1.6.4 Numerical Differential Equations

NDSolve[<i>eqns</i> ,	у,	$\{x,$	xmin,	$xmax$ }]
				solve numerically for the function y , with the independent variable x in the range $xmin$ to $xmax$
NDSolve[<i>eqns</i> ,	{ <i>y</i>	$_{1}, y_{2}$, }	{x, xmin, xmax}]
				solve a system of equations for the y_i

Numerical solution of ordinary differential equations.

This generates a numerical solution to the equation $y'(x) = y(x)$ with $0 < x < 2$. The result is given in terms of an InterpolatingFunction.	<pre>In[1]:= NDSolve[{y'[x] == y[x], y[0] == 1}, y, {x, 0, 2}] Out[1]= {{y -> InterpolatingFunction[{0., 2.}, <>]}}</pre>
Here is the value of $y(1.5)$.	In[2]:= y[1.5] /. % Out[2]= {4.48191}

With an algebraic equation such as $x^2 + 3x + 1 = 0$, each solution for *x* is simply a single number. For a differential equation, however, the solution is a *function*, rather than a single number. For example, in the equation y'(x) = y(x), you want to get an approximation to the function y(x) as the independent variable *x* varies over some range.

Mathematica represents numerical approximations to functions as InterpolatingFunction objects. These objects are functions which, when applied to a particular x, return the approximate value of y(x) at that point. The InterpolatingFunction effectively stores a table of values for $y(x_i)$, then interpolates this table to find an approximation to y(x) at the particular x you request.

y[x] /. solution use the list of rules for the function y to get values for y[x]
InterpolatingFunction[data][x]
evaluate an interpolated function at the point x
Plot[Evaluate[y[x] /. solution]], {x, xmin, xmax}]
plot the solution to a differential equation
Using results from NDSolve.

 This solves a system of two coupled differential equations.
 In[3] := NDSolve[{y'[x] == z[x], z'[x] == -y[x], y[0] == 0, z[0] == 1}, {y, z}, {x, 0, Pi}]

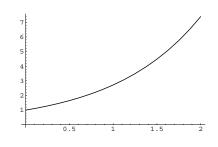
 Out[3] = {{y -> InterpolatingFunction[{0., 3.14159}, <>], z -> InterpolatingFunction[{0., 3.14159}, <>]}

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Here is a plot of the solution for y[x] found on line 1. Plot is discussed in Section 1.9.1.

In[4]:= z[2] /. %
Out[4]= {-0.416167}

In[5]:= Plot[Evaluate[y[x] /. %1], {x, 0, 2}]



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