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Problems Drive 1984

Andy Bernoff and Richard Pennington

(Answers on page 50, inside the back cover.)

1 The numbers 1 and 36 have the property that they are both square and triangular, ie they may be expressed as either m^2 or as $n(n+1)/2$ where m and n are integers. What are the next two such numbers?

2 The Archimedeans committee consists entirely of mathematicians (*this is a lie -Ed.*), and may thus be divided into *Pure* and *Applied* mathematicians, or alternatively into *Sane* and *Insane* mathematicians. Pure mathematicians always tell the truth about their beliefs, while Applied mathematicians invariably lie about their beliefs. The beliefs of Sane mathematicians are correct, while those of Insane mathematicians are incorrect.

The following conversation is overheard among three committee members:

A: "B is sane"
B: "C is pure"
C: "A is sane"
A: "B is applied"
B: "A is insane"
C: "B is pure"

Classify the three committee members as Pure or Applied and as Sane or Insane.

3 Calculate $\prod_{2}^{\infty} \cos \pi/2^k$

4 For which of the five Platonic solids (tetrahedron, cube, octahedron, dodecahedron and icosahedron) is it possible to assign a number to each face so that, although the numbers assigned are not identical, the sums of the numbers assigned to the faces meeting at each vertex are the same?

5 A rectangular box of noughts is entirely surrounded by a single layer (including corners) of crosses, as in the diagram:

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XXXXX
XOOOX
XOOOX
XXXXX
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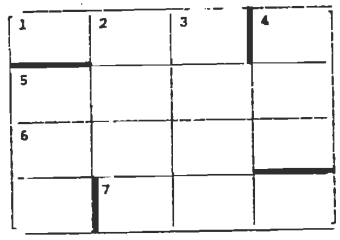
What are the possible shapes and sizes of the block of noughts if there are the same number of noughts as crosses?

6 Solve the following cross-number; m, n, p, q, r and s are integers and no number begins with a zero.



Across: (1) n^2 (5) s^3 (6) r^3 (7) $q(q+1)/2$
Down: (2) n^3+n (3) m^3+m (4) s^2 (5) p^2

7 In the market at Archimedea several kinds of fruit are on sale at a strictly positive integral number of garches per fruit (the currency is 100 garches to 1 eureka). The three people ahead of me in the queue make the following transactions:

oranges	apples	plums	bananas	tangerines	cost
2	2	2	5	3	66g
2	2	2	2	2	48g
5	0	0	9	5	E1

How much should I expect to pay for 12 oranges?

8 The differential equation

$$\frac{d^2y}{dx^2} + x\frac{dy}{dx} + \frac{y}{2} = 0$$

has two linearly independent solutions which may be referred to as A(x) and B(x). Find two linearly independent solutions of

$$\frac{d^2y}{dx^2} + x\frac{dy}{dx} + 1000001\,\frac{y}{2} = 0$$

9 What are the next three terms in each of the following series, and why?

(a) 6, 15, 35, 77, 143, 221
(b) 1, 18, 3, 8, 9, 13
(c) 1, 4, 12, 22, 34, 51

10 The ferries linking the five cities on the shores of the perfectly circular Lake Bathwater all travel in straight lines at a constant speed of 1000 Archimedean cubits per minute. Cauchyville is nearer to Archimedea than Demoivreton is to Archimedea. The ferry from Archimedea to Besselopolis takes 40 minutes. Cauchyville and Demoiveton are each equidistant from Archimedea and Besselopolis. The ferry routes from Cauchyville to Demoivreton and from Archimedea to Eulerberg cross at a point 20000 cubits from Cauchyville and 25000 cubits from Archimedea.

What is the radius of Lake Bathwater and when does the ferry which leaves Archimedea at midday arrive at Eulerberg?

11 Find all ordered pairs of integers (a,b) such that $0 < a < b < 100$ and $0.704 < a/b < 0.705$.

12 An Archimedean anthropologist on a remote island wishes to discover who is the tallest of the natives in the surrounding area. According to a quaint but strictly enforced local custom, he may meet the locals only two at a time. How many comparisons must he make in order to discover which of the 157 natives is the tallest, assuming that he works in such a way as to require as few comparisons as possible? No two of the natives are the same height.

Answers To The Quote Quiz

1 1.6 (Jan 39) by E.T.E., unaware of the Stalinist purges.
2 3.6 (Jan 40) by G.H. Hardy, who must now be turning rapidly in his grave. There's worse later in the same article.
3 4.8 (May 40) by D.J.H. *Eureka* was at that time edited jointly by Cambridge and London students.
4 5.9 (Jan 41) by H.A.E., but this could be any Liaison Committee report.
5 Likewise this is the generic Secretary's report.
6 22.9 (Oct 59) by Martin Fieldhouse.
7 29.3 (Oct 66) Colin Myerscough's editorial.
8 30.3 (Oct 67) and again.
9 31.3 (Oct 68) J.J. Barrett's editorial.
10 31.20 (Oct 68) Report of an "Any Questions" by Peter Johnstone.
11 35.1 (Oct 72) Joseph Conlon's editorial. 19xx=1970.
12 36.11 (Oct 73) W.L. Ferrar on life in 19xx=1912.
[*nn.mm is Eureka nn, page mm; general apologies to these authors for reminding them of their sometimes rather silly predictions!*]

The Mathematical Association

Rolph Schwarzenberger

The *Mathematical Association* was founded in 1871 as the *Association for the Improvement of Geometrical Teaching* and adopted its present title in 1897. It was the first subject teaching association to be founded in this country and its membership continues to consist mainly of those engaged in the teaching of mathematics at all levels from primary schools to universities.

The Association publishes three journals, the *Mathematical Gazette* (the most erudite - chiefly read by teachers in secondary schools and in colleges and univeristies), *Mathematics in School*, and *Mathematics Round the Country*. Two new journals, aimed at pupils rather than teachers, are planned.

The Association is responsible for the validation of three Diploma courses: *Mathematical Education* (for teachers of pupils in the age range 5-13), *Low Attainers* and *Heads of Department*. It organises regular weekend conferences and issues regular reports, booklists and posters. Members have the opportunity to participate in these activities through a system of local branches, through committees and working parties set up to consider particular issues, through committees running particular events (such as the national mathematical competitions and International Mathematical Olympiad) and through the annual conference at Easter.

Full details of these activities and of current subscription rates (which depend on the journals received) may be obtained from the Executive Secretary at the Association's headquarters, *259 London Road, Leicester, LE2 3BE, (0533) 703877*, or though the Archimedeans. There are reduced rates for students.

The Cambridge branch of the Mathematical Association meets about three times a term at the Cambridge Institute of Education in Shaftesbury Avenue. Members of this branch may attend the meetings of the Archimedeans and the College Societies for free, and members of the Archimedeans may attend branch meetings for free.

The 1985 Conference will be held in the University of Dundee from 10 to 13 April. The 1986 Conference will be held in Cambridge, with Hilary Shuard of Homerton College as President. It is hoped to have the active co-operation of the Archimedeans and the Faculty of Mathematics; arrangements for members of the Archimedeans to attend at a nominal charge are under discussion. -Ed.

Answers to Problems Drive

1 $1225 = 35^2 = 49 \times 50/2$ $\quad 41414 = 204^2 = 288 \times 289/2$

2 A is Applied and Sane
B is Pure and Insane
C is Applied and Insane

3 $2/\pi$

4 Possible for cube, octahedron and icosahedron

5 Block is 6x4 or 10x3 (either way round)

6

1 3	2 6	3 1	4 3
5 5	8	3	2
6 2	7	4	4
9	7 8	2	0

7 96 garches

8 The 500000th derivatives of $A(x)$ and $B(x)$, or linear combinations of these

9 (a) 323, 437, 667 products of consecutive primes
(b) 5, 4, 5 *ARCHIMEDES* with A=1, B=2, etc.
(c) 100, 121, 144 squares in base seven

10 The radius of Lake Bathwater is 42500 cubits;
the ferry arrives at 1.17 pm

11 $\frac{31}{44}$ $\frac{43}{61}$ $\frac{50}{71}$ $\frac{62}{88}$ $\frac{69}{98}$

12 156. He needs one *winner* and 156 *losers*