 726, 762, 770, 774, 780, 784, 800, 828, 868, 880, 884, 900, 920, 924, 942, 948, 954, 984, 1014, 1024, 1026, 1032, 1040, 1044, 1062, 1088, 1098, 1110, 1164, 1178, 1188, 1194, 1218, 1230, 1272, 1280, 1328, 1350, 1352, 1364, 1374, 1386, 1408, 1428, 1430, 1472, 1484, 1500, 1520, 1568, 1572, 1608, 1610, 1656, 1664, ...

The equation $\sigma(n) = m$ has just three solutions: 24, 42, 48, 60, 84,

A7372

#10

90, 224, 228, 234, 248, 270, 294, 324, 450, 468, 528, 558, 620, 640, 660, 810, 882, 888, 896, 968, 972, 1020, 1050, 1104, 1116, 1140, 1216, 1232, 1240, 1274, 1332, 1392, 1400, 1452, 1456, 1464, 1482, 1524, 1530, 1600, 1694, 1716, 1760, 1890, 1896, 1932, 1960, 1968, 2028, 2128, 2176, 2256, 2286, 2294, 2418, 2436, 2460, 2464, 2484, 2660, 2772, 2964, 3042, 3132, 3280, 3294, 3328, 3384, 3408, 3584, 3684, 3724, 3808, 3852, 3864, 3876, 3912, 3924, 3948, 3984, 3990, 4160, 4230, 4248, 4260, 4290, 4298, 4312, 4446, 4452, 4488, 4576, 4776, 4824, 4844, 4968, ...

The equation $\sigma(n) = m$ has just four solutions: 96, 120, 180, 312, 372,

#11

420, 434, 456, 540, 546, 560, 624, 702, 728, 798, 816, 930, 1064, 1120, 1170, 1404, 1632, 1638, 1674, 1710, 1776, 1792, 1944, 2100, 2240, 2544, 2560, 2664, 2760, 2800, 2844, 2856, 2940, 2952, 3000, 3040, 3048, 3060, 3080, 3096, 3108, 3224, 3432, 3492, 3510, 3564, 3768, 3822, 3920, 4004, 4140, 4356, 4424, 4572, 4644, 4650, 4656, 4712, 4836, 4914, 5004, 5088,

5120, 5130, 5320, 5496, 5568, 5640, 5652, 5670, 5724, 5832, 6200, 6288,
 6400, 6510, 6672, 6776, 6858, 6960, 7224, 7280, 7360, 7448, 7524, 7536,
 7650, 7688, 7704, 7872, 7944, 7968, 8060, 8244, 8256, 8460, ...

The equation $\sigma(n) = m$ has just five solutions: 72, 144, 192, 216, 588,

600, 648, 792, 936, 992, 1056, 1224, 1302, 1320, 1560, 1736, 1980, 2040,
 2088, 2112, 2268, 2448, 2730, 2790, 2912, 3038, 3136, 3312, 3472, 3520,
 3534, 3552, 3672, 3792, 3816, 3936, 4056, 4092, 4340, 4440, 4864, 4872,
 4920, 4960, 5082, 5334, 5600, 5796, 5904, 5940, 6096, 6156, 6768, 6936,
 7168, 7368, 7380, 7800, 7936, 8148, 8280, 8320, 8432, 8580, 8664, 8704,
 8856, 8904, 9180, 9312, 9432, 9552, 9648, 9660, 9768, 9900, 9920, 10032,
 10200, 10240, 10248, 10320, 10530, 10602, 10692, 10980, 10992, 11016, 11136,
 11256, 11400, 11440, 11700, 11844, 11928, 12012, 12152, 12192, 12264, 12400,
 12648, ...

The following is the first occurrence for n when $\phi(n) = k/2$ in which n
 is not a prime one less than n : 4, 8, 9, 15, 22, 21, 0, 32, 27, 25, 46, 35,

0, 58, 62, 51, 0, 57, 0, 55, 49, 69, 94, 65, 0, 106, 81, 87, 118, 77, 0, 85,
 134, 0, 142, 91, 0, 0, 158, 123, 166, 129, 0, 115, 0, 141, 0, 119, 0, 125,
 206, 159, 214, 133, 121, 145, 0, 177, 0, 143, 0, 0, 254, 255, 262, 161, 0,
 274, 278, 213, 0, 185, 0, 298, 302, 0, 0, 169, 0, 187, 243, 249, 334, 203,
 0, 346, 0, 267, 358, 209, 0, 235, 0, 0, 382, 221, 0, 394, 398, 275, 0, ...

The following is the last occurrence for n when $\phi(n) = k/2$: 2, 6, 12,

18, 30, 22, 42, 0, 60, 54, 66, 46, 90, 0, 58, 62, 120, 0, 126, 0, 150, 98,
 138, 94, 210, 0, 106, 162, 174, 118, 198, 0, 240, 134, 0, 142, 270, 0, 0,
 158, 330, 166, 294, 0, 276, 0, 282, 0, 420, 0, 250, 206, 318, 214, 378, 242,
 348, 0, 354, 0, 462, 0, 0, 254, 510, 262, 414, 0, 274, 278, 426, 0, 630, 0,
 298, 302, 0, 0, 474, 0, 660, 486, 498, 334, 588, 0, 346, 0, 690, 358, 594,
 0, 564, 0, 0, 382, 840, 0, 394, 398, 750, 0, ...

(In the preceding two sequences, the series of occurrences of Zeroes
 matches an earlier sequence presented at the bottom of page 3.)

The following is the first occurrence for n when $\sigma(n) = k$: 1, 0, 2, 3,

0, 5, 4, 7, 0, 0, 0, 6, 9, 13, 8, 0, 0, 10, 0, 19, 0, 0, 0, 14, 0, 0, 0,
 12, 0, 29, 16, 21, 0, 0, 0, 22, 0, 37, 18, 27, 0, 20, 0, 43, 0, 0, 0, 33, 0,
 0, 0, 0, 0, 34, 0, 28, 49, 0, 0, 24, 0, 61, 32, 0, 0, 0, 0, 67, 0, 0, 0, 30,
 0, 73, 0, 0, 0, 45, 0, 57, 0, 0, 0, 44, 0, 0, 0, 0, 0, 40, 36, 0, 50, 0, 0,
 42, 0, 52, 0, 0, 0, ...

The following is the first occurrence for n when $\sigma(n) = k$ less the
 Zeroes: 1, 2, 3, 5, 4, 7, 6, 9, 13, 8, 10, 19, 14, 12, 29, 16, 21, 22, 37,

18, 27, 20, 43, 33, 34, 28, 49, 24, 61, 32, 67, 30, 73, 45, 57, 44, 40, 36,
 50, 42, 52, 101, 63, 85, 109, 91, 74, 54, 81, 48, 68, 64, 93, 86, 121, 137,
 76, 66, 149, 111, 99, 157, 133, 106, 163, 60, 98, 173, 129, 88, 117, 169,
 80, 105, 193, 72, 197, 199, 134, 104, 211, 102, 100, 146, 84, 147, 229, 90,
 114, 241, 112, 96, 128, 217, 257, 171, 215, 148, 136, 201, 277, ...

To expand on your Sequence Nbr. 1215, $\phi(n) = \phi(n+1)$: "1, 3, 15, 104,
 164, 194, 255, 495, 584, 975, 2204, 2625, 2834, 3255, 3705, 5186, 5187,"

10604, 11714, 13365, 18315, 22935, 25545, 32864, 38804, 39524, 46215, 48704,
 49215, 49335, 56864, 57584, 57645, 64004, 65535, 73124, 105524, 107864,
 123824, 131144, 164175, 184635, 198315, 214334, 215775, 256274, 286995,
 307395, 319275, 347324, 388245, 397485, 407924, 415275, 454124, 491535,
 524432, 525986, 546272, 568815, 589407, 679496, 686985, 840255, 914175,
 936494, 952575, 983775, 1025504, 1091684, 1231424, 1259642, 1276904,
 1390724, 1405845, 1574727, 1659585, 1759874, 1788254, 1925564, 2123583,
 2200694, 2388044, 2521694, 2539004, 2619705, 2648204, 2759925, 2792144,
 2822715, 2847584, 3104744, 3137355, 3170936, 3240614, 3289934, 3653564,
 3693525, 3794834, 3877184, 3988424, 4002405, 4034744, ...

To expand on your Sequence Number 1328, $\phi(n) = \phi(n+2)$: "1, 4, 7, 8,
 10, 26, 32, 70, 74, 122, 146, 308, 314, 386, 512, 554, 572, 626, 635, 728,
 794, 842, 910, 914, 1015, 1082,"

1228, 1322, 1330, 1346, 1466, 1514, 1608, 1754, 1994, 2132, 2170, 2186,
 2306, 2402, 2426, 2474, 2590, 2642, 2695, 2762, 2906, 3242, 3314, 3506,
 3746, 3866, 3986, 4034, 4274, 4292, 4338, 4682, 4946, 5114, 5186, 5594,
 5714, 5834, 5950, 6122, 6434, 6497, 6506, 6626, 6764, 7034, 7466, 8042,
 8114, 8354, 8522, 8546, 8714, 8882, 9100, 9122, 9242, 9758, 9866, 10154,
 10202, 10226, 10307, 10466, 10826, 10874, 11162, 11402, 12074, 12146, 12212,
 12242, 12266, 12317, 12434, ...

As long as we are on this train of thought, then the next logical
 sequence is to include the occurrences of n when $\sigma(n+1) = \sigma(n)$, and it is
 as follows: 14, 206, 957, 1334, 1364, 1634, 2685, 2974, 4364, 14841, 18873,

19358, 20145, 24957, 33998, 36566, 42818, 56564, 64665, 74918, 79826, 79833,
 84134, 92685, 109214, 111506, 116937, 122073, 138237, 147454, 161001,
 162602, 166934, 174717, 190773, 193893, 201597, 230390, 274533, 289454,

#11

A2961

Move terms!
 %R AS1 840.
 $\sigma(n+1) = \sigma(n)$

Qleed add to cat 25!

347738, 383594, 416577, 422073, 430137, 438993, 440013, 445874, 455373,
 484173, 522621, 544334, 605985, 621027, 649154, 655005, 685995, 695313,
 739556, 792855, 937425, 949634, 1154174, 1174305, 1187361, 1207358, 1238965,
 1642154, 1670955, 1765664, 1857513, 2168906, 2284814, 2305557, 2913105,
 3296864, 3477435, 3571905, 3582224, 3682622, 3726009, 4328937, 4473782,
 4481985, 4701537, 4795155, 5002335, 5003738, 5181045, 5351175, 5446425,
 5459024, 5517458, 6309387, 6431732, 6444873, 6514995, 6771405, 7192917,
 7263944, 7796438, 7845386, 7955492, ...

Continuing is to include the occurrences of n when $\sigma(n+2) = \sigma(n)$, and

A7373 it is as follows: 33, 54, 284, 366, 834, 848, 918, 1240, 1504, 2910, 2913,

New!

#12

3304, 4148, 4187, 6110, 6902, 7169, 7912, 9359, 10250, 10540, 12565, 15085,
 17272, 17814, 19004, 19688, 21410, 21461, 24881, 25019, 26609, 28124, 30592,
 30788, 31484, 38210, 38982, 39786, 40310, 45354, 46863, 49225, 51835, 53106,
 53963, 55286, 59987, 76360, 77057, 81055, 83094, 94996, 95392, 96728,
 101101, 117570, 117858, 121394, 124758, 127585, 143369, 147340, 149149,
 149750, 150419, 163936, 167560, 170114, 170561, 173920, 175796, 181384,
 197260, 205727, 215069, 220817, 239954, 278920, 280787, 292315, 293656,
 319955, 334540, 334983, 336505, 344416, 359454, 360325, 360685, 370435,
 388074, 418307, 434433, 463218, 472323, 477904, 510340, 516026, 543453,
 564857, ...

%0 1,1
 %R ASI
 840

~~first and last~~ $\sigma(n+2) = \sigma(n)$

From page 235, the first occurrence of k when $n - \phi(n) = k$, and it is

as follows: 3, 4, 9, 6, 25, 10, 15, 12, 21, 0, 35, 18, 33, 26, 39, 24, 65,

34, 51, 38, 45, 30, 95, 36, 69, 0, 63, 52, 161, 42, 87, 48, 93, 0, 75, 54,
 217, 74, 99, 76, 185, 82, 123, 60, 117, 66, 215, 72, 141, 0, 235, 0, 329,
 78, 159, 98, 105, 0, 371, 84, 177, 122, 135, 96, 305, 90, 427, 134, 201,
 102, 335, 108, 213, 146, 207, 148, 245, 114, 511, 152, 189, 130, 395, 164,
 165, 0, 415, 120, 581, 126, 267, 132, 261, 138, 623, 144, 1501, 194, 195, 0,
 485, ...

The sequence of k 's when $n - \phi(n) = k$ has no solutions (the Zeros

above), and it is as follows: 10, 26, 34, 50, 52, 58, 86, 100, 116, 122,

130, 134, 146, 154, 170, 172, 186, 202, 206, 218, 222, 232, 244, 260, 266,
 268, 274, 290, 292, 298, 310, 326, 340, 344, 346, 362, 366, 372, 386, 394,
 404, 412, 436, 466, 470, 474, 482, 490, 518, 520, 532, 534, 536, 546, 554,
 562, 566, 580, 584, 596, 626, 634, 650, 652, 666, 680, 686, 688, 698, 706,
 722, 724, 730, 732, 746, 772, 778, 786, 794, 808, 818, 834, 842, 850, 872,
 874, 902, 906, 914, 922, 926, 932, 940, 962, 964, 974, 980, 986, 1018, 1036,
 1038, ...

The first occurrence of k when $\sigma(n) - n = k$, and it is as follows:

No
 I don't think so,
 but
 check
 Biele
 for
 these
 3

2, 0, 4, 9, 0, 6, 8, 10, 15, 14, 21, 121, 27, 22, 16, 12, 39, 289, 65, 34, 18, 20, 57, 529, 95, 46, 69, 28, 115, 841, 32, 58, 45, 62, 93, 24, 155, 1369, 217, 44, 63, 30, 50, 82, 123, 52, 129, 2209, 75, 40, 141, 0, 235, 42, 36, 106, 99, 68, 265, 3481, 371, 118, 64, 56, 117, 54, 305, 4489, 427, 134, 201, 5041, 98, 70, 213, 48, 219, 66, 365, 6241, 147, 158, 237, 6889, 395, 166, 105, 0, 171, 78, 581, 88, 267, 116, 445, 0, 245, 9409, 1501, 124, 291, ...



The sequence of k 's when $\sigma(n) - n = k$ has no solutions (the Zeros above), and this is also the "untouchable" numbers of Paul Erdos. Wells, page 125. It is as follows: 2, 5, 52, 88, 96, 120, 124, 146, 162, 188, 206,

210, 216, 238, 246, 248, 262, 268, 276, 288, 290, 292, 304, 306, 322, 324, 326, 336, 342, 372, 406, 408, 426, 430, 448, 472, 474, 498, 516, 518, 520, 530, 540, 552, 556, 562, 576, 584, 612, 624, 626, 628, 658, 668, 670, 708, 714, 718, 726, 732, 738, 748, 750, 756, 766, 768, 782, 784, 792, 802, 804, 818, 836, 848, 852, 872, 892, 894, 896, 898, 902, 926, 934, 936, 964, 966, 976, 982, 996, 1002, 1028, 1044, 1046, 1060, 1068, 1074, 1078, 1080, 1102, 1116, 1128, ...

AS114
~~ASS~~
 No
 Done



If a number is in parenthesis then that is not the value but the limit to which the test was run on my HP-71B. On the other hand, if the integer presented is Zero, then there is no possible answer. Often, a particular series maybe divided by two to save some room or to make clearer the sequence involved. The references for the above are in your bibliography as S11, BE3 and AS1, plus "The Penguin Dictionary of Curious and Interesting Numbers," David Wells, Middlesex, England, 1986. If at some future date, I run across a filler or greatly extend the limits, I will forward the same to you.

Sequentially yours,

Robert G. Wilson v
 Ph.D., ATP/CF&GI

RGWv:hp110+
 Quotes:

"God invented 1,2 and 3, and man invented all the rest."
 The author is unknown to me, but is a great quote for a book on numerical sequences. Maybe I'm thinking of the following quote.

"God himself made the whole numbers: everything else is the work of man." Leopold Kronecker

"The trouble with integers is that we have examined only the small ones." Ronald Graham

"The primary source of all mathematics are the integers." Herman Minkowski

"I am ill at these numbers." Wm. Shakespeare (Hamlet)