DIAGNOSTICS FOR EVALUATION OF SUPERPOPULATION MODELS FOR VARIANCE ESTIMATION UNDER SYSTEMATIC SAMPLING

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Key Words: Confidence interval coverage rates and widths; Design effects and misspecification effects; Finite population inference; Implicit stratification; Multiple random starts; Superpopulation model

1. Introduction

Systematic sampling frequently is used in practical applications because it may lead to estimators that are more efficient than those constructed under simple random sampling. One common form, known as single random start systematic sampling, the indices of the selected sample units are determined by a single random integer, to which one adds a set of predetermined integers, which usually are evenly spaced. Two fundamental problems with the single-random-start design are that it does not lead to the construction of standard design-based variance estimators (which in turn leads to the use of model-based variance estimators, as discussed in Cochran, 1977, Chapter 8 and Wolter, 1985, Chapter 7); and it cannot be readily adapted to accommodate uncertainty regarding constraints on data collection costs. Systematic sampling with multiple random starts can help one address each of these issues. For some general background on multiple-random-start systematic sampling, see, Iachan (1983), Zinger (1980) and references cited therein.

Of special interest is the fact that when one has a multiple-random-start systematic design, one may develop diagnostics in the following areas.

(a) Empirical comparison of model-based and design-based variance estimators.

(b) Examination of superpopulation model conditions (e.g., random order, correlated error, linear trend or implicit stratification conditions) generally used to develop model-based variance estimators.

(c) Evaluation of the relative efficiencies (as reflected in, e.g., mean confidence interval widths) of inference obtained, respectively, from design-based and model-based variance estimators, subject to satisfactory results from (a) and (b).

Technical issues associated with (a) through (c) requires detailed development beyond the space limitations of the current paper, and will be considered elsewhere. The remainder of this paper will present a detailed description of one specific application of multiple-random-start systematic sampling, and will highlight several features of the resulting data that may require the extension of standard superpopulation model-based approaches to systematic sampling.

2. The BLS Procurement Elapsed Time Study: Background and Goals

The BLS Procurement Office initiated a Procurement Elapsed Time Study to compare the times required for processing purchase orders (POs) in Fiscal Years (FYs) 1998 and 2000, respectively. Fiscal Year 1998 covered October 1, 1997 through September 30, 1998; and Fiscal Year 2000 covered October 1, 1999 through September 30, 2000.

The PO process is divided into the following five steps as defined by the Procurement Office. For the current study, the measurements of principal interest were the lengths of time required for each of these steps. Although the five steps were defined using slightly different terms in FY1998 and FY2000, the Procurement Office considers these steps to be equivalent.

Step A: Time elapsed from the date that the requisition (REQ) is initiated, to the date that REQ reaches the Branch of Printing, Procurement and Property Management (BPPPM).

Step B: Time elapsed from the date that REQ reaches the BPPPM to the date that REQ is approved.

Step C: Time elapsed from the date that the PO number is assigned, to the date that the PO is dispatched.
Step D: Time elapsed from the date the items are received to the date the items are delivered to the requesting office.

Step E: The total PO processing time. This includes the time used in Steps A, B, C and Step D, plus the time vendors used between the end of step C and the start of step D. The end of step B (date REQ is approved) was not necessarily simultaneous with the start of step C (date the PO number is assigned) in some cases, so there was a time gap between steps B and C.

3. Two Subpopulations: Goods and Services

The BLS Procurement Office was interested in evaluating elapsed times separately for purchase orders classified as Goods and Services, respectively. This classification is based on a specific accounting code known as the DOL object class; object classes that begin with the digits 26 or 31 are classified as goods, while object classes that begin with the digits 23 or 35 are classified as services. Items classified as Goods can be used as long as they are in good working condition, while Services are used for specific period. For example, pens, furniture or computers are classified as Goods, while rental items, telephone services and maintenance agreements are classified as Services.

Each purchase order in this study was classified uniquely as either a Good or a Service. However, we were not able to determine whether a given purchase order was a Good or Service before the sample was selected. Consequently, we were not able to use the Goods/Services classification as an initial stratification variable; and the subpopulation sizes for Goods and Services, respectively, are unknown.

Note that the Procurement Office tends to buy services like copier maintenance near the beginning of the fiscal year but does not report having 'received' the service until the end of the contract period, which is often close to the end of the fiscal year. Consequently, the times elapsed in steps D and E for Services is not directly relevant to assessment of the performance of the procurement process, and thus are excluded from the current study.

4. Sample Design

Development of the sample design was influenced primarily by two factors: the potential for temporal variation in purchase order patterns; and limitations on information regarding feasible sample sizes. First, there was a potential for concern that the distribution of elapsed times for one or more of steps A through E may vary over the fiscal year. For example, there were suggestions that at certain points in the budget cycle, the procurement office may receive more purchase orders, which in turn may lead to an increase in the time required to process the PO. In addition, there was also concern that the relative proportion of purchase orders for goods and services might vary with time. However, discussion with the procurement office did not produce strong prior indications of the specific months for which the purchase order submission rate or the goods-to-services ratio would be especially high. Consequently, it was not considered appropriate to stratify explicitly on specific months, for example. Instead, in keeping with the "implicit stratification" ideas reviewed in Section 1, we chose to use a systematic sample design in which the population units were ordered according to their purchase order numbers.

Second, the procurement office staff had relatively limited time available for data collection, and did not have firm prior information on the mean time per purchase order that would be required for data collection. Thus, it was not possible to determine a priori the total number of sample units for which we could collect the elapsed-time data. Consequently, it was important to implement the sample design in a form that would both: (i) allow a high degree of flexibility in the number of purchase orders for which we collected data; and (ii) maintain the statistical features generally associated with systematic sample designs, e.g., implicit stratification and approximately unbiased point estimation.

We were able to satisfy both of criteria (i) and (ii) through a systematic sample design with multiple random starts. Specifically, for each year (1998 and 2000) we selected 12 systematic samples of size 30 each through the following steps.

1) Number the populations of purchase orders for each year. Population sizes for FY1998 and FY2000 are 550 and 1307 respectively.

2) For FY 1998, select 12 numbers u_i through simple random sampling without replacement from the set of integers from 1 to 19. For FY 2000, select 12 numbers v_i through simple random sampling without replacement from the integers from 1 to 44.

3) Then the indices of units in the i th systematic sample selected for FY1998 are: $u_i + 19(j-1)$ for $j = 1,...,30$. Similarly, the indices of units in the i th systematic sample for FY2000 are: $v_i + 44(j-1)$ for $j = 1,...,30$. We omit $j = 30$ if $u_i + 19(j-1) \geq 550$ for FY1998; or if $v_i + 44(j-1) \geq 1307$ for FY2000.

The BLS Procurement Office was asked to collect data sequentially for the i -th systematic samples for 1998 and 2000, $i = 1,...,12$ as time permitted. Ultimately, data were collected for the first eight systematic samples. Under mild regularity conditions, this stopping rule allowed the construction of approximately unbiased point estimators based on the collected systematic sample data. Note that we chose "12" as a convenient maximum number of sample clusters. The BLS procurement Office was certain that it would not have time to collect data for more than 12 sample clusters per year, but was not certain about the exact number of sample clusters for which it could complete data collection. We could have produced a random permutation of $\{1,...,44\}$, and used the stopping rule at a complete cluster when a data collector ran out of the time.

5. Data Collection Costs

As noted in Section 2.3, we did not have any prior information regarding the time required for collection of data for each selected purchase order. Record keeping during the current study led to the cost results reported in Table 1. Note that for locating, organizing, and numbering the original files, the time cost per PO in the population was (4.50 hours)/(550 POs)=0.0082 hours/PO for FY 1998, and (20 hours)/(1307 POs) =0.0153 hours/PO for FY 2000. The mean time per PO for FY 2000 was higher than for FY 1998 because during the data collection period, some relevant paper records for FY 2000 were also being used for other purposes, and thus took longer to locate.

The overall mean of data collection and data entry cost per PO for the sample was (20.5 hours)/(232 POs)=0.0884 hours/PO for FY 1998, and (27.75 hours)/(236 POs) =0.118 hours/PO for FY 2000. The mean time for data collection and data entry per PO was longer for FY 2000 than for FY 1998 because for FY 2000 the relevant electronic files were organized in a way that made the required data less accessible.

Table 1: Data Collection Cost

	FY 1998	FY 2000
Pop size	550	1307
Sample size	232	236
Locate, org, number files	4.50hours	20.00 hours
Sample data collection	12.50 hours	19.75 hours
Sample data entry	8.00 hours	8.00 hours

6. Elapsed Time Data

Table 2 displays the elapsed time data applicable to steps A , B and C in FY2000. The first through eighth columns, respectively, report: CL, the systematic sample (cluster) number (1 through 8); PO, the purchase order number (as assigned by the BLS Procurement Office); TP, type of purchase (G or S, for Goods or Services, respectively); DR, the date the requisition (REQ) was initiated; DB, the date the REQ reached the BPPPM; DA, the date REQ was approved; AS, the date the PO number was assigned; and DD, the date the PO was dispatched. For some POs, some of the abovementioned dates were not recorded, and are marked with an asterisk (*) in Table 2 below. Discussions with the BLS Purchasing Office did not identify any specific reasons for this missingness.

One may calculate the elapsed times for steps A, B and C directly from the columns of this table. The elapsed time for step A equals (DB– DR +1); the elapsed time for step B equals (DA– DB + 1); and the elapsed time for step C equals (DD – AS +1). Note that under the BLS procurement office rules, holidays and weekend days are included in the calculation of elapsed time. For example, if a requisition is initiated on Friday and reaches the BPPPM on the following

Monday, then the elapsed time for step A is recorded as four days. The dates October 1, 1999 through September 30, 2000 were labeled 1 through 366, respectively. Thus, November 1, 1999 is day 32, December 1, 1999 is day 62, and so on.

Graphical and analytic exploration (which will be detailed in other publications) of the purchase order data identified five features of the data that may be of special relevance in development and evaluation of appropriate variance estimators. First, the purchase orders were not distributed uniformly over time; as the year progressed, the temporal density of the purchase orders generally increased, at least until the final month. Consequently, in discussion of a linear trend model, there are nontrivial distinctions between a trend that is linear in time and a trend that is linear in the purchase order number. This in turn has implications for lack of fit of a proposed linear trend model, and for the properties of the associated error terms. Second, in a small number of cases, systematic sample sort order induced by the purchase order numbers is not identical to the temporal order in which purchase order numbers were assigned (as indicated by the recorded date AS). Anecdotal evidence suggests that this type of imperfection is often encountered in the selection of systematic samples from administrative records. Third, Step A for Goods, Step D for Goods, and Step E for Goods displayed some evidence of a linear trend. Fourth, Step D for Goods, and Step E for Goods displayed nontrivial patterns of heteroscedasticity. Finally, Step B and Step C for Goods each displayed one fairly prominent outlier out of 155 sample units respectively. Step C for Services also displayed one fairly prominent outlier out of 81 sample units, and Step E for Goods displayed five fairly prominent outliers out of 124 sample units. For such cases, customary assumptions of normal error terms may be problematic, which in turn may have implications for assessment of the stability of the associated variance estimators.

Acknowledgements

The authors thank Lesley Ward for providing the data reported in Tables 1 and 2, and for comments on the data collection process for the BLS Procurement Elapsed Time Study. The views expressed in this paper are those of the authors and do not necessarily reflect the policies of the U.S. Bureau of Labor Statistics.

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Table 2: Data for Steps A, B and C in FY 2000

CL	PO	TP	DR	DB	DA	AS	DD
1	2019	S	8	13	126	126	148
1	2066	S	34	54	54	54	64
1	2119	S	69	82	83	83	84
1	2163	G	90	90	97	97	109
1	2208	G	33	36	116	116	116
1	2260	S	116	119	127	127	137
1	2304	G	111	127	139	139	140
1	2354	G	97	105	116	116	151
1	2399	G	159	166	168	168	170
1	2443	G	175	175	186	186	194
1	2489	S	200	200	200	204	204
1	2535	G	160	166	214	214	228
1	2579	G	221	222	229	229	231
1	2624	G	238	238	243	243	248
1	2669	S	245	246	257	257	261
1	2718	G	190	194	267	267	299
1	2763	S	*	*	*	275	278
1	2809	G	288	291	291	291	296
1	2858	S	266	287	299	299	299
1	2902	G	298	302	302	302	307
1	2947	S	309	309	309	309	312
1	2994	G	306	307	313	313	321
1	3038	G	308	308	320	320	321
1	3085	G	309	312	323	323	329
1	3134	G	309	327	327	327	352
1	3180	G	315	315	331	331	335
1	3228	S	309	309	337	337	338
1	3276	G	302	302	344	345	350
1	3322	G	287	287	354	354	356

1	3367	G	362	362	363	363	364
2	2034	S	1	20	20	22	22
2	2083	G	57	57	61	62	64
2	2134	G	81	91	98	98	284
2	2179	S	98	99	105	105	109
2	2224	S	91	96	120	120	125
2	2275	S	*	*	*	137	138
2	2319	G	125	131	145	145	151
2	2369	S	153	153	159	159	162
2	2414	S	172	172	172	172	174
2	2459	S	168	175	190	190	191
2	2504	G	197	197	197	202	210
2	2550	G	217	217	219	219	228
2	2594	G	188	188	231	231	263
2	2639	S	243	244	251	251	293
2	2684	S	256	256	260	260	264
2	2733	G	270	271	273	295	299
2	2780	G	249	263	281	281	293
2	2825	S	292	292	293	293	312
2	2873	G	181	181	301	301	366
2	2917	G	302	302	307	307	321
2	2962	S	309	309	313	313	320
2	3009	G	307	307	317	317	322
2	3055	G	308	308	309	310	435
2	3100	G	308	308	324	324	337
2	3149	G	327	328	328	328	336
2	3197	G	295	300	309	329	335
2	3246	G	323	323	342	342	351
2	3292	G	307	308	348	348	354
2	3338	S	307	309	358	358	365
3	2005	S	12	13	13	13	13
3	2051	S	26	27	48	48	48
3	2100	S	53	53	69	69	69
3	2149	S	1	1	102	102	191
3	2194	S	97	106	110	110	112
3	2242	S	30	35	127	127	133
3	2290	G	127	131	138	138	140
3	2339	G	130	139	146	151	151
3	2384	S	145	146	166	166	186
3	2429	S	124	148	180	180	180
3	2474	G	188	189	191	191	194
3	2519	G	162	175	207	207	211
3	2565	G	222	222	223	223	261
3	2610	S	210	210	239	239	242
3	2655	S	237	237	253	253	258
3	2700	G	221	238	263	263	264
3	2749	S	263	263	273	273	278
3	2795	G	279	281	285	285	286
3	2843	G	293	295	295	295	349
3	2888	S	292	295	302	302	307
3	2933	G	299	309	309	309	312
3	2979	S	305	306	314	314	356
3	3024	S	298	308	319	319	365
3	3071	G	273	277	323	323	336
3	3119	G	309	312	327	327	365
3	3165	S	309	315	327	329	329
3	3213	G	309	309	335	335	348
3	3262	G	302	312	319	319	351
3	3307	G	347	348	348	350	358
3	3353	G	359	359	359	359	364
4	2031	S	13	20	20	22	22
4	2080	G	34	54	62	62	62
4	2131	G	78	82	98	98	98
4	2176	G	99	102	104	104	104
4	2221	S	*	*	*	119	119
4	2316	G	116	118	132	132	141
4	2366	G	139	140	159	159	189
4	2411	S	167	172	172	172	174
4	2456	S	180	183	190	190	194
4	2501	S	*	*	*	202	209
4	2547	G	215	216	218	218	229
4	2591	S	208	208	230	230	231
4	2636	G	236	236	251	251	254
4	2681	G	249	249	259	259	261
4	2730	S	238	270	270	279	284
4	2777	G	270	271	274	277	288
4	2822	S	237	243	293	293	295
4	2870	G	295	298	300	300	302
4	2914	G	300	300	305	305	307
4	2959	G	308	309	312	312	320
4	3006	G	307	309	317	317	320
4	3052	G	312	312	322	322	329
4	3097	G	308	308	324	324	333
4	3146	G	327	328	328	328	336
4	3194	G	307	308	334	334	364
4	3243	G	272	272	342	342	354
4	3289	G	312	315	341	341	354
4	3335	S	298	299	357	357	364
5	2026	S	7	12	19	19	19
5	2075	S	43	43	61	61	106
5	2126	S	81	82	89	89	89
5	2171	S	41	41	103	103	104
5	2216	G	119	119	119	119	131
5	2267	G	127	127	132	132	137
5	2311	G	127	130	138	140	141
5	2361	G	124	124	125	153	160
5	2406	G	161	166	168	168	170
5	2451	G	175	180	189	189	204
5	2496	G	186	187	201	201	204
5	2542	G	182	182	216	216	231
5	2586	G	218	222	229	229	237
5	2631	S	*	*	*	250	*
5	2676	G	256	257	258	258	261
5	2725	G	266	266	270	270	284

5	2770	G	271	271	272	272	284
5	2817	G	292	292	292	292	295
5	2865	G	291	291	298	300	301
5	2909	G	298	300	303	303	305
5	2954	G	302	306	306	306	312
5	3001	G	294	294	316	316	319
5	3047	G	285	287	320	320	322
5	3092	S	288	288	323	323	366
5	3141	G	302	302	328	328	352
5	3188	G	112	116	333	333	361
5	3237	G	312	315	341	341	348
5	3283	G	307	308	347	347	351
5	3330	S	270	327	345	356	361
5	3374	G	327	328	328	364	364
6	2029	S	13	20	20	20	20
6	2078	G	49	57	61	61	64
6	2129	G	33	33	97	97	104
6	2174	G	36	36	104	104	109
6	2219	G	119	119	119	119	126
6	2270	G	127	127	132	132	137
6	2314	S	103	111	141	141	141
6	2364	G	159	159	159	159	160
6	2409	G	166	166	170	170	180
6	2499	G	174	175	201	201	204
6	2545	G	106	141	191	191	225
6	2589	G	146	173	230	230	237
6	2634	G	214	214	237	250	254
6	2679	G	251	251	257	257	260
6	2728	S	267	267	270	270	278
6	2775	G	260	263	281	281	291
6	2820	G	292	292	293	293	299
6	2868	S	299	299	300	300	301
6	2912	G	299	301	303	303	305
6	2957	S	175	225	312	312	364
6	3004	G	306	306	317	317	320
6	3050	G	308	308	313	321	322
6	3095	G	260	260	323	323	350
6	3144	G	309	309	327	328	329
6	3192	G	223	223	334	334	342
6	3241	S	307	308	341	341	351
6	3286	G	307	307	347	347	351
6	3333	G	312	312	356	356	361
6	3378	S	309	309	327	365	366
7	2020	S	18	18	49	49	50
7	2067	G	47	50	54	54	64
7	2120	S	76	82	83	83	84
7	2165	S	18	18	103	103	112
7	2209	G	33	36	116	116	116
7	2261	S	98	106	127	127	127
7	2305	G	111	127	139	139	140
7	2355	G	99	104	148	148	153
7	2400	S	158	158	166	168	168
7	2444	G	173	175	187	187	194
7	2490	G	169	169	200	200	229
7	2536	G	189	195	210	215	235
7	2580	G	221	221	229	229	235
7	2625	S	168	193	210	245	*
7	2670	G	251	251	257	257	261
7	2719	G	179	181	210	231	288
7	2764	S	*	*	*	275	298
7	2810	S	273	284	284	284	291
7	2859	G	295	298	299	299	300
7	2903	G	286	292	302	302	342
7	2948	G	309	309	310	310	320
7	2995	G	307	307	309	313	322
7	3039	G	302	302	320	320	320
7	3086	G	309	309	323	323	329
7	3135	G	312	319	327	327	329
7	3181	G	301	321	331	331	358
7	3229	S	314	314	337	337	351
7	3277	G	308	308	344	347	365
7	3323	G	328	337	354	354	358
7	3368	G	362	363	363	363	365
8	2028	G	22	22	22	22	22
8	2077	G	43	43	61	61	64
8	2128	S	88	88	95	95	95
8	2173	G	103	103	103	103	126
8	2218	S	119	119	119	119	125
8	2269	G	127	130	132	132	137
8	2313	S	139	140	141	141	141
8	2363	S	154	154	154	154	155
8	2408	S	146	159	168	168	186
8	2453	G	168	169	189	189	237
8	2498	G	186	187	201	201	211
8	2544	S	211	211	218	218	229
8	2588	S	208	208	230	230	231
8	2633	S	243	250	250	250	254
8	2678	G	253	256	258	258	261
8	2727	G	267	267	270	270	287
8	2773	G	260	263	274	274	286
8	2819	G	284	285	292	292	298
8	2867	G	298	298	298	300	302
8	2911	G	299	299	303	303	307
8	2956	G	302	309	310	310	320
8	3003	G	306	309	317	317	322
8	3049	G	306	306	314	314	336
8	3094	S	312	312	323	323	331
8	3143	G	307	307	327	328	329
8	3191	S	225	295	334	334	365
8	3240	S	273	273	341	341	349
8	3285	G	302	307	347	347	351
8	3332	G	293	293	356	356	364
8	3376	S	365	365	365	365	366