10 Years of Comparative Results: Chained C-CPI-U vs. Regular CPI-U October 2013

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Abstract

In February, 2013, the BLS calculated and published its twelfth annual set of C-CPI-U indexes (for the 12 months of 2011) and its eleventh annual set of C-CPI-U indexes for 12-month price changes. This paper will concentrate on the last ten years of this series. The C-CPI-U (Chained Consumer Price Index – Urban) is calculated and published every year, with a one to two year lag, using a Tornqvist formula, and its set of weights are updated yearly, so that a unique set of monthly weights are available for both time t as well as for time t–n. The C-CPI-U can thus be labeled a "superlative" index. By contrast the Regular CPI-U uses weights that are, at a minimum, at least two years old, and uses a Laspeyres formula as its final high-level estimator. The set of All-US–All-Items Chained C-CPI-U index results continue to diverge – lower, but more slowly – from Regular CPI-U index results. We investigate the nature of this divergence.

Key Words: Tornqvist; Superlative Index

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1. Chained C-CPI-U vs. Regular CPI-U

The Official, or Regular, CPI-U is not a superlative index, and does not use a superlative index formula. The Official CPI-U uses a Modified Laspeyres formula for its second and final stage of index calculation, which is not a superlative formula. The Chained C-CPI-U, on the other hand, does employ a superlative formula: the Tornqvist.

(1) Modified Laspeyres:

[illegible]

with [illegible] a 2-3 year-old weight.

(2) Tornqvist: [illegible]

where [illegible] is an expenditure share.

Both formulas draw on the exact same set of IX's, which are the lower-level indexes estimated by the PRC (Price Relative Calculation) for each Item(i) – Area(a) cell. The Item-Area price relatives that move the respective IX's are a combination (Hybrid) of Geomeans and Laspeyres formulas, complete with their own set of sampling weights at the unique price level. Both of the index-level formulas above also draw from the same set of expenditure weights, but with only the Tornqvist weights being timely weights.

So, the regular CPI claims to know yesterday's prices (at time t-1), even today's prices (at time t), and also claims to know a set of (2-3 year-old) weights. The CPI is able to collect today's prices, but not today's weights, in a timely fashion. The CPI calculates and publishes, for examples, April's CPI in mid-May, using April's (t) and February's (t-1) prices, while the weights used are, at a minimum, two years old. In order to call an index "superlative", what is required are not only today's prices but today's weights. In other words, for an index, or an index formula, to be "superlative", all four ingredients – yesterday's prices, yesterday's weights, today's prices and today's weights – must be available. Using the "superlative" Tornqvist formula, albeit with a real-time lag time of 1 to 2 years, the BLS has gathered together the four necessary ingredients and, so, has been able to produce a "superlative" index.

One last caveat on the superlative nature of the C-CPI-U (Tornqvist) estimates. The superlative nature of Tornqvist comes from the use of a set of unique monthly weights, detailed above at the item-area level, for both time period t and time period t–1. These monthly weights are smoothed weights, but they do represent a unique monthly weight for that particular month for that particular item in one particular area. The "smoothed" aspect of these weights mitigates considerably the purity of this uniqueness, but the superlative character of the Tornqvist formally remains intact. The two weights, at times t and t–1, are unique, but roughly 90% of the information content of the one is shared by the other. Moreover, each weight, by itself, is a smoothed construct involving the averaging of the item-area's weight back over the 11 prior months and the averaging of the item-area's weight across all the areas. This is a lot of smoothing, but the uniqueness of the monthly item-area weight is preserved. The other obvious mitigating factor is the non-superlative nature of the lower-level indexes that are used in the Tornqvist estimates. BLS has wisely chosen to formally call the C-CPI-U a Chained CPI and not a Superlative CPI, even while informally retaining the right to call the Tornqvist results "superlative".

A "superlative" formula, like the Tornqvist, is generally expected to produce a lower index than an index that uses a Laspeyres formula. According to classical price index theory, the Laspeyres formula, under homothetic assumptions, will provide an upper bound for a Konus (Cost of Living) Index --- with the Paasche formula providing a lower bound. The Tornqvist formula, along with the Fisher Ideal (or a perfectly parameterized CES formula), provides a close approximation to a true cost-of-living index (i.e., closest to a Konus), and as such is expected to produce a consistently lower index than an index employing a Laspeyres formula. The Boskin Commission's 1996 "Final Report on the Advisory Commission to Study the Consumer Price Index" estimated the (upper-level) "substitution bias" between a Superlative and a Laspeyres index for a 12-month price change to be "no more than 0.4 percentage points per annum" for the All-US—All-Items index. Now that we have more than ten years worth of "superlative" results we can compare the two indexes and see how the divergence is or is not holding up.

Fig 1

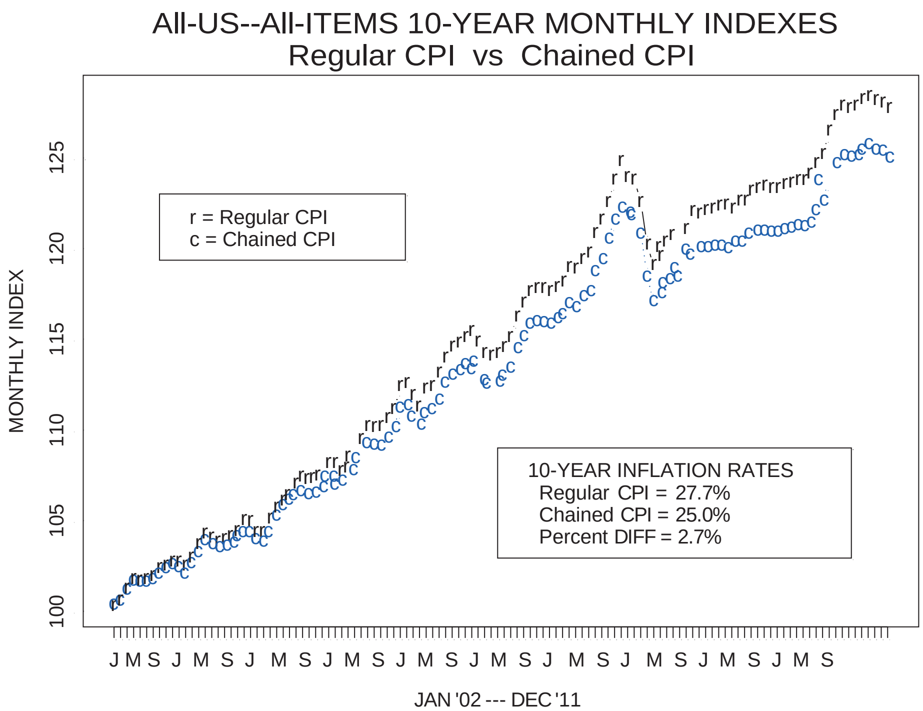


Fig 1 displays the last ten years of the Chained (C-CPI-U) Index as compared with the Regular (Official) CPI-U Index, for the same set of 60 months, for the All-US—All-Items category. Starting our comparative index series at January 2002, we end up, at the end of ten years, with a Chained Index of 125.0355 compared with a Regular Index of 127.7352. The 10-year percentage difference is 2.7%. (In real terms, as when we visit the issue of U.S. Govt. COLA adjustments later on, this means that a monthly $1000 payment beginning in January 2002 would, by the end of 2011, be up to $1,250.35 or $1,277.35, respectively.)

Breaking down these differences year by year, we can better able see how Regular CPI is comparing with its "superlative" counterpart, Chained CPI:

Table 1. Yearly Percentage Differences (R - C)

	DIFF (%)			DIFF (%)
2002	0.30%		2007	0.42%
2003	0.19%		2008	-0.13%
2004	0.17%		2009	0.26%
2005	0.45%		2010	0.20%
2006	0.30%		2011	0.03%

First note that in the course of one of these years (2008) that the end-of-year chained index was actually higher than the regular, official CPI index. This was the year of the financial collapse, when prices, along with so many other things, plummeted downwards. As we will be able to see in some subsequent graphs, this plunged the CPI into temporary deflationary territory. Thus, it would appear that, in the neighborhood of zero to negative inflation, the regular CPI matches or even tracks lower than the chained CPI. This 2008 result may or may not be an anomaly. Quite possibly the way a Geomeans formula performs in this non-positive inflation range is comparatively different than the way a Laspeyres formula does.

What is more germane is whether the current (i.e., regular) CPI is diverging from the chained CPI less or more over the years, and perhaps more to the point, whether the percentage difference between the two indexes hovers in that "less than 0.4%" range that the Boskin Commission predicted. Well, as a general rule, discounting the "flipped" year of 2008, we see only 2 out of the remaining 9 years with differences above 0.4, while the other 7 are comfortably and properly hovering in the 0.15 to 0.40 range. Looking at those last four years of percentage differences, a case could certainly be made that the two indexes are diverging more slowly than before, but that does not make a case for their converging. Odds are, as inflation re-adjusts to its more-recent long-term rate of 2-3% annual inflation, the annual difference between the two indexes will remain in the 0.15 to 0.40 range of percentage annual difference.

2. Chained CPI Standard Errors vs. Regular CPI Standard Errors

Unofficially, the CPI program produces monthly 1-, 2-, 6- and 12-month standard errors for most all of its chained CPI published results. These monthly standard errors are produced annually in the month or so after the release of the next year's Chained Index estimates. We will use this set of chained CPI standard errors to update the significant differences graphs (see below) for our two indexes over these same ten years of comparative results.

The C-CPI-U standard errors use the same basic SRG (Stratified Random Group) methodology to produce their results. (Remember, STD ERROR = SQRT [VAR].)

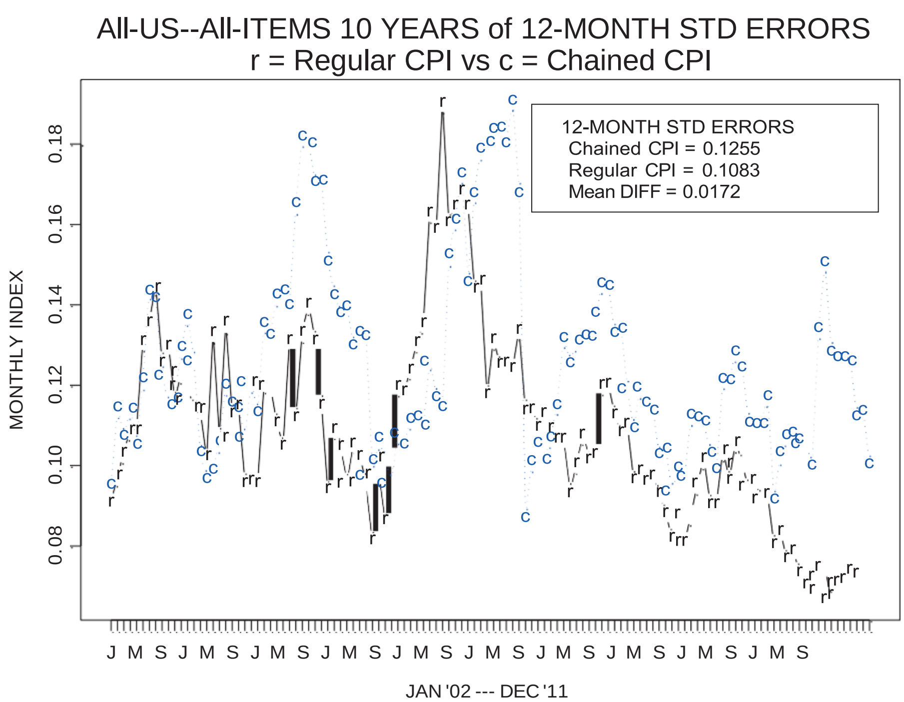
(3) $$\mathrm{VAR}(I, A, t, t-k) = \sum_{a \in A} \frac{1}{N_a (N_a - 1)} \sum_{r=1}^{N_a} \left[PC(I, A, t, t-k, a, r) - PC(I, A, t, t-k, fs) \right]^2$$

This is the SRG variance formula for both Regular and Chained CPI (fs means full-sample). The differences arise in the Price Change (PC) formulas used for each. With t – k now always being t – 12, and an r index added to accommodate the required replicates, the two PC formulas are

$PC_{CPI\text{-}U} = (PREL^{ML}_{I,A,t-12,t,r} - 1) * 100$ and $PC_{CPI\text{-}U} = (PREL^{TQ}_{I,A,t-12,t,r} - 1) * 100$

The two PREL (Price Relative) formulas are from formulas (1) and (2) above. The comparative results for the ten years of C-CPI versus C-CPI-U standard errors are as follows:

Fig 2



The C-CPI-U SE's run fairly consistently above the CPI-U SE's, except around 2005 and a few times in late 2006. The last year (2011) of comparative differences might be grounds for concern, since the Regular

CPI SE's in 2011 dropped rather markedly while the Chained CPI SE's in that same 2011 time period rose. Curiously enough, the Standard Deviations of these two sets of SE's are nearly identical: 0.024125 (Regular) versus 0.024170 (Chained).

3. Significant Differences – 12-Month PCs (CPI-U vs. C-CPI-U)

In order to estimate the difference variance between our two estimators, we might choose to utilize the new Chained CPI-U standard errors to construct our confidence intervals month by month, but since we could just as easily use the Regular CPI-U standard errors to construct confidence intervals, clearly neither methodology is optimal. What we need is a proper set of variances estimates for the differences themselves.

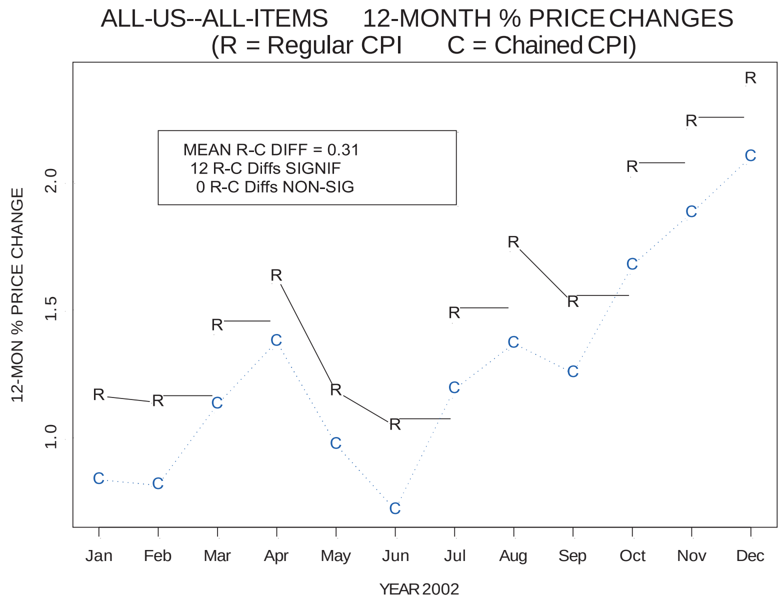
Since the BLS variance formula for both Regular CPI-U (R) and Chained CPI-U (C) price change is Eq (3) above, a natural variance estimator for the difference between the two percent price change estimates would be:

(4) VAR (RC) =

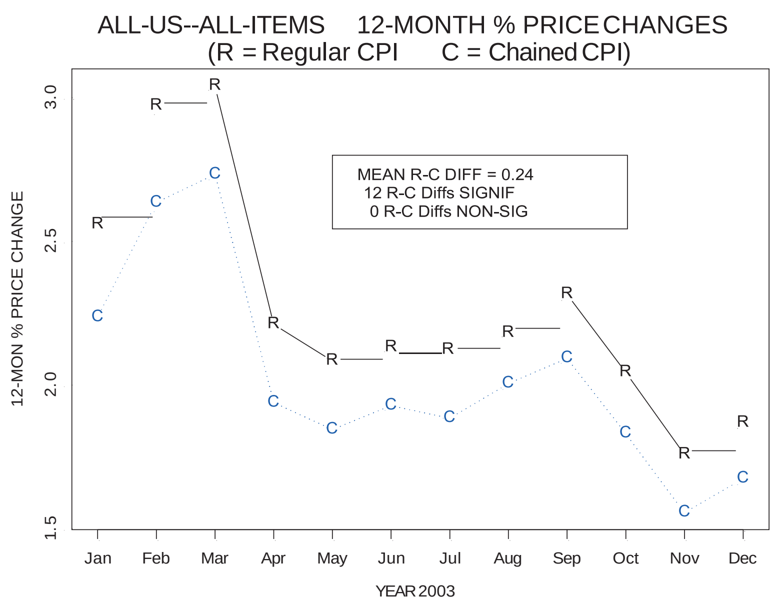
$$\sum_{a \in A} \frac{1}{N_a (N_a - 1)} \sum_{r=1}^{N_a} \left[\left[PC_R(I,a,r) - PC_R(I,f) \right] - \left[PC_C(I,a,r) - PC_C(I,f) \right] \right]^2$$

The constructions of the various replicate (r) percent price changes (PC) follow the rubrics for the respective Regular and Chained ("Superlative") estimates, as applied using Stratified Random Group (SRG) methods, with I = Item, A = Area, a = area random group, and N_a = number of replicates in each a. The difference estimator is, of course, estimating zero. (Standard error estimates are simply the square roots of these variance estimates.)

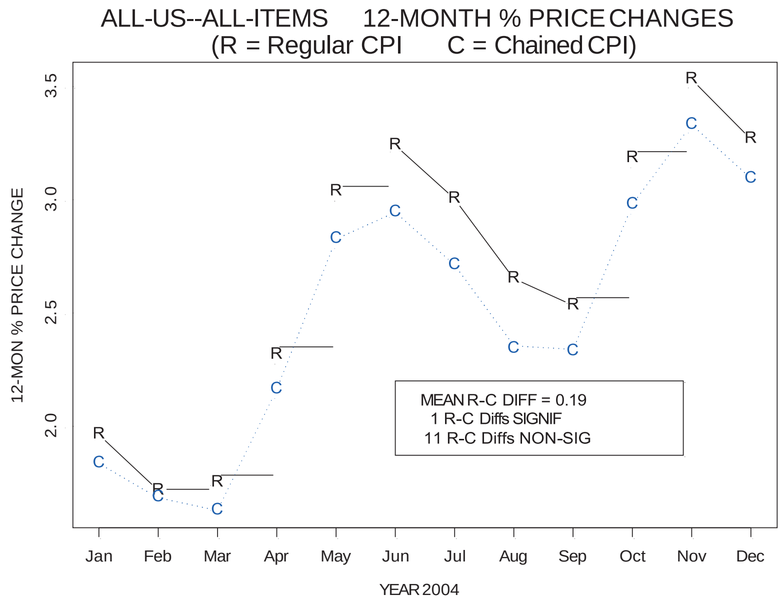
On the following five pages, we graphically present ten years (2002-2011) of comparative results for the CPI's All-US—All-Items category. The ten graphs display the monthly 12-month percent price change differences for each year; the ten accompanying tables display the p-values that result from applying Eq. (4) to the two sets of these All-US—All-Items estimates. The null hypothesis for our two-sided significance tests is H : R = C, with α = .025.



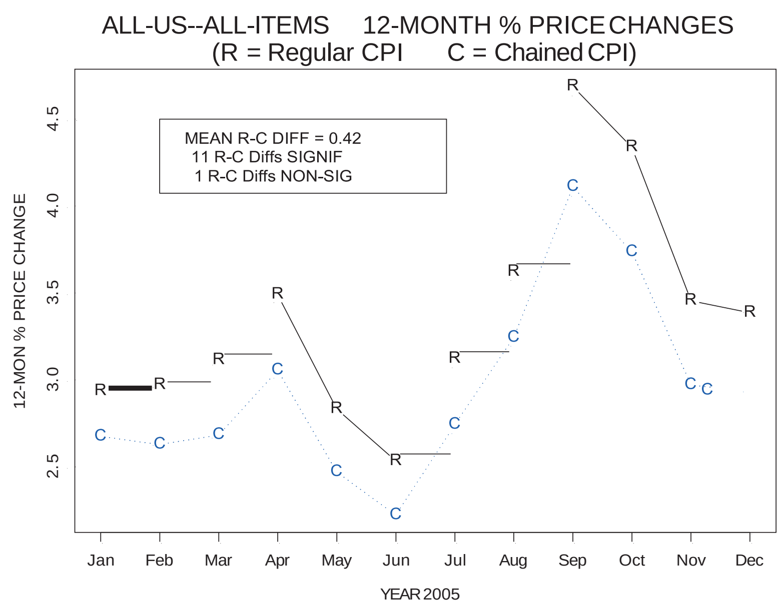
2002	JAN	FEB	MAR	APR	MAY	JUN	JUL	AUG	SEP	OCT	NOV	DEC
PVAL	0.000	0.000	0.001	0.012	0.004	0.002	0.010	0.004	0.023	0.001	0.000	0.002
SIGNIF	YES	YES	YES	YES	YES	YES	YES	YES	YES	YES	YES	YES



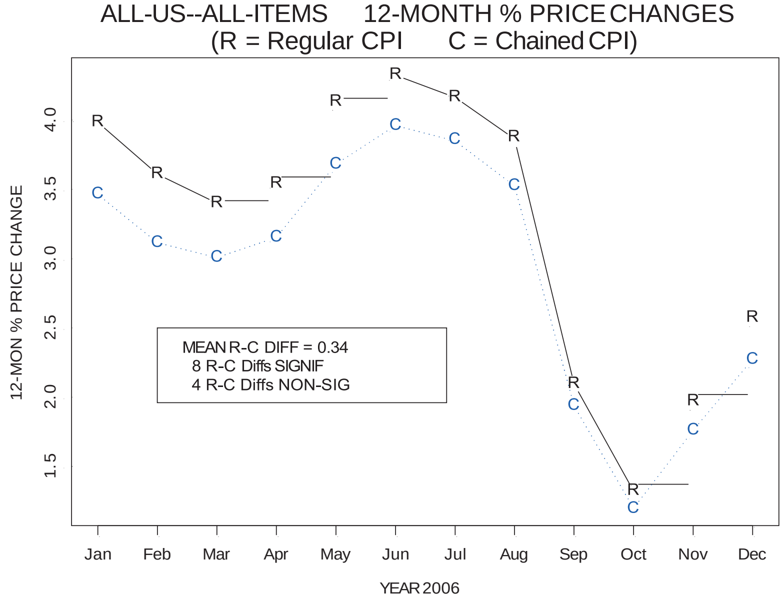
2003	JAN	FEB	MAR	APR	MAY	JUN	JUL	AUG	SEP	OCT	NOV	DEC
PVAL	0.001	0.000	0.000	0.000	0.034	0.004	0.018	0.036	0.016	0.011	0.010	0.024
SIGNIF	YES	YES	YES	YES	YES	YES	YES	YES	YES	YES	YES	YES



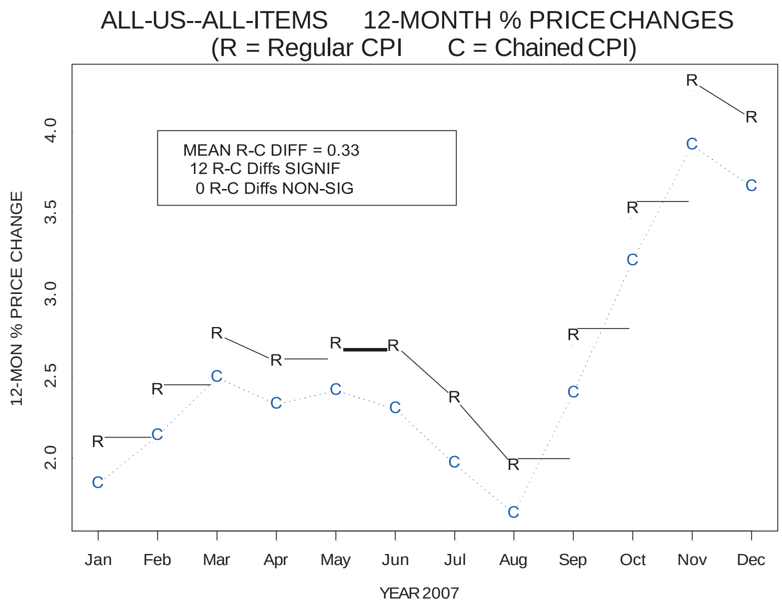
2004	JAN	FEB	MAR	APR	MAY	JUN	JUL	AUG	SEP	OCT	NOV	DEC
PVAL	0.094	0.669	0.217	0.217	0.163	0.049	0.107	0.107	0.262	0.236	0.185	0.269
SIGNIF	NO	NO	NO	NO	NO	YES	NO	NO	NO	NO	NO	NO



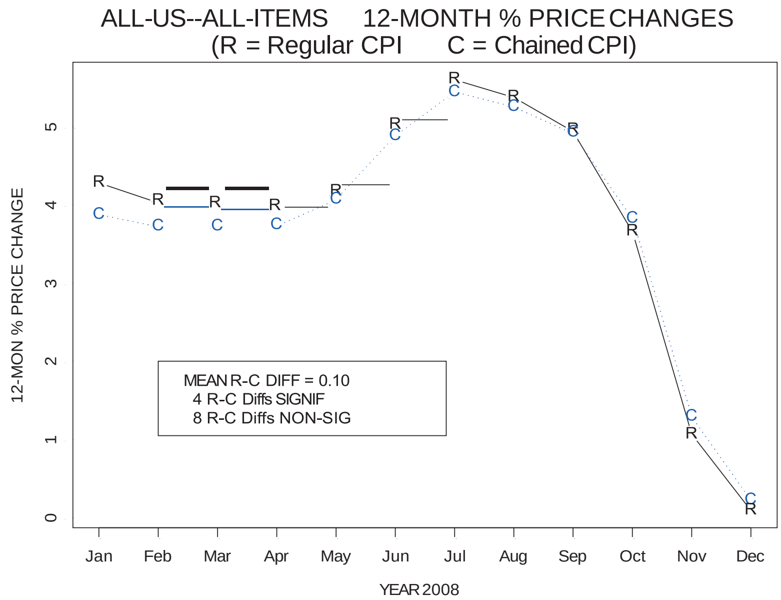
2005	JAN	FEB	MAR	APR	MAY	JUN	JUL	AUG	SEP	OCT	NOV	DEC
PVAL	0.065	0.013	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
SIGNIF	NO	YES	YES	YES	YES	YES	YES	YES	YES	YES	YES	YES



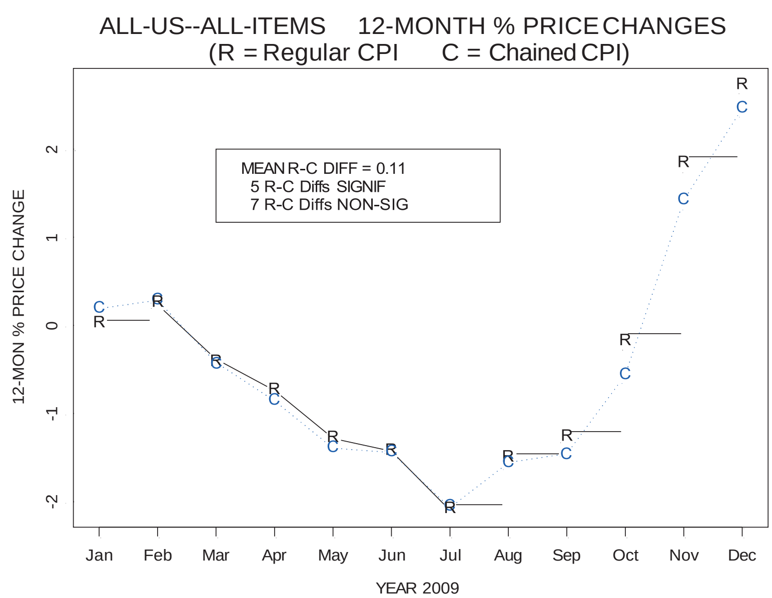
2006	JAN	FEB	MAR	APR	MAY	JUN	JUL	AUG	SEP	OCT	NOV	DEC
PVAL	0.000	0.000	0.002	0.005	0.004	0.032	0.085	0.045	0.343	0.353	0.096	0.003
SIGNIF	YES	YES	YES	YES	YES	YES	NO	YES	NO	NO	NO	YES



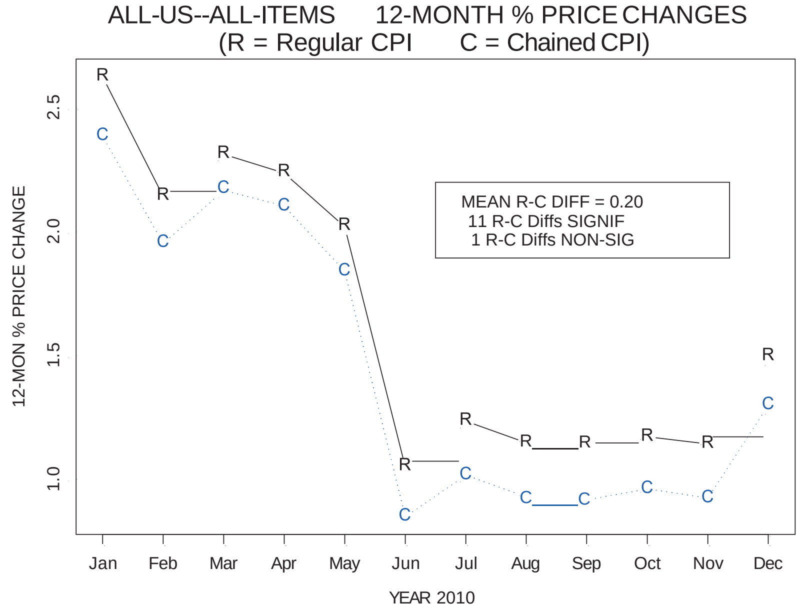
2007	JAN	FEB	MAR	APR	MAY	JUN	JUL	AUG	SEP	OCT	NOV	DEC
PVAL	0.016	0.009	0.009	0.019	0.009	0.000	0.000	0.000	0.000	0.000	0.000	0.000
SIGNIF	YES	YES	YES	YES	YES	YES	YES	YES	YES	YES	YES	YES



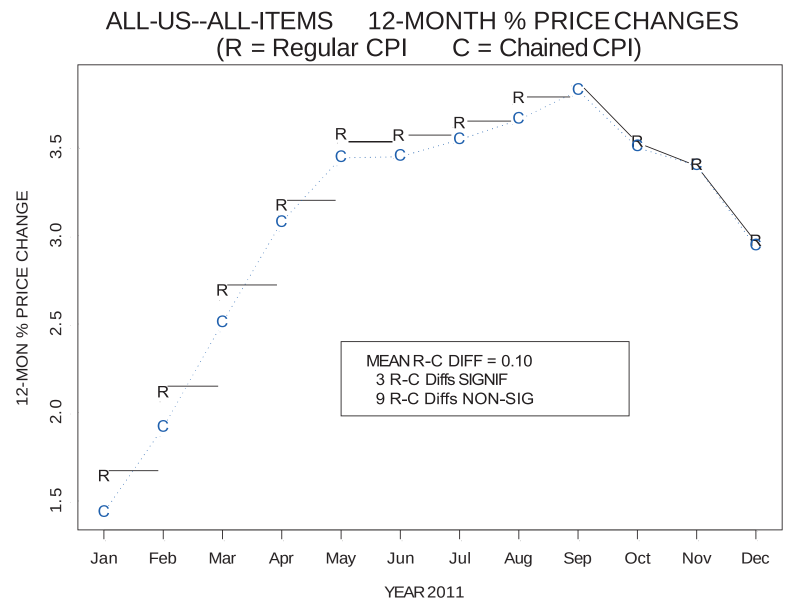
2008	JAN	FEB	MAR	APR	MAY	JUN	JUL	AUG	SEP	OCT	NOV	DEC
PVAL	0.003	0.091	0.233	0.535	0.572	0.835	0.984	0.637	0.158	0.000	0.000	0.000
SIGNIF	YES	NO	NO	NO	NO	NO	NO	NO	NO	YES	YES	YES



2009	JAN	FEB	MAR	APR	MAY	JUN	JUL	AUG	SEP	OCT	NOV	DEC
PVAL	0.022	0.501	0.591	0.156	0.167	0.837	0.708	0.358	0.032	0.000	0.000	0.001
SIGNIF	YES	NO	NO	NO	NO	NO	NO	NO	YES	YES	YES	YES



2010	JAN	FEB	MAR	APR	MAY	JUN	JUL	AUG	SEP	OCT	NOV	DEC
PVAL	0.005	0.015	0.052	0.029	0.002	0.000	0.001	0.003	0.001	0.006	0.012	0.019
SIGNIF	YES	YES	NO	YES	YES	YES	YES	YES	YES	YES	YES	YES



2011	JAN	FEB	MAR	APR	MAY	JUN	JUL	AUG	SEP	OCT	NOV	DEC
PVAL	0.015	0.016	0.036	0.463	0.265	0.320	0.433	0.293	0.668	0.762	0.986	0.769
SIGNIF	YES	YES	YES	NO	NO	NO	NO	NO	NO	NO	NO	NO

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For our significance tests, we are assuming our Diffs (R—C) are an independent sample from an N (μ_{Diff} , σ^2) distribution. Thus, we calculate our p-values by standardizing our N (μ_{Diff} , σ^2) results into z-scores:

[illegible]

(PNORM is an R function that transforms a z-score into an N distribution probability.)

To illustrate how the difference variance estimates are produced, we can observe one year (2011) of results, using $\alpha = .025$ to form 2-sided Confidence Intervals.

Table 2. Difference Variance Results

Year	Mon	μ_{Diff}	σ	CI_{Lower}	CI_{Upper}	Pval
2011	JAN	0.199	0.082	0.039	0.359	0.015
2011	FEB	0.194	0.081	0.036	0.352	0.016
2011	MAR	0.176	0.084	0.012	0.341	0.036
2011	APR	0.088	0.120	-0.147	0.322	0.463
2011	MAY	0.130	0.117	-0.099	0.359	0.265
2011	JUN	0.113	0.113	-0.109	0.335	0.320
2011	JUL	0.087	0.110	-0.130	0.303	0.433
2011	AUG	0.117	0.111	-0.101	0.335	0.293
2011	SEP	0.049	0.115	-0.176	0.274	0.668
2011	OCT	0.029	0.097	-0.161	0.220	0.762
2011	NOV	-0.002	0.099	-0.196	0.192	0.986
2011	DEC	0.028	0.096	-0.159	0.215	0.769

Note that when the confidence interval contains zero, then the difference is not significant, with the three significant results (p-value < 0.05) occurring when zero is not contained in the confidence interval.

The results from all these significance tests are a mixed bag. Overall, the count is 79 months where the 12-month price change differences are significantly different, with 41 of the months where the differences are not significance. So, roughly 2/3 of the 120 months from 2002 through 2011 show significant differences, with 1/3 of the monthly differences not significant. There is no noticeable trend to these differences, though each given year seems to lean heavily one way or the other. In the last year of our comparative results (2011), we find the last nine months all quite clearly not significantly different, with one of those differences (Nov ’11) showing Chained CPI-U higher than its Regular CPI-U 12-month price change counterpart. (Note also that for ten months running, from May ’08 through Feb ’09, this same unexpected result occurred. These were the near-deflationary months leading up to, and including the first six months following the Financial Crash in Fall ‘08.) These yearly ups and downs in the comparative differences can best viewed in this simple yearly table:

Table 3. Comparative Result by Year

YEAR	Yearly Significance Level
2002	SIGNIFICANT (12 vs. 0)
2003	SIGNIFICANT (12 vs. 0)
2004	NON-SIGNIFICANT (1 vs. 11)
2005	SIGNIFICANT (11 vs. 1)
2006	Mostly SIGNIFICANT (8 vs. 4)
2007	SIGNIFICANT (12 vs. 0)
2008	Mostly NON-SIGNIFICANT (4 vs. 8)
2009	Mostly NON-SIGNIFICANT (5 vs. 7)
2010	SIGNIFICANT (11 vs. 1)
2011	Mostly NON-SIGNIFICANT (3 vs. 9)

The Chained ("Superlative") CPI-U remains significantly lower than the Regular CPI-U a full 2/3 of the time. However, not only are a good 1/3 of the comparative differences not significantly different, but full 1/4 of those differences find the Regular CPI-U's 12-month percent price change actually higher than its "superlative" counterpart. The differences between the two 12-month price change estimates appear to be shrinking over time, but past is not necessarily prologue in this particular comparative game. The next (2012) Chained Tornqvist ("superlative") results could as easily as not all be again significantly different from the Regular CPI-U results.

6. Summary

- BLS's "Superlative" Final Tornqvist Index, C-CPI-U, has now more than 10 years worth of data behind it
- C-CPI-U continues, as expected, to track lower than CPI-U, but in recent years the gap has narrowed (with the Great Recession year of 2008 finding CPI-U actually lower over that year than C-CPI-U)
- C-CPI-U Standard Errors have increased comparatively to CPI-U SEs in the last 5 years, with an average difference greater than 0.25 in the last 5 years compared with less than 0.01 difference in the earlier 5 years
- Significant Differences between C-CPI-U and CPI-U 12-Month % Price Changes have decreased over 10 years, with 64 out of 120 months compared showing non-significance, especially in the more recent years