Characteristics of a Model-based Variance Measure for X-11 Seasonal Adjustment October 2010

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1. Introduction

Building from the work of Wolter and Monsour (1981), Pfeffermann (1994) and Bell and Kramer (1999) develop variance measures for X-11 seasonal adjustment which account for both time series features and sampling error. Scott, Pfeffermann, and Sverchkov (2011) summarize methods and a body of empirical work. This paper investigates a puzzling, unsatisfying property that sometimes occurs in time series of variance or standard deviation estimates stemming from the measures. Figure 1 contains graphs of standard deviations for error in seasonal adjustment from two methods for three economic time series. Before explaining these graphs in detail, we focus simply on the shapes. Figure 1a shows values in blue increasing from the center to the ends of the series. In Figure 1b, both methods show a dip during the last year, with one staying below central values throughout the last year and the other increasing to values at the ends exceeding the central values. For the series in Figure 1c, the methods have similar shapes, with dips toward the ends and end values well below the central values. The natural view of seasonal adjustment is that there is more uncertainty at the ends of the series than in the center, that is,

$$SDA(end) > SDA(center) . \text{ (1)}$$

This paper explores when and why deviations from this property occur, as in Figure 1c.

We begin by considering the conventional decomposition of an economic time series Y into a trend or trend-cycle, a seasonal effect, and an irregular or noise term, or, alternatively, into a seasonally adjusted series and a seasonal component,

$$Y = T + S + I = A + S .$$

Typically, the population or signal values Y are unknown and the data are obtained from a sample survey, implying that the observed series is $y = Y + \varepsilon$, ε representing the sampling error. Seasonal adjustment is usually carried out by augmenting the series with ARIMA model extrapolations. Let

$$\hat{y} = [\hat{y}_{-m+1}, \cdots, \hat{y}_0, y_1, \cdots, y_n, \hat{y}_{n+1}, \cdots, \hat{y}_{n+m}]' = [\hat{y}'_b, y'_o, \hat{y}'_f]'$$

define the $(n+2m) \times 1$ vector consisting of m backcasts, n observed values, and m forecasts. The seasonally adjusted estimator is $\hat{A} = \Omega \hat{y}$, where Ω is the $n \times (n+2m)$ matrix of the X-11 seasonal adjustment filter. Bell and Kramer (1999) define the target seasonally adjusted series to be $A^* = \Omega Y$, the seasonally adjusted series that would be obtained if Y were observed for all $n+2m$ time points, and the variance measure to come from the error in estimating this target,

$$Var(A^* - \hat{A}) = Var(\Omega Y - \Omega \hat{y}) = \Omega Var(Y - \hat{y})\Omega' .$$

Application of the method requires modeling the signal series Y with an ARIMA model and the sampling error ε with a (stationary) ARMA model. This measure accounts for variability of the sampling errors and for backcast and forecast error. The authors
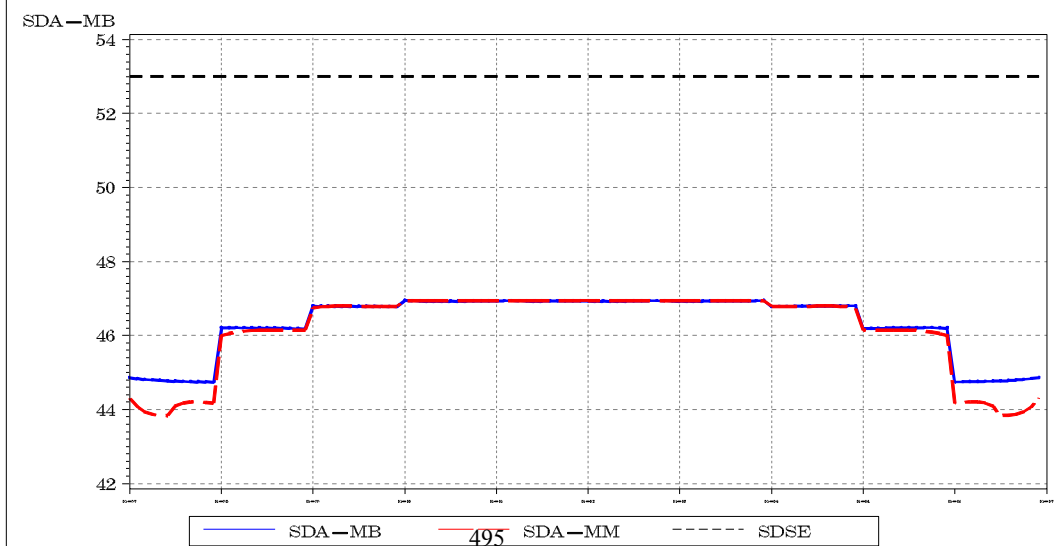
Figure 1. MB and MM Standard Deviations for Seasonal Adjustment Error

a. Series = MD00
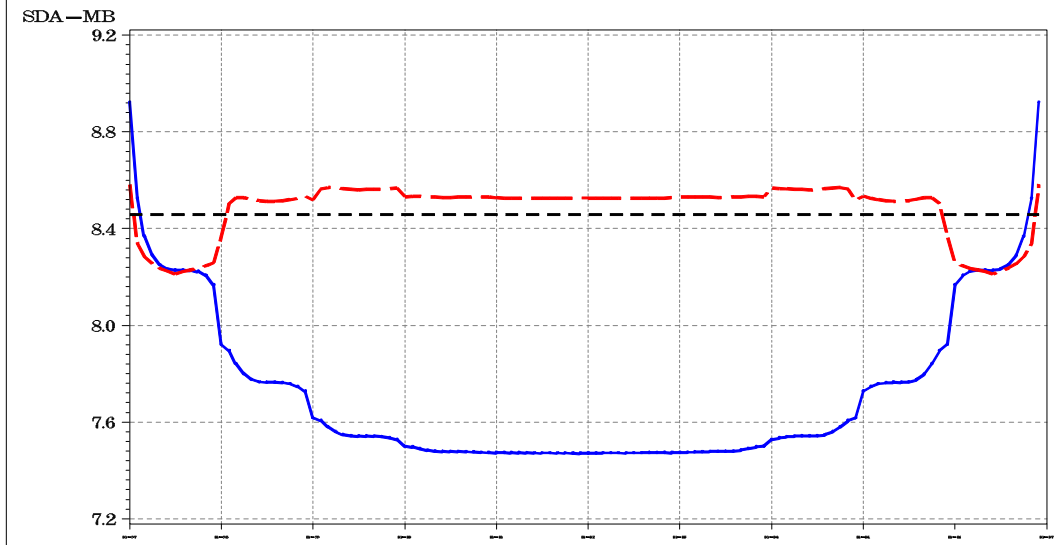
b. Series = MN00
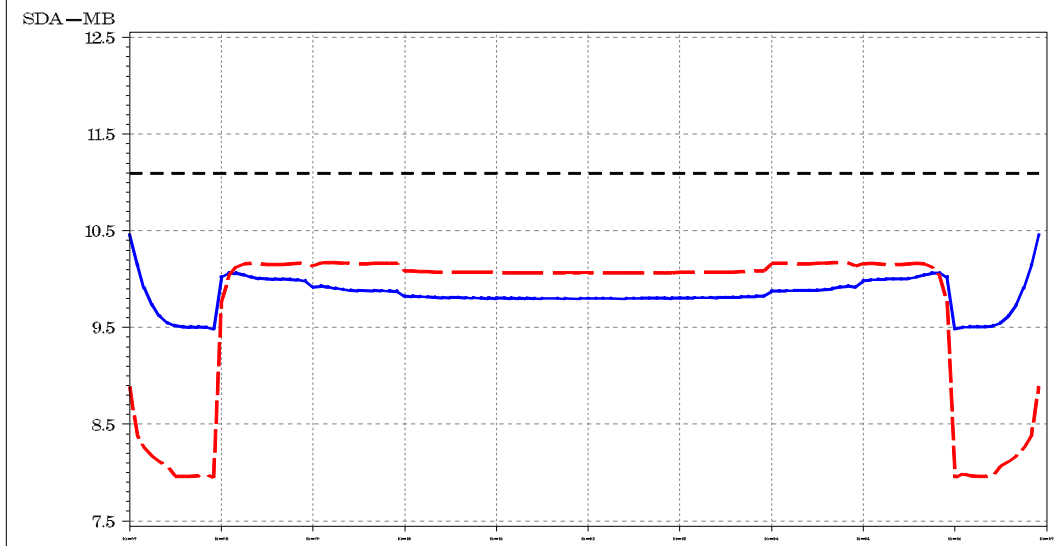
c. Series = MN10

SDA—MB   SDA—MM   SDSE

provide all the neces sary details for cal culating $Var(Y - \hat{y})$. Hereaft er we r efer to this method as the model-based method (MB).

Pfeffermann (1994) proposes a moments-matching (MM) method for variance estimation and defines the target series to be $\tilde{A} = Y - \tilde{S}$, where $\tilde{S} = \Omega(Y - I)\Omega = (T + S)$ and Ω_s is the X-11 seasonal filter matrix. Pfeffe rmann (1994) assumes that the irregular component and the sampling error are stationary and mutually independent, implying that for the combined error $e_t = \varepsilon_t + I_t$

$$\gamma_k = Cov(e_t, e_{t-k}) = Cov(\varepsilon_t, \varepsilon_{t-k}) + Cov(I_t, I_{t-k}) \equiv \lambda_k + v_k, \quad k = 0,1,\dots .$$

The following approximation is developed in the paper:

$$Var(\tilde{A}_t - \hat{A}_t) \approx Var(\sum_{k=1}^{n} w_{tk}^{(S)} e_k) + \lambda_0 (1 - 2w_{tt}^{(S)}) - 2\sum_{k \neq t} w_{tk}^{(S)} \lambda_{t-k} ,$$

where the co efficients $\{w_{tk}^{(S)}\}$ are the weights of the X-11 seasonal filter for time t. This measure acco unts for the variabilit y of the sam pling error and the irregular com ponent. Assuming the availability of t he variance a nd autocovariances of the sam pling errors, estimation of the variance mea sure requires esti mating the vector γ of autocova riances γ_k. Pfefferm ann develops these estimate s based on a sy stem of linear equations involving empirical m oments of the es timated irreg ular from ap plication of t he X-11 method.

Each graph in Figure 1 contains SDA_{MB}, SDA_{MM}, and $SDSE$, square roots of the respective va riance measures and the s ampling error (SE) standard deviation, assu med constant. In Figure 1a, SDA_{MM} is larger than SDA_{MB} in the center of the series, which illustrates the extra error term which the MM method includes. In all three graphs, we see $SDA_{MB} > SDA_{MM}$ at the ends of the series, showing that the MB method captures more fully forecast-b ackcast error. As alr eady m entioned, for the series in Figure 1c, both measures fall below the central values at the end.

Section 2 presents empirical results for employment series from the U.S. Bureau of Labor Statistics (BL S), which help e xplain the behavior seen in Figure 1 . Section 3 goes into some details of calculations for the m odel-based method. Section 4 characterizes end-behavior for a typical example and a final section summarizes results.

2. Empirical End-behavior of Variance Measures for Employment Series

The results in Figure 1 come fro m three of abou t 145 i ndustry em ployment series appearing in BLS's monthly *Employment Situation* press release. A large-scale empirical test of the MB and MM methods has been carried out on monthly change for these series. The data come fro m BLS's Current Employment Statistics (CES) program , a monthly survey of over 300,000 establishments. In a ddition to its large size, the survey has the advantage of annual pop ulation figur es fro m an external source, the Unemployment Insurance (UI) program. Employers report m onthly employment on quarterly tax forms. With a 10-month lag, survey estimates are benchmarked to population UI values. If t is the current month, the employment estimate y_t comes from a "link-relative" estimator,

$$y_t = Y_0 \cdot r_1 \cdot r_2 \cdots r_t ,$$

where Y_0 is the latest avail able bench mark, subsequent subscripts denote num ber of months away from the benchmark, and

$$r_j = \sum_{i \in M_j} w_{ij} y_{ij} \Big/ \sum_{i \in M_j} w_{i,j-1} y_{i,j-1}$$

is the ratio of weighted em ployment in m onths j and $j-1$, with y_{ij} representing the number of em ployees in establish ment i in m onth j and M_j the set of establish ments reporting in both months j and $j-1$. M ost national employment series are seasonally adjusted multiplicatively, which fits wi th modeling the series on the log scale. Monthly change takes the simple form

$$\log(y_t) - \log(y_{t-1}) = \log(y_t / y_{t-1}) = \log(r_t).$$

Sampling error standard deviations and auto correlations are co mputed each month using the BRR method for various statistics, including these log ratios. Figure 2 is a scatterplot of a set of absolute log ratios (m ultiplied by 10^4) and the sam pling error standard deviations for the Durable Manufactu ring se ries (MD00). The two largest standard deviations oc cur for large log ra tios, but, overall, t here is ve ry little pattern. T his holds for most of t he series examined, which has le d us to assu me a co nstant variance for th e sampling error. Lag 1 SE autocorrelation es timates vary consi derably but t end to be negative. Au tocorrelations at other lags are also variable, with m edians close to 0. This leads us to adopt an MA(1) model for all series. We compute median lag 1 autocorrelation and roun d to the nearest . 05, except for roundi ng all magnitudes below .075 to 0 and allowing a maximum magnitude of .20.

Table 1b shows that property (1) holds for the MB method for Manufacturing, Durable Goods (MD00) and for Manufacturing, N ondurable Goods (MN00), but not for Petroleum & Coal Product s (MN10), in agreement with Fig ure 1. For the MM method, (1) holds onl y for MD00 and then onl y by 0.7%. Table 1a shows modeling information used for seas onal adjustment and calculation of the two variance measures. For MD00, estimates of the disturb ance variance fo r both t he signal model and the irregular component exceed the SE disturbance variance. Sam pling error is more pro minent for MN00, with the irregular co mponent estima ted to be quite small, and dominant for MN10, for which no irregular is identified. It a ppears that (1) is influenced by the amount of sampling error.

To obtain a measure of SE size, it is fr uitful to difference the observed series. Let $\delta(B)$ be the differencing operator in the ARIMA model for the signal. Then,

$$w_t = \delta(B) y_t = u_t + v_t, \ (2)$$

where $u_t = \psi(B) a_t, v_t = \delta(B) \varepsilon_t$, and $\psi(B)$ represents the ARMA model for the differenced signal. An im portant point in the derivations of Bell and Kramer (1999) is that error in forecasting the series is a f unction of error in forecasting the differenced series,

$$y_f - \hat{y}_f = C(w_f - \hat{w}_f),$$

for some matrix C. The differenced seri es is stationar y and we can justifiably consider variances. A natural measure of the SE contribution is the "differenced variance ratio,"

$$DVR = Var\big[\delta(B)\varepsilon_t\big] \big/ Var(w_t). \ (3)$$

Table 1. Results for MD00, MN00, and MN10
a. Modeling Information

Series	Sampling Error		Irregular	Signal	
	$\hat{\rho}_1$	$\hat{\sigma}_\varepsilon^2$	$\hat{\sigma}_I^2$	$\hat{\sigma}_a^2$	Model
MD00	-.10	71.5	126.1	294.7	(101)(011)
MN00	-.10	123.1	3.6	180.7	(101)(011)
MN10	0	2807.7	0	145.9	(101)(011)

b. Key SDA Statistics from the MB and MM Methods

Series	SDSE	SDA_{MB}	$SDA_{MB} - SDSE$ (%)	SDA_{MM}	$SDA_{MM} - SDSE$ (%)	End-Ctr (%) MB	MM
MD00							
Center	8.46	7.47	-11.6	8.53	+0.8	19.4	0.7
End	8.46	8.92	+5.5	8.58	+1.5		
MN00							
Center	11.09	9.80	-11.6	10.06	-9.3	6.7	-11.7
End	11.09	10.46	-5.8	8.89	-19.9		
MN10							
Center	52.99	46.94	-11.4	46.94	-11.4	-4.4	-5.6
End	52.99	44.86	-15.3	44.30	-16.4		

Table 2. Frequency Table for End Behavior, by Relative Size of Sampling Error

		SDA(end) > SDA(center)		SDA(end) > SDSE		SDA(end)
Total		MB	MM	MB	MM	MB>MM
DVR						
≤ 1/2	38 (30%)	38 (100%)	11 (29%)	12 (32%)	12 (32%)	26 (68%)
>1/2	90 (70%)	2(2%)	0 (0%)	0 (0%)	0 (0%)	80 (89%)
Total	128	40 (31%)	11 (9%)	12 (9%)	12 (9%)	106 (83%)

Table 2 shows results for property (1) for 128 series, overall and by DVR size category. Overall, only 31% of the series satisfy (1) with the MB method and a scant 9% with MM. However, with $DVR \leq 1/2$, the condition is satisfied for 100%   of series with MB an d 29% with MM. With $DVR > 1/2$, only two series satisfy the condition and only with the MB method; these series both have $DVR = .51$. Significantly, SDA_{MB} exceeds SDA_{MM} at the ends 68% of the time when DVR $\leq$1/2 and 83% overall. The exceptions are ca ses where a large irregular is estimated. The characteristics observed here are inherent in the measures, not the result of estimation. Previously conducted simulation experiments with the MM method confirm the kinds of sh apes noted here; the MB measure comes entirely from models and the X-11 filters.

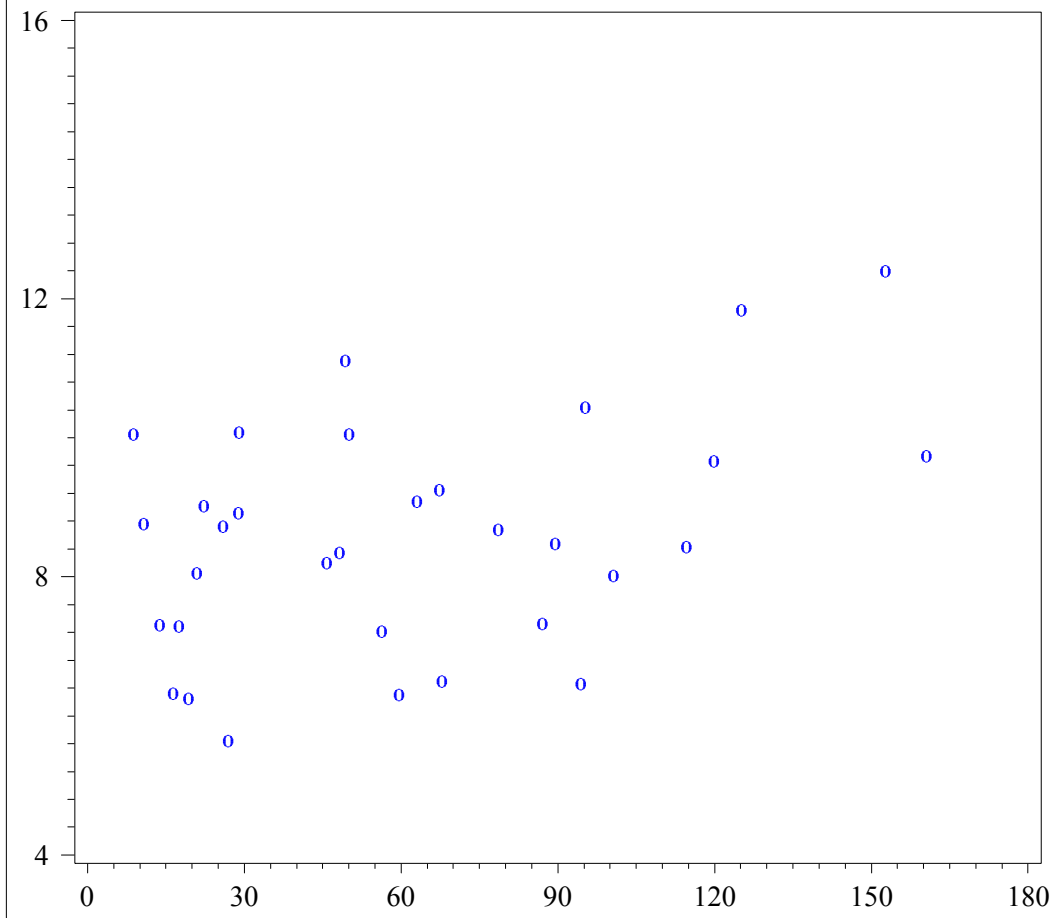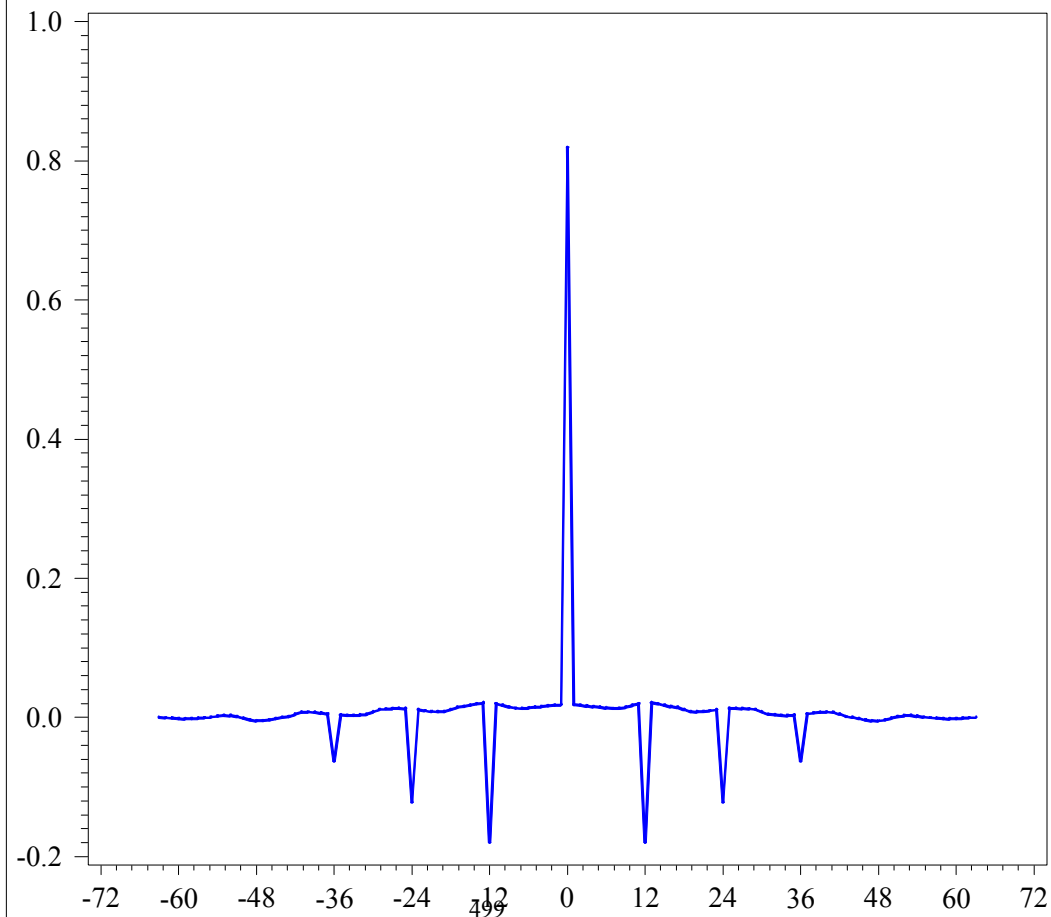Figure 2. Scatterplot of Observed Series vs. SE Standard Deviations, MD00

Figure 3. X-11 Seasonal Adjustment Filter, 13-point Henderson, 3x5 Seasonal

The main finding in Table 2 is disappointing. Property (1) fails most of the time. Yet the breakdown by DVR size suggests that when th e sampling error is not too large, one or both measures may indeed satisfy (1). Among the 1 28 press release series are 17 h ighly aggregated series, including 1   3 "supe rsectors" defined un der t he current i ndustry classification system and Total Private (Sect or) Employment, the most highly aggregated series for wh ich SE information is available.   Eleven of these ser ies satisfy property (1) with the MB method and 5 with MM.   Overall, the empirical results suggest that the MB method is likely to satisfy the intuitive property $SDA(end) > SDA(center)$, as long as the SE contribution is less than half.

3. Key Calculations for the Model-based (MB) Method

This section examines in some detail the matrix calculations for the MB method, in order to put the em pirical results of the previ ous section on a stronger fo oting. Foll owing the notation and formulas of Bell and Kram er (1999), the equation for the covari ance matrix of $-Y$ $\hat{y}$ is

$$Var(Y - \widehat{y}) = Var(\varepsilon) + Var\begin{bmatrix} b \\ 0 \\ f \end{bmatrix} - Cov(\begin{bmatrix} b \\ 0 \\ f \end{bmatrix}, \varepsilon) - Cov(\varepsilon, \begin{bmatrix} b \\ 0 \\ f \end{bmatrix})$$

where $b = y_b - \hat{y}_b$ and $f = y_f - \hat{y}_f$, so that   t he covarianc e matrix of the s easonal adjustment error $A * -\hat{A}$ is

$$VARA = Var[\Omega(Y - \widehat{y})] = \Omega Var(Y - \widehat{y})\Omega` = \Omega Var(\varepsilon)\Omega`+VFA + CVA .$$

In this last equation, VFA denotes the contribution of fo recast and backcast errors   and CVA the contribut ion of the covariance term   be tween t hese error s a nd sa mpling error. Let's focus on the diagonal elements. With a stationary model for the sampling error, the first term i s constant. Thus, m ovement in $SDA_{MB}^2(t) = VARA[t,t]$ acros s ti me co mes entirely from $NET = VFA + CVA$. If the obs erved series i s sufficiently long, the central seasonally adjusted values don' t depend on forecasts or backcasts, so the contribution of NET in the center is 0. In particular, this means

$$(\quad) \qquad SDA_{MB} \ end \ > SDA_{MB}(center) \Leftrightarrow NET(end) > 0 .$$

X-11 seasonal adjustment filter matrix Ω

Figure 3 graphs the weights of the X-11 seasonal adjustment filter using the 13-point Henderson trend filter and the "X-11 default" seasonal filter. The latter refers to use of the 3x3 and 3x5 filters in successive iterations of the basic X-11 calculations. The overall filter is symmetric and has length 169. The key weights are 0.82 at the center and -.18, -.12, and -.06 at distances 12, 24, and 36 from the center. For the example in the next section, we take *n*=169 and *m*=84. This means that (1) the central time point *t=85* doesn't depend on backcasts and forecasts and (2) forecasts do not depend on backcasts and vice versa. The matrix Ω is $(n \times \ n + 2m)$; it is convenient to break it down as

$$\Omega = [\Omega_1 \quad \Omega_2 \quad \Omega_3] ,$$

where Ω_1 and Ω_3 are $n \times m$ and Ω_2 is $n \times n$. It is useful to pin point where the key weights fall in these matrices. Each row contains a full set of filter weights. The last row of Ω is 0 thr ough col umn 168, has the left part of t he filter in c olumns 169- 252, the central weight .82 in col umn 253, and the right part in colu   mns 254-33 7. Table 3 contains the columns where the key weights fall for this last row in Ω, Ω_2, and Ω_3.

Table 3. Large Filter Weights: Column Locations in Row 169 (last row)

Wt.		-.06 -.12 -.18		.82		-.18 -.12 -.06	
	Dimension						
Ω	169x337	217 229	241	253	265	277	289
Ω_2	169x169	133 145	157	169	-	-	-
Ω_3	169x84	- - - - 12				24	36

Formulas for variance and covariance terms

We present som e key for mulas, which are i ngredients to unders tanding beha vior of the MB measure. A technical report (Scott, 2010) contains details.

Proposition 1 (Brockwell and Davis, 1991).
Asymptotically, the forecast error covariance matrix is given by
$$VF = Var(f) = \sigma^2 \Psi \Psi`,$$
where Ψ is lower triangular and, starting from the d iagonal, colu mns contain coefficients from $\psi(z) = \theta(z)/\phi(z)$.

Proposition 2.
(i) $f = y_f - \hat{y}_f = C(w_f - \hat{w}_f)$

(ii) $\hat{w}_f = \Sigma_{32}\Sigma_{22}^{-1}w_o$, where $\Sigma_{32}(= Cov \ w_f, w_o)$, $\Sigma_{33} = Var(w_o)$

(iii) $Cov(f,\varepsilon) = \sigma_\varepsilon^2 [0 \quad H \quad I_m]$

Remarks.
1. The matrix C is basically the inverse to a differencing matrix.
2. The m atrix H include s a contributi on $-CCov(\hat{w}_f, \varepsilon_o)$. From ii), H clearly depends on bot h the signal and the sam pling error, but we claim th at $Cov(f,\varepsilon)$ is primarily a function of the sampling error.

Proposition 3.
(i). The forecast error contribution to VARA is $VFA = \Omega_3 VF\Omega_3`$
(ii). The diagonal terms of VFA are $VFA[t,t] = VA1(t) + VA2(t)$, where

$$VA1(t) = \sum_{i=1}^{m}\Omega_3^2[t,i]VF[i,i], \quad VA2(t) = \sum_{i \neq k}\Omega_3[t,i]\Omega_3[t,k]VF[i,k]$$

(iii). A simple approximation at the last time point for the second term is
$$\sigma^{-2}VA2(169) \approx 2\Omega_3[169,12](\Omega_3[169,24] + \Omega_3[169,36])VF[12,24]$$
$$+2\Omega_3[169,24]\Omega_3[169,36]VF[24,36]$$

Proposition 4.
(i). The contribution of the covariance ter m i n the forecast period is
$$CVA = -(CVA1 + CVA2), \text{ with}$$

$$CVA1 = \Omega_3 H\Omega_2` + (\Omega_3 H\Omega_2`)`, \quad CVA2 = 2\sigma_\varepsilon^2\Omega_3\Omega_3` = 2\sigma_\varepsilon^2\sum_{h=1}^{m}\Omega_3^2[t,h]. \quad (4)$$
(ii). For $t > n/2$,

$$\Omega_3 H \Omega_2`[t,t] \approx \sum\nolimits_{j=1}^{3} \Omega_3[t,t-n+12j]\sum\nolimits_{k=0}^{3}\Omega_2[t,t-12k]H[t-n+12j,t-12k] \quad (5)$$

In this last form   ula,   we adopt the conventi   on $\Omega_3[t,t-n+12j]=0$ w henever $t-n+12j<1$. As t moves backward from 169, the large weights m ove left in the rows of Ω, which   means that f ewer ter ms are nonzero. For $t<134$, the approxim ation becomes 0.

4. Characterization of End-behavior – an Example

Using form ulas in Section 3,   our str ategy is t o approximate variances or standard deviations of the MB m easure in term s of separate contributions of signal and sampling error. We carry out the analysis for an

EXAMPLE. The observed series is $y = Y + \varepsilon$ and the models for the signal Y and the sampling error ε are

$$u_t = (1-B^{12})Y_t = (1-\theta_1 B)(1-\theta_{12}B^{12})a_t, \quad a_t \sim WN(0,\sigma_a^2)$$
$$\varepsilon_t \sim WN(0,\sigma_\varepsilon^2).$$

Parameter values are $\theta_1 = -.2, \theta_{12} = .5, \sigma_a^2 = 100, \sigma_\varepsilon^2 = 64$; in agreement wi th Section 3, the series length and forec ast period length are n=169 and m=84, respectively. The se models are close to those select ed for EH00, Education & Healt h Services, one of the supersectors treated in Section 2. The signal model is like an airline model, except that it lacks a first difference. Since the em ployment data in Section 2 are monthly change in logs, the above m odel fits when log employ ment follows an airline model This example does satisfy (1). For the last time point, our variance measure is

$$SDA_{MB}^2(169) = \text{SE contribution}(169) + VFA(169) + CVA(169)$$
$$= 50.232 + 18.635 - 17.482 = 51.386$$

and at the   end $NET = 18.635 - 17.481 = 1.153 > 0$ as might be expected, since $DVR = 128/258 = .496$. The SDA_{MB} values are 7.09, and 7. 17 for t he center an d end time points, both below the SE standard deviation $\sigma_\varepsilon = 8$.

We now proceed to seek an explanation more directly in terms of the signal and sampling error co mponents, as described in (2).   The differenced signal u and the diffe renced sampling error v have variances 130 and 128, respectively. Autocovariances for u, v and w appear in Table 4 and forecast error variances (or MSE's) in Table 5. As expected, the forecast error variance for w increases with forecast lead. The 'Su m' column in T able 5 provides an approxim ation for $Var(w_f)$ b y su mming values of the co mponents. Th e values increa se, like the e xact values i n th e last colum n, but miss a contribution from $Cov(\hat{w},\varepsilon)$ which is affected by the signal component. The approximation does have two advantages:
   (1) it is a simple way to separate component contributions,
   (2) it show s us that as   the forecast lead increas es the varianc e is increa singly
          dominated by the signal contribution.
We see that 60% of the total co mes from the signal at lead 1, increasing gradually to 80% at lead 84. We now compute NET at any time point past the center of the series as
   (   ) $NET = VFA\ signal + VFA(SE) - CVA$. (6)

Table 4. Autocovariances for w and Its Components					Table 5. Forecast Error MSE's for Selected Leads				
Lag	u	v	w		Lead	u	v	Sum	w
0	130	128	258		1	100	64	164	183
1	25	0	25		12	104	64	168	185
2	0	0	0		13	129	64	193	211
⋮					25	155	64	219	237
10	0	0	0		37	181	64	245	263
11	-10	0	-10		49	207	64	271	288
12	-52	-64	-116		61	233	64	297	314
13	-10	0	-10		73	259	64	323	340
14	0	0	0		84	260	64	324	341

Since (*VFA signal*) is th e dom inant forecast vari ance ter m a nd $Cov(f,\varepsilon)$ depends primarily on sampling error, it now beco mes apparent that shape can vary considerably according to the relative size of the components.

For the covariance ter m CVA, we apply Pr oposition 4. From (5), u sing the weights information from Table 3 and properties of H,

$$CVA1(169)/(2\sigma_\varepsilon^2) \approx$$
$$(-.18-.12-.06)(.82 \cdot H[12,169]-.18 \cdot H[12,157]-.12 \cdot H[12,145]-.06 \cdot H[12,133])$$
$$= -.36 \cdot (.82(-.372)-.18(-.232)-.12(-.146)-.06(-.092)) = .0869$$

Based on this approxim ation the total contribut ion of $CVA1$ to covariance is $-.174 \cdot \sigma_\varepsilon^2$, fairly close to the true value $-.159 \cdot \sigma_\varepsilon^2$. The terms 1-11 and 13-23 contribute $+.014 \cdot \sigma_\varepsilon^2$. With this correction, the approxim ation beco mes $-.160 \cdot \sigma_\varepsilon^2$. Also, $CVA2[169,169] = .114 \cdot \sigma_\varepsilon^2$. Thus, from Proposition 4 the total covariance contribut ion for 169 , even using the rough approximation, is

$$CVA(169) = -(2 \cdot CVA1 + CVA2) = -(.114+.174) \cdot \sigma_\varepsilon^2 = -.288\sigma_\varepsilon^2,$$

compared to the true value -.273 σ_ε^2. What is significant is that the weight $\Omega_2[169,169] = .82$, by far the la rgest magnitude weight in the filter (cf. Figure 3), oc curs in the $CVA1$ part of the calculati on and nowhere el se. In fact, it occurs in $CVA1(t)$ only for the last 12 points in the series. Since th e covariance ter m i s negative, this explain s why a dip is liable to occur over the last 12 points.

We can further pin down these findings. B y factoring out t he differenced variance fro m each term in (6),

$$NET = VFA(signal) + VFA(SE) - CVA = a\sigma_u^2 + b\sigma_v^2 - c\sigma_v^2,$$

for some coefficients a, b, and c. A little algebra y ields a criteri on for pr operty (1) in terms of DVR, as defined in (3):

$$NET > 0 \Leftrightarrow DVR = \sigma_v^2/\sigma_w^2 < a/(a+c-b) \qquad (7)$$

For our example, we have $a = .1107$, $b = .0332$, and $c = .1366$, so

$$SDA(end) > SDA(center) \Leftrightarrow DVR < .1107/(.1107+.1366-.0332) = .517.$$

This is satisf ied for our exa mple, since DVR=128/258=0.496. As a further check, we carry out the analysis with $\sigma_\varepsilon^2 = (8.5)^2 = 72.25$ and other p arameters un changed, so t hat DVR=.526. We find $NET = .029$, positive but very close to 0. We see the criterion doesn't quite work. There are approximations involved. We've adjusted the two forecast

variance ter ms to add to the exact va lue and th e last term is n ot strictly a f unction of sampling error. Also, a, b, and c change with the disturbance variances. Usi ng values $a = .1118$, $b = .0336$, and $c = .1340$ from the calculations with $\sigma_\varepsilon = 8.5$, the criterion becomes

$$NET(169) > 0 \Leftrightarrow DVR < .1118 / (.1118 + .1340 - .0336) = .527 ,$$

which appears very close to being co rrect. This gives us the sense that (7) works approximately (and maybe iteratively) in identifying when $SDA(end) > SDA(center)$.

Our machinery allows us to analy ze the dip at t=169-11=158 as well. At t=158, the signal represents 76% of the forecast variance. Exact calculations yield

$$NET(158) = 12.929 + 4.084 - 17.622 = -.609.$$

We also have the approximation

$$NET(158) = .0995\sigma_u^2 + .0319\sigma_v^2 - .1377\sigma_v^2 .$$

From this equation for NET, we find

$$NET(158) > 0 \Leftrightarrow DVR < .0995 / (.0995 + .1377 - .0319) = .485 .$$

Computing variances with $\sigma_\varepsilon^2 = 7.75^2 = 60.0625$ (and DVR=.480), we find

$$NET(158) = 12.856 + 3.811 - 16.710 = -.043 .$$

From calculations for this case, the refined criterion becomes

$$NET(158) > 0 \Leftrightarrow DVR < .0989 / (.0989 + .1391 - .0317) = .479 ,$$

which appears about right. Given the signal m odel, $SDA_{MB}(t)$ can be expe cted to stay above SDA_{MB} at the center as long as DVR is below .479, which corresponds closel y to $\sigma_\varepsilon^2 < 60$.

We have treated onl y a pa rticular model form and set of MA param eters. However, the example seems strong enough to make the following characterization of SDA for change, at least when the airline model fits the original series. In moving from the center of the series to the end, the forecast error varian ce makes an increasing positive contribution to SDA and its size depends increasingly o n th e signal near the end of the s eries. The covariance term in SDA_{MB}^2 is neg ative and depends mostly on the sam pling error. Both variance and covariance effects are strongest during the last 12 months. The covariance term CVA jumps sharply at n-11 and is relatively stable up to the end, while the variance term grows slowly but steadily right up to the last time point n. Th us, their net has a minimum at n-11 and tends to increase from there to n. Given signal and SE models, we can find a cutoff value for

(1) SDA_{MB} staying entirely above its central value or nearly so,

(2) $SDA_{MB}(end) > SDA_{MB}(center)$

in terms of the relative size of the SE. DVR, the relative contribution of sampling error to total variance on the differenced sc ale, is inform ative. $SDA_{MB}(end) > SDA_{MB}(center)$ corresponds fairly closely to DVR<0.5.

5. Summary

The model-based (MB) and m oments-matching (MM) m ethods bot h provide conceptually reasonable variance me asures for error in seasonal adjustment which account for sampling error. The main advantage of the MB method over the MM method is that it capt ures more ful ly the uncertainty at the end of the series, which is t he most important poi nt in time. Both methods tend to have unnatural d ips in t he first and last years of the series. These dips are partic ularly pr onounced whe n the sam pling error is dominant. The model-based (MB) method is likely to provide satisfactory measures of variance, including havin g end values great er than central values, when the relative contribution of the sampling error is less than half the variance of the differenced serie s, as measured by the "DVR" statistic, defined in Section 2. Both the large-scale empirical study and the theoretical findings for a typical example support this conclusion.

For the em ployment chan ge application presented in Section 2, we feel that we hav e established that the model-based (MB) method is tenable for use in assessing significance of monthly change, as long as the SE contribution is not too large. This is in fact the case for a majority of the highly aggregated series.

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References

Bell, William . R. and Matthew Kram er (1999). Toward variances for X-11 seasonal adjustments. *Survey Methodology* 25. 13-29.

Brockwell, Peter J. and Richard A. Dav is (1991). *Time Series: Theory and Methods*, 2d ed., Springer-Verlag, New York.

Pfeffermann, D. (1994). "A general method for estim ating the variances of X-11 seasonally adjusted estimators. *Journal of Time Series Analysis* 15: 85-116.

Scott, S., M. Sverchkov, and D. Pfefferm ann (2011). Measuring uncertaint y in X-11 seasonal adjust ment. In *Economic Time Series: Modeling and Seasonality*", Bell, William R., Scott H. Holan, and Tucker S. McElro y eds., Chapman & Hall/ CRC Press.

Wolter, K. M. and N. J. Monsour (1 981). On the problem of variance estim ation for a deseasonalized series. In *Current Topics in Survey Sampling*, D. Krewski, R. Platek, and J. N. K. Rao eds. Academic Press: New York. 367-403.